SCIREA Journal of Physics
ISSN: 2706-8862
http://www.scirea.org/journal/Physics
October 23, 2022
Volume 7, Issue 5, October 2022
https://doi.org/10.54647/physics14491

29.58% (31.26)% -Limiting Highest Efficiencies obtained in the $n^+(p^+) - p(n)$ Crystalline InP Junction Solar Cells at T=300 K, Due to The Effects of Heavy (Low) Doping and Impurity Size

H. Van Cong, K. C. Ho-Huynh Thi*, C. T. Huynh-Pivet*, A. Pivet*, C. V. Huynh*, A. L. Pivet*, and I. Pivet*

Université de Perpignan Via Domitia, Laboratoire de Mathématiques et Physique (LAMPS), EA 4217, Département de Physique, 52, Avenue Paul Alduy, F-66 860 Perpignan, France.

van-cong.huynh@univ-perp.fr ; huynhvc@outlook.fr

* Groupe de Physique Théorique, 20 Rue du Col de LLi, F-66100 Perpignan, France.

Abstract

In the $n^+(p^+) - p(n)$ crystalline InP-junction solar cells, by basing on a same treatment method, and for a same heavy (low) doping effect, as those investigate in our recent papers [1, 2], but using now a new expression, obtained for the static relative dielectric constant $\varepsilon(r_{d(a)})$, determined exactly in the effective Bohr model, as that given in Eq. (1c), representing the donor (acceptor) d(a)-radius $r_{d(a)}$ – effect, or the $\varepsilon(r_{d(a)})$ – effect, suggesting further that, for an increasing $r_{d(a)}$, $\varepsilon(r_{d(a)})$ decreases, as showed in Table 1, according to the increase in photovoltaic efficiency η, as that observed in Tables 2 and 3, we finally get, in our present paper, for highest values of $r_{d(a)}$, the limiting highest efficiency results of such $n^+(p^+) - p(n)$ crystalline InP-junction solar cells, **η=29.58% (31.26%)**, respectively.

Furthermore, one notes that our present value: **η=31.26%** can also be compared with the corresponding one, **η=30.6%**, investigated by Raj et al. [4], using a p-i-ZnO sample.

Keywords: donor (acceptor)-size effect; heavily doped emitter region; photovoltaic conversion factor; open circuit voltage; photovoltaic conversion efficiency

1. Introduction

In our recent papers (RP) [1, 2], by basing on: (**i**) the heavy doping and impurity size effects, which affect the total carrier-minority saturation current density $J_{oI(II)} \equiv J_{En(p)o} + J_{Bp(n)o}$, $J_{En(p)o}$ and $J_{Bp(n)o}$ being the carrier-minority saturation current densities, injected respectively into the heavily doped donor (acceptor) InP emitter region-and-lightly doped acceptor (donor) InP base region, HD[d(a)- InP]ER-LD[a(d)- InP]BR, of $n^+(p^+) - p(n)$ junction solar cells, **(ii)** an effective Gaussian donor (acceptor)-density profile $\rho_{d(a)}$ to determine $J_{En(p)o}$[1, 2, 13, 18-20, 22], and **(iii)** the use of two fixed experimental points, we investigated the photovoltaic conversion factor $n_{I(II)}$, the short circuit current density $J_{scI(II)}$, the fill factor $F_{I(II)}$, and finally the efficiency $\eta_{I(II)}$ [1- 45]. These physical quantities were expressed as functions of the open circuit voltage V_{oc}, and of various parameters such as: the emitter thickness W, high donor (acceptor) density $N_{d(a)}$, surface recombination velocity S, given in the HD[d(a)- InP]ER, and low acceptor (donor) density $N_{a(d)}$, given in the LD[a(d)- InP]BR.

Then, in our present paper, by basing on a same treatment method, and for a same heavy (low) doping effect, as those given in RP, but using now a new expression, obtained for the relative dielectric constant $\varepsilon(r_{d(a)})$, being determined exactly in the effective Bohr model, as that given in Eq. (1c), which represents the donor (acceptor) d(a)-radius $r_{d(a)}$ −effect or the $\varepsilon(r_{d(a)})$ − effect, suggesting further that, for an increasing $r_{d(a)}$, $\varepsilon(r_{d(a)})$ decreases, as showed in Table 1, according to the increase in photovoltaic efficiency η, as observed in Tables 2 and 3, we finally get, in our present paper, for highest values of $r_{d(a)}$, the limiting highest efficiency results of such $n^+(p^+) - p(n)$ crystalline InP-junction solar cells, **η=29.58% (31.26%)**, respectively.

Furthermore, one notes here that our present value: **η=31.26%** can also be compared with the corresponding one, **η=30.6%**, investigated by Raj et al. [4], using a p-i-ZnO sample.

In Section 2, all the results energy-band-structure parameters for d(a)- InP systems are reported in Table 1, and the expressions for $J_{En(p)o}$ are also reported, so that we can determine the total (or dark) carrier-minority saturation current density $J_{oI(II)} \equiv J_{En(p)o} + J_{Bp(n)o}$, $J_{Bp(n)o}$ being determined in Eq. (C1) of the Appendix C. In Section 3, the photovoltaic effect is investigated. Finally, some numerical results and concluding remarks are given and discussed in Section 4.

2. Energy-Band-Structure Parameters and dark minority-carrier saturation current density, due to impurity-size and heavy doping effects

Now, we present the effects of donor (acceptor) [d(a)]-size and heavy doping, taken on the energy-band-structure parameters, and investigate the minority-carrier saturation current densities, as follows.

2.1. Effect of d(a)-size

In d(a)- InP systems at T=0 K, since the d(a)-radius $r_{d(a)}$, in tetrahedral covalent bonds is usually either larger or smaller than the P(In) atom-radii $r_{P(In)}$, a local mechanical strain (or deformation potential energy) is induced, according to a compression (dilation) for $r_{d(a)} > r_{P(In)}$ ($r_{d(a)} < r_{P(In)} = r_{do(ao)} = 0.110\ (0.144)$ nm), respectively, due to the d(a)-size effect [42]. Then, we have shown [8] that this $r_{d(a)}$-effect affects the changes in all the energy-band-structure parameters, expressed in terms of the relative dielectric constant $\varepsilon(r_{d(a)})$, as given in the following.

First, we note that in the InP [8] the static relative dielectric constant of the intrinsic silicon is equal to: $\varepsilon(r_{P(In)}) = 12.37$, the relative effective electron (hole) mass in conduction (valence) bands yield: $(m_c/m_o) = 0.015$ and $(m_v/m_o) = \frac{0.021+0.39}{2} = 0.2055$, the unperturbed intrinsic band gap at 0K, $E_{go}(r_{do(ao)}) = 1.42$ eV, the effective donor (acceptor)-ionization energy in absolute values: $E_{do}(r_{do}) = \frac{13600\times(m_c/m_o)}{(\varepsilon(r_{do}))^2}$ meV $= 1.333$ meV, and $E_{ao}(r_{ao}) = \frac{13600\times(m_v/m_o)}{(\varepsilon(r_{ao}))^2}$ meV $= 18.26$ meV, and the isothermal bulk modulus are defined, for the n(p)-type InP, by: $B_n \equiv \frac{E_{do}}{(4\pi/3)\times(r_P)^3} = 3.831 \times 10^7$ (N/m^2), and finally, $B_p \equiv \frac{E_{ao}}{(4\pi/3)\times(r_{In})^3} = 2.339 \times 10^8$ (N/m^2).

Therefore, at $r_{d(a)} = r_{do(ao)}$, the boundary conditions are found to be, for the impurity-atom volume V, $V_{do(ao)} = (4\pi/3) \times (r_{do(ro)})^3$, the pressure p, $p_o = 0$, and the deformation potential energy or the strain energy σ, $\sigma_o = 0$. Further, the two important equations [42], needed to

determine the σ-variation $\Delta\sigma \equiv \sigma - \sigma_o = \sigma$, are defined by: $\frac{dp}{dV} = -\frac{B}{V}$ and $p = -\frac{d\sigma}{dV}$. giving: $\frac{d}{dV}\left(\frac{d\sigma}{dV}\right) = \frac{B}{V}$. Then, in the n(p)-type InP, by an integration, one gets:

$$(\Delta\sigma)_{n(p)} = B_{n(p)} \times (V - V_{do(ao)}) \times \ln\left(\frac{V}{V_{do(ao)}}\right) = E_{do(ao)} \times \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 \geq 0. \quad (1a)$$

Furthermore, we also shown [42] that, as $r_{d(a)} > r_{do(ao)}$ ($r_{d(a)} < r_{do(ao)}$), the compression (dilatation) corresponding the repulsive (attractive) force increases (decreases) the energy gap $E_{gn(gp)}(r_{d(a)})$ and the effective donor(acceptor)-ionization energy $E_{d(a)}(r_{d(a)})$ in absolute values, obtained in the effective Bohr model, which is represented by: $+(-)\ (\Delta\sigma)_{n(p)}$, respectively. That gives:

$$E_{gn(gp)} - E_{go} = E_{d(a)} - E_{do(ao)} = E_{do(ao)} \times \left[\left(\frac{\varepsilon(r_{do(ao)})}{\varepsilon(r_{d(a)})}\right)^2 - 1\right] = +\ (\Delta\sigma)_{n(p)}, \text{ for } r_{d(a)} \geq r_{do(ao)}, \text{ and}$$

$$E_{gn(gp)} - E_{go} = E_{d(a)} - E_{do(ao)} = E_{do(ao)} \times \left[\left(\frac{\varepsilon(r_{do(ao)})}{\varepsilon(r_{d(a)})}\right)^2 - 1\right] = -\ (\Delta\sigma)_{n(p)}, \text{ for } r_{d(a)} \leq r_{do(ao)}. \quad (1b)$$

Then, from Equations (1a, 1b), the exact expression of static relative dielectric constant $\varepsilon(r_{d(a)})$, is given by:

$$\varepsilon(r_{d(a)}) = \frac{\varepsilon(r_{do(ao)})}{\sqrt{1 + \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3}} \leq \varepsilon(r_{do(ao)}), \text{ for } r_{d(a)} \geq r_{do(ao)}, \text{ and}$$

$$\varepsilon(r_{d(a)}) = \frac{\varepsilon(r_{do(ao)})}{\sqrt{1 - \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3}} \geq \varepsilon(r_{do(ao)}), \text{ for } r_{d(a)} \leq r_{do(ao)}, \quad (1c)$$

which can be compared with its approximated and simple form, proposed in our recent papers (RP) [1, 2], by:

$$\varepsilon_{RP}(r_{d(a)}) \simeq \varepsilon(r_{do(ao)}) \times \left(\frac{r_{do(ao)}}{r_{d(a)}}\right)^{4.377\ (4.7)}, \quad \varepsilon(r_{do(ao)} = 12.37. \quad (1d)$$

For example, in the n-type InP and for $r_{do(P)} = 0.110$ nm and $r_{d(Sn)} = 0.140$ nm, we obtain respectively from Equations (1c, 1d), $\varepsilon(r_{Sn})$=9.30 $\gg \varepsilon_{RP}(r_{Sn}) = 4.30$, meaning that $\varepsilon_{RP}(r_{d(a)})$, used in our RP, are found to be inaccurate.

Finally, it should be remarked that, with increasing $r_{d(a)}$, $\varepsilon(r_{d(a)})$, determined in Eq. (1c), decreases, according to the increase in $E_{go}(r_{d(a)})$ and $E_{do(ao)}(r_d)$, determined in Eq. (1b), as those observed in the following Table 1.

Table 1. Impurity size effects, taken on the effective dielectric constant $\varepsilon(r_{d(a)})$, being determined in Eq. (1c), the effective donor(acceptor)-ionization energy, $E_{d(a)}(r_{d(a)})$, in absolute values, and the band gap, $E_{gn(p)}(r_{d(a)})$, at T=0K, determined both in Eq. (1b), and finally, the intrinsic band gap, $E_{gin(p)}(T = 300K, r_{d(a)})$ and the intrinsic carrier concentration $n_{in(p)}(T = 300K, r_{d(a)})$, determined respectively in Equations (A4) and (A5) of the Appendix A

Donor	$d_o \equiv P$	As	Te	Sb	Sn
r_d (nm) [8] (↗)	0.110	0.118	0.132	0.136	0.140
$\varepsilon(r_d)$ (↘)	**12.37** [8]	12.07	10.46	9.88	9.30
$E_d(r_d)$ in meV (↗)	1.33	1.40	1.86	2.09	2.36
$E_{gn}(r_d)$ in meV (↗)	1420 [8]	1420.1	1420.53	1420.75	1421.02
$E_{gin}(300K, r_d)$ in meV (↗)	1323.48	1323.55	1324.01	1324.24	1324.51
$n_{in}(300K, r_d)$ (↘) in 10^6 cm^{-3}	2.51	2.51	2.49	2.48	2.47

Acceptor	$a_o \equiv In$	Mg	Al	Ga
r_a (nm) [8] (↘)	0.144	0.140	0.126	0.126
$\varepsilon(r_a)$ (↗)	**12.37** [8]	12.41	13.28	13.28
$E_a(r_a)$ in meV (↘)	18.26	18.14	15.85	15.85
$E_{gp}(r_a)$ in meV (↘)	1420 [8]	1419.9	1417.6	1417.6
$E_{gip}(300K, r_a)$ in meV (↘)	1323.5	1323.4	1321.1	1321.1
$n_{ip}(300K, r_a)$ (↗) in 10^6 cm^{-3}	2.51	2.52	2.64	2.64

In summary, those effects of $N_{d(a)}$-heavy doping and $r_{d(a)}$- impurity size [or the $\varepsilon(r_{d(a)})$ −effect], given in the HD[d(a)-InP]ER, and those of $N_{a(d)}$-low doping in the LD[a(d)- InP]BR, affect all the minority-carrier transport properties, as those given in the Appendix A, B and C, and also in the following equations.

2.2. Total minority-carrier saturation current density at 300K

The total carrier-minority saturation current density is defined by:

$$J_{oI(II)} \equiv J_{En(p)o} + J_{Bp(n)o}, \tag{2}$$

where $J_{Bp(n)o}$ is the minority-electron (hole) saturation current density injected into the LD[a(d)-InP]BR, being determined in Eq. (C1) of the Appendix C, and $J_{En(p)o}$ is the minority-hole saturation-current density injected into the HD[d(a)- InP]ER.

In the non-uniformly and heavily doped emitter region of d(a)- InP devices, the effective Gaussian d(a)-density profile or the d(a) (majority-e(h)) density, is defined in the HD[d(a)- InP]ER-width W:

$$\rho_{d(a)}(x) = N_{d(a)} \times \exp\left\{-\left(\frac{x}{W}\right)^2 \times \ln\left[\frac{N_{d(a)}}{N_{d(a)o}(W)}\right]\right\} \equiv N_{d(a)} \times \left[\frac{N_{d(a)}}{N_{d(a)o}(W)}\right]^{-\left(\frac{x}{W}\right)^2}, \quad 0 \le x \le W,$$

$$N_{d(a)o}(W) \equiv 7.9 \times 10^{17}\ (2 \times 10^5) \times \exp\left\{-\left(\frac{W}{184.2\ (1)10^{-7}\ cm}\right)^{1.066\ (0.5)}\right\}\ (cm^{-3}), \tag{3}$$

where $\rho_{d(a)}(x = 0) = N_{d(a)}$ is the surface d(a)-density, and at the emitter-base junction, $\rho_{d(a)}(x = W) = N_{d(a)o}(W)$, decreasing with increasing W [1, 2, 13]. Further, the "effective doping density" is defined by:

$$N_{d(a)eff.}(x, r_{d(a)}) \equiv \rho_{d(a)}(x)/\exp\left[\frac{\Delta E_{ga\ n(p)}(\rho_{d(a)}(x), r_{d(a)})}{k_B T}\right],$$

$$N_{d(a)eff.}(x = 0, r_{d(a)}) \equiv \frac{N_{d(a)}}{\exp\left[\frac{\Delta E_{ga\ n(p)}(N_{d(a)}, r_{d(a)})}{k_B T}\right]} \text{ and } N_{d(a)eff.}(x = W,\ r_{d(a)}) \equiv \frac{N_{d(a)o}(W)}{\exp\left[\frac{\Delta E_{ga\ n(p)}(N_{d(a)o}(W), r_{d(a)})}{k_B T}\right]}, \tag{4}$$

where $\Delta E_{ga\ n(p)}$ are determined in Equations (B4, B5) of the Appendix B.

Then, under low-level injection, in the absence of external generation, and for the steady-state case, we can define the minority-h(e) density by:

$$p_o(x)[n_o(x)] \equiv \frac{n_{in(p)}^2}{N_{d(a)eff.}(x,\ r_{d(a)})}, \tag{5}$$

where $n_{in(p)}^2$ is determined in (A5) of the Appendix A and a normalized excess minority-h(e) density u(x) or a relative deviation between $p(x)[n(x)]$ and $p_o(x)[n_o(x)]$, by [22, 25]:

$$u(x) \equiv \frac{p(x)[n(x)] - p_o(x)[n_o(x)])}{p_o(x)[n_o(x)]}, \tag{6}$$

which must verify the two following boundary conditions proposed by Shockley as [6]:

$$u(x = 0) \equiv \frac{-J_h(x=0)[J_e(x=0)]}{eS \times p_o(x=0)[n_o(x=0)]}, \tag{7}$$

$$u(x = W) = \exp\left(\frac{V}{n_{I(II)}(V) \times V_T}\right) - 1. \tag{8}$$

Here, $n_{I(II)}(V)$ is a photovoltaic conversion factor determined in Equations (27, 28), S $(\frac{cm}{s})$ is the surface recombination velocity at the emitter contact, V is the applied voltage, $V_T \equiv (k_B T/e)$ is the thermal voltage, and the minority-hole (electron) current density $J_{h(e)}(x)$.

Further, as developed in RP, from the Fick's law for minority hole (electron)-diffusion equations [8, 12]:

$$J_{h(e)}(x) = \frac{-e(+e)\times n_i^2}{F_{h(e)}(x)} \times \frac{du(x)}{dx} = \frac{-e(+e)n_{in(p)}^2 D_{h(e)}(x)}{N_{d(a)eff.}(x)} \times \frac{du(x)}{dx}, \quad (9)$$

where $N_{d(a)eff.}$ is given in Eq. (4), $D_{h(e)}$ and $F_{h(e)}$ are determined respectively in Equations (C3, C2, C6) of the Appendix C, and from the minority-hole (electron) continuity equation [8, 12]:

$$\frac{dJ_{h(e)}(x)}{dx} = -e(+e) \times n_{i\,n(p)}^2 \times \frac{u(x)}{F_{h(e)}(x)\times L_{h(e)}^2} = -e(+e) \times n_{i\,n(p)}^2 \times \frac{u(x)}{N_{d(a)eff.}(x)\times \tau_{h(e)E}}, \quad (10)$$

where $L_{h(e)}$ and $\tau_{h(e)E}$ are defined respectively in Equations (C7, C8) of the Appendix C, one finally obtains the following second-order differential equation as [22]:

$$\frac{d^2u(x)}{dx^2} - \frac{dF_{h(e)}(x)}{dx} \times \frac{du(x)}{dx} - \frac{u(x)}{L_{h(e)}^2(x)} = 0. \quad (11)$$

Then, taking into account the two boundary conditions (7, 8), one thus gets the general solution of this Eq. (11), as [22]:

$$u(x) = \frac{\sinh(P(x)) + I(W,S)\times\cosh(P(x))}{\sinh(P(W)) + I(W,S)\times\cosh(P(W))} \times \left(\exp\left(\frac{V}{n_{I(II)}(V)\times V_T}\right) - 1\right), \; I(W,S) = \frac{D_{h(e)}(N_o(W))}{S\times L_{h(e)}(N_o(W))}. \quad (12)$$

where the function $n_{I(II)}(V)$ is the photovoltaic conversion factor, determined in Eq. (29). Further, since $\frac{dP(x)}{dx} \equiv C \times F_{h(e)}(x) = \frac{1}{L_{h(e)}(x)}$, $C = 2.0893\times 10^{-30}$ (cm^4/s), for the crystalline Si, being an empirical parameter, chosen for each crystalline semiconductor, P(x) is thus found to be defined by:

$$P(x) \equiv \int_0^x \frac{dx}{L_{h(e)}(x)}), \; 0 \le x \le W, \; P(x = W) \equiv (\frac{1}{W} \times \int_0^W \frac{dx}{L_{h(e)}(x)}) \times W \equiv \frac{W}{L_{h(e)eff.}} = \frac{L_{h(e)}}{L_{h(e)eff.}} \times \frac{W}{L_{h(e)}}, \quad (13)$$

where $L_{h(e)eff.}$ is the effective minority-hole (electron) diffusion length. Further, from Eq. (9, 13), the minority-hole (electron) current density injected into the HD[d(a)- InP]ER is found to be determined by:

$$J_{h(e)}(x, W, N_{d(a)}, r_{d(a)}, S, V) = -J_{Eno}(x, W, N_d, r_d, S)\,[J_{Epo}(x, W, N_a, r_a, S)] \times \left(\exp\left(\frac{V}{n_{I(II)}(V)\times V_T}\right) - 1\right), \quad (14)$$

where $J_{En(p)o}$ is the saturation minority-hole (electron) current density,

$$J_{En(p)o}(x, W, N_{d(a)}, r_{d(a)}, S) = \frac{en_{i\,n(p)}^2 \times D_{h(e)}}{N_{d(a)eff.}\times L_{h(e)}} \times \frac{\cosh(P(x)) + I(W,S)\times\sinh(P(x))}{\sinh(P(W)) + I(W,S)\times\cosh(P(W))}. \quad (15)$$

Here, the intrinsic carrier concentration $n_{i\,n(p)}$ is computed by Eq. (A5) of the Appendix A, and the effective doping density $N_{d(a)eff.}$ is determined in Eq. (4), the minority-hole (electron) diffusion coefficient $D_{e(h)}$ and minority-hole (electron) diffusion length $L_{h(e)}$ are given respectively in Equations (C2, C3, C7) of the Appendix C, and the factor $I(W,S)$ is determined by:

$$I(W,S)=\frac{D_{h(e)}(N_{d(a)o}(W))}{S\times L_{h(e)}(N_{d(a)o}(W))}, \tag{16}$$

where $N_{d(a)o}(W)$ is determined in Eq. (3).

Further, one remarks that: (i) from Equations (12, 14-16) one obtains: $u(x=0)\equiv\frac{-J_h(x=0)[J_e(x=0)]}{eS\times p_o(x=0)[n_o(x=0)]}$, which is just the first boundary condition given in Eq. (7), and then, (ii) Eq. (12) yields: $u(x=W)=\exp\left(\frac{V}{n_{I(II)}(V)\times V_T}\right)-1$, being the second boundary condition given in Eq. (8).

In the following, we will denote $P(W)$ and $I(W,S)$ by P and I, for a simplicity. So, Eq. (15) gives:

$$J_{En(p)o}\left(x=0,W,N_{d(a)},r_{d(a)},S\right)=\frac{en^2_{i\,n(p)}\times D_{h(e)}}{N_{d(a)eff.}\times L_{h(e)}}\times\frac{1}{\sinh(P)+I\times\cosh(P)}, \tag{17}$$

$$J_{En(p)o}\left(x=W,W,N_{d(a)},r_{d(a)},S\right)=\frac{en^2_{i\,n(p)}\times D_{h(e)}}{N_{d(a)eff.}\times L_{h(e)}}\times\frac{\cosh(P)+I\times\sinh(P)}{\sinh(P)+I\times\cosh(P)}. \tag{18}$$

$$\frac{J_{h(e)}\left(x=0,W,N_{d(a)},r_{d(a)},S,V\right)}{J_{h(e)}\left(x=W,W,N_{d(a)},r_{d(a)},S,V\right)}\equiv\frac{J_{En(p)o}\left(x=0,W,N_{d(a)},r_{d(a)},S\right)}{J_{En(p)o}\left(x=W,W,N_{d(a)},r_{d(a)},S\right)}=\frac{1}{\cosh(P)+I\times\sinh(P)}. \tag{19}$$

Now, if defining the effective excess minority-hole (electron) charge storage in the emitter region by [22]:

$Q_{h(e)eff.}(x=W,N_{d(a)},r_{d(a)})\equiv\int_0^W+e(-e)\times u(x)\times p_o(x)[n_o(x)]\times\frac{\tau_{h(e)E}(N_{d(a)},r_{d(a)})}{\tau_{h(e)E}(\rho_{d(a)}(x),r_{d(a)})}dx$, and the effective minority-hole transit time by: $\tau_{teff.}(x=W,W,N_{d(a)},r_{d(a)},S)\equiv Q_{h(e)\,eff.}(x=W,N_{d(a)},r_{d(a)})/J_{En(p)o}\left(x=W,W,N_{d(a)},r_{d(a)},S\right)$, one can define, from Equations (10, 19), the reduced effective minority-hole transit time:

$$\frac{\tau_{teff.}\left(x=W,W,N_{d(a)},r_{d(a)},S\right)}{\tau_{h(e)E}}\equiv 1-\frac{J_{En(p)o}\left(x=0,W,N_{d(a)},r_{d(a)},S\right)}{J_{En(p)o}\left(x=W,W,N_{d(a)},r_{d(a)},S\right)}=1-\frac{1}{\cosh(P)+I\times\sinh(P)}. \tag{20}$$

Now, some important results can be obtained and discussed below.

As $P\ll 1$ (or $W\ll L_{h,eff.}$) and $S\to\infty$, $I\equiv I(W,S)=\frac{D_h(N_o(W))}{S\times L_h(N_o(W))}\to 0$, from Eq. (20), one has: $\frac{\tau_{t,eff.}\left(x=W,W,N_{d(a)},r_{d(a)},S\right)}{\tau_{h(e)E}}\to 0$, suggesting a completely transparent emitter region (CTER)-case, where, from Eq. (18), one obtains:

$$J_{En(p)o}\left(x=W, N_{d(a)}, r_{d(a)}, S\to\infty\right)\to\frac{en^2_{i\,n(p)}\times D_{h(e)}}{N_{d(a)eff.}\times L_{h(e)}}\times\frac{1}{P(W)}, \quad (21a)$$

and then, as $P \gg 1$ (or $W \gg L_{h,eff.}$) and $S \to 0$, $I \equiv I(W,S) = \frac{D_h(N_o(W))}{S\times L_h(N_o(W))} \to \infty$, from Eq. (20), one has: $\frac{\tau_{teff.}\left(x=W,W,N_{d(a)},r_{d(a)},S\right)}{\tau_{h(e)E}} \to 1$, suggesting a completely opaque emitter region (COER)-case, where, from Eq. (18), one gets:

$$J_{En(p)o}\left(x=W, N_{d(a)}, r_{d(a)}, S\to 0\right)\to\frac{en^2_{i\,n(p)}\times D_{h(e)}}{N_{d(a)eff.}\times L_{h(e)}}\times\tanh(P). \quad (21b)$$

In summary, in the $n^+(p^+)-p(n)$ junction solar cells, the dark carrier-minority saturation current density J_o, defined in Eq. (2), is now replaced by $J_{oI(II)}$, for a good presentation, and rewritten by:

$$J_{oI(II)}\left(W, N_{d(a)}, r_{d(a)}, S, N_{a(d)}, r_{a(d)}\right)\equiv J_{En(p)o}(W, N_{d(a)}, r_{d(a)}, S)+J_{Bp(n)o}(N_{a(d)}, r_{a(d)}), \quad (22)$$

where $J_{En(p)o}$ and $J_{Bp(n)o}$ are determined respectively in Equations (18) and (C1) of the Appendix C.

3. Photovoltaic conversion effect at 300K

Here, in the $n^+(p^+)-p(n)$ junction solar cells, denoted respectively by I(II), and for physical conditions:

$$W=0.0044\ \mu m, N_{P(In)}=10^{19}\ (10^{20})cm^{-3}, r_{P(In)}, S=10^{50}\frac{cm}{s}, N_{In(P)}=10^{17}(10^{16})\ cm^{-3}, r_{In(P)}, \quad (23)$$

we propose, at given $V_{ocI1(2)}$ and $V_{ocII1(2)}$, the experimental results of the short circuit current density $J_{scI(II)}$, fill factor $F_{I(II)}$, and photovoltaic conversion factor $\eta_{I(II)}$, in order to formulate our following treatment method of two fixe experimental points [3, 4], for the n^+-p junction,

$V_{ocI1(I2)}=729.9\ (909)\ mV$, $J_{scI1(I2)}=26.64\ (30.47)\ mA/cm^2$, $F_{I1(I2)}=77.04\ (83.9)\ \%$, $\eta_{I1(I2)}=15\ (23.24)\ \%$, and for the p^+-n junction,

$V_{ocII1(II2)}=785\ (960)\ mV$, $J_{scII1(II2)}=30.5\ (30.4)\ mA/cm^2$, $F_{II1(II2)}=80.1\ (87.4)\ \%$, $\eta_{II1(II2)}=19.2\ (25.5)\ \%$. (24)

First of all, we define the net current density J at T=300 K, obtained for the infinite shunt resistance, and expressed as a function of the applied voltage V, flowing through the $n^+(p^+)-p(n)$ junction of InP solar cells, by [1, 2, 5-10]:

$$J(V)\equiv J_{ph.}(V)-J_{oI(II)}\times\left(e^{X_{I(II)}(V)}-1\right),\ X_{I(II)}(V)\equiv\frac{V}{n_{I(II)}(V)\times V_T},\ V_T\equiv\frac{k_BT}{e}=25.85\ mV, \quad (25)$$

where the function $n_{I(II)}(V)$ is the photovoltaic conversion factor, noting that as $V = V_{oc}$, $J(V) = 0$, the photocurrent density is defined by: $J_{ph.}(V = V_{oc}) \equiv J_{scI(II)}\left(W, N_{d(a)}, r_{d(a)}, S, N_{a(d)}, r_{a(d)}, V_{oc}\right)$, for $V_{oc} \geq V_{ocI(II)1}$. Therefore, the photovoltaic conversion effect occurs, according to:

$$J_{scI(II)}\left(W, N_{d(a)}, r_{d(a)}, S, N_{a(d)}, r_{a(d)}, V_{oc}\right) \equiv J_{oI(II)}\left(W, N_{d(a)}, r_{d(a)}, S, N_{a(d)}, r_{a(d)}\right) \times \left(e^{X_{I(II)}(V_{oc})} - 1\right), \quad (26)$$

where $n_{I(II)}(V_{oc}) \equiv n_{I(II)}\left(W, N_{d(a)}, r_{d(a)}, S, N_{a(d)}, r_{a(d)}, V_{oc}\right)$, and $X_{I(II)}(V_{oc}) \equiv \frac{V_{oc}}{n_{I(II)}(V_{oc}) \times V_T}$.

Here, one remarks that (i) for a given V_{oc}, both $n_{I(II)}$ and $J_{oI(II)}$ have the same variations, obtained in the same physical conditions, as observed in many cases, given in Ref. [1], and (ii) the function $\left(e^{X_{I(II)}(V_{oc})} - 1\right)$ or the PVCF $n_{I(II)}$, representing the photovoltaic conversion effect, thus converts the light, represented by $J_{scI(II)}$, into the electricity, by $J_{oI(II)}$.

Further, from Equations (22, 26), we obtain for the $n^+ - p$ junction:

$$n_{I1(2)}\left(W, N_d, r_d, S, N_a, r_a, V_{ocI1(2)}, J_{scI1(2)}\right) \equiv \frac{V_{ocI1(2)}}{V_T} \times \frac{1}{\ln\left(\frac{J_{scI1(2)}}{J_{oI}} + 1\right)} \equiv n_{I1(2)}(V_{ocI1(2)}, J_{scI1(2)}),$$

and we then propose:

$$n_I(W, N_d, r_d, S, N_a, r_a, V_{oc}) = n_{I1}(V_{ocI1}, J_{scI1}) + n_{I2}(V_{ocI2}, J_{scI2}) \times \left(\frac{V_{oc}}{V_{ocI1}} - 1\right)^{1.16621}, \quad (27)$$

being valid for any values of $(W, N_d, r_d, S, N_a, r_a, V_{oc} \geq V_{ocI1})$.

Furthermore, for the $p^+ - n$ junction,

$$n_{II1(2)}\left(W, N_a, r_a, S, N_d, r_d, V_{ocII1(2)}, J_{scII1(2)}\right) \equiv \frac{V_{ocII1(2)}}{V_T} \times \frac{1}{\ln\left(\frac{J_{scII1(2)}}{J_{oII}} + 1\right)} \equiv n_{II1(2)}(V_{ocII1(2)}, J_{scII1(2)}),$$

and then,

$$n_{II}(W, N_a, r_a, S, N_d, r_d, V_{oc}) = n_{II1}(V_{ocI1}, J_{scI1}) + n_{II2}(V_{ocII2}, J_{scII2}) \times \left(\frac{V_{oc}}{V_{ocII1}} - 1\right)^{1.13383}, \quad (28)$$

being valid for any values of $(W, N_a, r_a, S, N_d, r_d, V_{oc} \geq V_{ocII1})$.

Therefore, from Equations (23, 24, 27, 28), one obtains, $n_{I1(II1)} = 0.721024$ (0.7855) at $V_{ocI1(II1)} = \mathbf{729.9}$ (785) mV, and $n_{I2(II2)} = 0.894876$ (0.9607) at $V_{ocI2(II2)} = 909$ (960) mV, respectively, for $\mathbf{n^+(p^+) - p(n)}$ junction solar cells.

Thus, X_I defined from Eq. (26) now becomes for the $n^+ - p$ junction:

$X_I(W, N_d, r_d, S, N_a, r_a, V_{oc}) \equiv \frac{V_{oc}}{n_I(W,N_d,r_d,S,N_a,r_a,V_{oc})\times V_T}$, and therefore, we can determine the values of the fill factors $F_{I1(I2)}$ at $V_{oc} = V_{ocI1(I2)}$ by [1, 2]:

$F_{I1(I2)}\left(W, N_d, r_d, S, N_a, r_a, V_{ocI1(I2)}\right) = \frac{X_I(W,N_d,r_d,S,N_a,r_a,V_{ocI1(I2)})-\ln[X_I(W,N_d,r_d,S,N_a,r_a,V_{ocI1(I2)})+0.72]}{X_I(W,N_d,r_d,S,N_a,r_a,V_{ocI1(I2)})+6.88486\ (3.1423)} \equiv F_{I1(I2)}\left(V_{oc} = V_{ocI1(I2)}\right)$, for a presentation simplicity, and further, the fill factor F_I can be computed by:

$$F_I(W, N_d, r_d, S, N_a, r_a, V_{oc}) = F_{I1}(V_{ocI1}) + F_{I2}(V_{ocI2}) \times \left(\frac{V_{oc}}{V_{ocI1}} - 1\right)^{1.782196}, \quad (29)$$

which is valid for any values of $(W, N_d, r_d, S, N_a, r_a, V_{oc} \geq V_{ocI1})$.

Then, also from Eq. (26), we can define for the $p^+ - n$ junction:

$X_{II}(W, N_a, r_a, S, N_d, r_d, V_{oc}) \equiv \frac{V_{oc}}{n_{II}(W,N_a,r_a,S,N_d,r_d,V_{oc})\times V_T}$, where $n_{II}(W, N_a, r_a, S, N_d, r_d, V_{oc})$ is determined in Eq. (28). Therefore, we can determine the values of the fill factors $F_{II1(II2)}$ at $V_{oc} = V_{ocII1(II2)}$ as:

$F_{II1(II2)}\left(W, N_a, r_a, S, N_d, r_d, V_{ocII1(II2)}\right) = \frac{X_{II}(W,N_a,r_a,S,N_d,r_d,V_{ocII1(II2)})-\ln[X_{II}(W,N_a,r_a,S,N_d,r_d,V_{ocII1(II2)})+0.72]}{X_{II}(W,N_a,r_a,S,N_d,r_d,V_{ocII1(II2)})+5.0173\ (1.3695)} \equiv F_{II1(2)}\left(V_{ocII1(II2)}\right)$, for a presentation simplicity, and further, the fill factor F_{II} is determined by:

$$F_{II}(W, N_a, r_a, S, N_d, r_d, V_{oc}) = F_{II1}(V_{ocII1}) + F_{II2}(V_{ocII2}) \times \left(\frac{V_{oc}}{V_{ocII1}} - 1\right)^{1.6541}, \quad (30)$$

being valid for any values of $(W, N_a, r_a, S, N_d, r_d, V_{oc} \geq V_{ocII1})$.

Then, with physical conditions given in Eq. (23), our numerical calculation shows that we obtain the same values of $J_{scI1(I2)}$ and $F_{I1(I2)}$ at $V_{ocI1(I2)} = 729.9\ (909)$ mV, and $J_{scII1(II2)}$ and $F_{II1(II2)}$ at $V_{ocII1(II2)} = 785\ (960)$ mV, as those given in Eq. (24).

Finally, the efficiency $\eta_{I(II)}$ can be defined in the $n^+(p^+) - p(n)$ junction solar cells, by:

$$\eta_{I(II)}\left(W, N_{d(a)}, r_{d(a)}, S, N_{a(d)}, r_{a(d)}, V_{oc}\right) \equiv \frac{J_{scI(II)}\times V_{oc}\times F_{I(II)}}{P_{in.}}, \quad (31)$$

where, $J_{scI(II)}$ and $F_{I(II)}$ are determined respectively in Equations (26, 29, 30), being assumed to be obtained at 1 sun illumination or at AM1.5G spectrum ($P_{in.} = 0.100\ \frac{W}{cm^2}$) [1, 2, 26-29]. Then, from Equations (31, 24), we get the numerical results of η, by using this assumption: $P_{in.} = 0.100\ \frac{W}{cm^2}$, and their relative errors in absolute values (RE), calculated by using the experimental results of $\eta_{I1(I2)}$ and $\eta_{II1(II2)}$ given in Eq. (24),

-for the $n^+ - p$ junction at $V_{ocI1(I2)} = 729.9\ (909)$ mV, $\eta_{I1(I2)} = 14.981\%\ (23.238\%)$, with RE = $1.3 \times 10^{-3}(7.8 \times 10^{-5})$, and

-for the $p^+ - n$ junction at $V_{ocII1(II2)} = 785\ (960)$ mV, $\eta_{II1(II2)} = 19.201\%\ (25.516\%)$, with RE =$6.2 \times 10^{-5}(6.2 \times 10^{-4})$.

4. Numerical results and concluding remarks

We will respectively consider the two following cases.

4.1. HD [Sn − **InP**] **ER** − LD[Ga (Mg, In) − **InP**] **BR** −**cases**

Here, for those 3 $(n^+ - p) -$ junctions: $[(Sn - Ga), (Sn - Mg), (Sn - In)]$, respectively, we propose the following physical conditions as:

$W = 15\ \mu m,\ N_d = 10^{19}\ cm^{-3}, S\ = 100\ (cm/s\ ),$ and $N_a = 10^{17}\ cm^{-3}.$ (32)

Then, from Eq. (20), one respectively obtains: $\frac{\tau_{teff.}}{\tau_{hE}} = (0.32, 0.33, 0.33)$, according to the lightly transparent condition, and from Eq. (18), $J_{Eno} = (0, 0, 0)\ \left(\frac{A}{cm^2}\right)$. Further, one respectively gets from Eq. (C1) of the Appendix C: $J_{Bpo} = (3.11, 2.66, 2.63) \times 10^{-19}\left(\frac{A}{cm^2}\right)$. Then, from Eq. (22), one obtains respectively: $J_{oI} = (3.11, 2.66, 2.63) \times 10^{-19}\ \left(\frac{A}{cm^2}\right) = J_{Bpo}$, and from the following Table 2, for example, at $V_{oc} = 821$ mV, n_I = (0.803; 0.801; 0.800) and η_I = (29.48; 29.57; **29.58**) %, suggesting that, with increasing r_a, or with decreasing ε_a, due to the acceptor-size effect, both J_{oI} and n_I decrease, while η_I increases, being new obtained results.

Table 2. In the HD[Sn-InP] **ER**-LD[Ga (Mg, In)-InP] **BR** and for physical conditions given in Eq. (32), our numerical results of n_I, J_{scI}, F_I, and η_I, are computed by using Equations (27, 26, 29, 31), respectively. Here, on notes that, for a given V_{oc} and with increasing $r_{d(a)}$, the function n_I decreases, while other functions J_{scI}, F_I, and η_I increase, being due to the impurity size $r_{d(a)}$-effect, suggesting thus our new obtained results.

V_{oc}(mV)	n_I	$J_{scI}(\frac{mA}{cm^2})$	F_I(%)	η_I(%)
960	0.954; 0.954; 0.954	21.21; 21.21; 21.21	87.7; 87.7; 87.7	17.87; 17.87; 17.87
909	0.899; 0.895; 0.895	30.46; 30.47; 30.47	83.8; 83.9; 83.9	23.21; 23.24; 23.24
900	0.888; 0.885; 0.885	32.24; 32.26; 32.26	83.2; 83.3; 83.3	24.14; 24.18; 24.18
825	0.807; 0.804; 0.804	45.08; 45.18; 45.18	79.2; 79.2; 79.3	29.45; 29.54; 29.54
821	**0.803; 0.801; 0.800**	45.45; 45.54; 45.55	79.0; 79.0; 79.1	**29.48; 29.57**; **29.58**
817	0.799; 0.796; 0.796	45.75; 45.84; 45.85	78.9; 78.9; 78.9	29.47; 29.56; 29.57

815	0.797; 0.794; 0.794	45.88; 45.97; 45.98	78.8; 78.8; 78.9	29.46; 29.55; 29.55
800	0.782; 0.779; 0.779	46.25; 46.35; 46.36	78.2; 78.3; 78.3	28.95; 29.04; 29.05
785	0.768; 0.765; 0.765	45.40; 45.50; 45.51	77.8; 77.9; 77.9	27.73; 27.81; 27.82
729.9	0.724; 0.721; 0.721	26.63; 26.64; 26.64	76.9; 77.0; 77.0	14.96; 14.98; 14.98

4.2. HD $[\mathrm{In}-\mathrm{InP}]$ **ER** $-$ LD$[$ P (As, Sb, Sn) $-$ **InP** $]$ **BR** $-$**cases**

Here, for those 4 $(p^{+}-n)-$ junctions: $[(\mathrm{In}-\mathrm{P}),(\mathrm{In}-\mathrm{As}),(\mathrm{In}-\mathrm{Sb}),(\mathrm{In}-\mathrm{Sn})]$, respectively, we propose the following physical conditions as:

$$W = 15\ \mu m,\ N_a = 10^{20}\ cm^{-3}, S\ = 100\ (cm/s\ ),\ \text{and}\ N_d = 10^{16}\ cm^{-3}. \quad (33)$$

Then, from Eq. (20) , one respectively obtains: $\frac{\tau_{teff.}}{\tau_{hE}} = (0, 0, 0, 0)$, suggesting the completely transparent condition, and from Eq. (18), $J_{Epo} = (0, 0, 0, 0)\ \left(\frac{A}{cm^2}\right)$. Further, one respectively gets from Eq. (C1) of the Appendix C: $J_{Bno} = (4.96, 4.83, 3.85, 3.59) \times 10^{-19}\left(\frac{A}{cm^2}\right)$. Then, from Eq. (22), one obtains respectively:

$J_{oII} = (4.96, 4.83, 3.85, 3.59) \times 10^{-19}\ \left(\frac{A}{cm^2}\right) = J_{Bno}$, and from the following Table 3, for example, at $V_{oc} = 868\ mV$, n_{II} = (0.861; 0.860; 0.855; 0.853) and η_{II} = (31.16; 31.17; 31.24; **31.26**) %, meaning that, with increasing r_d, or with decreasing ε_d, due to the donor-size effect, both J_{oII} and n_{II} decrease, according to the increase in η_{II}, being also new obtained results.

Table 3. In the HD[In-InP] **ER**-LD[P(As, Sb, Sn)-InP] **BR** and for physical conditions given in Eq. (33), our numerical results of n_{II}, J_{scII}, F_{II}, and η_{II}, are computed by using Equations (28, 26, 30, 31), respectively. Here, on notes that, for a given V_{oc} and with increasing $r_{a(d)}$, the function n_{II} decreases, while other functions J_{scII}, F_{II}, and η_{II} increase, being due to the impurity size $r_{a(d)}$-effect, suggesting thus our new obtained results.

$V_{oc}(mV)$	n_{II}	$J_{scII}(\frac{mA}{cm^2})$	$F_{II}(\%)$	$\eta_{II}(\%)$
960	0.960; 0.959; 0.954; 0.952	30.39; 30.39; 30.40; 30.40	87.4; 87.4; 87.5; 87.5	25.64; 25.64; 25.65; 25.65
940	0.937; 0.937; 0.932; 0.930	33.71; 33.71; 33.73; 33.74	86.1; 86.1; 86.1; 86.2	27.42; 27.42; 27.44; 27.45
875	0.868; 0.867; 0.862; 0.860	42.84; 42.85; 42.94; 42.97	82.5; 82.5; 82.6; 82.7	31.12; 31.13; 31.20; 31.22
868	**0.861; 0.860; 0.855; 0.853**	43.40; 43.41; 43.50; 43.53	82.2; 82.2; 82.3; 82.4	**31.16**; **31.17; 31.24**; **31.26**
860	0.852; 0.851; 0.847; 0.845	43.86; 43.87; 43.97; 44.00	81.9; 81.9; 82.0; 82.0	31.08; 31.09; 31.16; 31.18
820	0.814; 0.813; 0.808; 0.807	42.07; 42.08; 42.16; 42.19	80.6; 80.6; 80.7; 80.7	27.98; 27.99; 28.05; 28.07
810	0.805; 0.804; 0.799; 0.798	40.15; 40.16; 40.23; 40.25	80.4; 80.4; 80.5; 80.5	26.31; 26.32; 26.36; 26.38
800	0.796; 0.795; 0.791; 0.789	37.39; 37.40; 37.45; 37.46	80.2; 80.2; 80.3; 80.4	24.15; 24.16; 24.19; 24.20

785	0.785; 0.784; 0.780; 0.779	30.50; 30.50; 30.50; 30.50	80.1; 80.1; 80.2; 80.2	19.29; 19.29; 19.30; 19.30

Acknowledgments: We thank Drs M. Cayrol and J. Sulian for their technical helps.

Appendix

Appendix A. Fermi Energy

In the n(p)-type InP crystal, the Fermi energy $E_{Fn}(-E_{Fp})$, obtained for any T and donor density N, being investigated in our previous paper, with a precision of the order of 2.11×10^{-4} [39, 40], is now summarized in the following.

First of all, we define the reduced electron density by:

$$u\equiv\frac{N_{d(a)}}{N_{c(v)}},\ N_c(T)=2\times1\times\left(\frac{m_c\times k_BT}{2\pi\hbar^2}\right)^{\frac{3}{2}}\ (cm^{-3}),\ N_v(T)=2\times1\times\left(\frac{m_v\times k_BT}{2\pi\hbar^2}\right)^{\frac{3}{2}}\ (cm^{-3}). \quad (A1)$$

Here, $N_{c(v)}$ is the conduction (valence)-band density of states, respectively, m_c is the effective mass of the electron in n-type InP can be defined by [8]:

$$m_c=0.015\times m_o, \quad (A2)$$

where $m_o=9.1096\times10^{-28}\ g$ is the electron rest mass, the averaged effective mass of the hole in the p-type InP yields [8]:

$$(m_v/m_o)=\frac{0.021+0.39}{2}=0.2055, \quad (A3)$$

and finally, $E_{gin(p)}(T,r_{d(a)})$ is the intrinsic band gap in the InP -crystal, due to the T-dependent carrier-lattice interaction-effect, by [1, 2, 33, 34]:

$$E_{gin(p)}(T,r_{d(a)})=E_{gn(p)}(r_{d(a)})-\frac{5.405\times10^{-4}\times T^2}{T+204}. \quad (A4)$$

Here, $E_{gn(p)}(r_{d(a)})$ is determined in Eq. (1b), due to the d(a)-size effect.

Furthermore, in the n(p)-type InP, one can define the intrinsic carrier concentration $n_{in(p)}$ by:

$$n_{i\,n(p)}^2(T, r_{d(a)}) \equiv N_c(T) \times N_v(T) \times \exp\left(\frac{-E_{gin(p)}(T, r_{d(a)})}{k_B T}\right). \quad \text{(A5)}$$

Then, denoting the reduced Fermi energy in the n(p)-type semiconductor, respectively, by $\frac{E_{Fn}(u)}{k_B T}(\frac{-E_{Fp}(u)}{k_B T})$,

being accurate to within 10^{-7} , we have [39] :

$$\frac{E_{Fn}(u)}{k_B T}(\frac{-E_{Fp}(u)}{k_B T}) = \frac{G(u)+Au^B F(u)}{1+Au^B}, \; A = 0.0005372 \text{ and } B = 4.82842262 \quad \text{(A6)}$$

where

$$F(u) = au^{\frac{2}{3}}\left(1 + bu^{-\frac{4}{3}} + cu^{-\frac{8}{3}}\right)^{-\frac{2}{3}}, \quad a = [(3\sqrt{\pi}/4) \times u]^{2/3}, \quad b = \frac{1}{8}\left(\frac{\pi}{a}\right)^2 \quad \text{and} \quad c = \frac{62.3739855}{1920}\left(\frac{\pi}{a}\right)^4$$

and

$$G(u) \simeq Ln(u) + 2^{-\frac{3}{2}} \times u \times e^{-du}; \; d = 2^{3/2}\left[\frac{1}{\sqrt{27}} - \frac{3}{16}\right] > 0.$$

Here, one notes that: (i) $\frac{E_{Fn}(u \gg 1)}{k_B T}(\frac{-E_{Fp}(u \gg 1)}{k_B T}) > 1$, according to the HD[d(a)-InP]ER-case, or to the degenerate case, Eq. (A6) is reduced to the function F(u), and (ii) $\frac{E_{Fn}(u \ll 1)}{k_B T}(\frac{-E_{Fp}(u \ll 1)}{k_B T}) < -1$, to the LD[a(d)- InP]BR-case, or to the non-degenerate case, Eq. (A6) is reduced to the function G(u), respectively.

(i) In the HD[d(a)- InP]ER-case for $N_{d(a)} = 10^{19}(10^{20})\ cm^{-3}$, we respectively get: $\frac{E_{Fn}}{k_B T}(\frac{-E_{Fp}}{k_B T}) = 43.66\ (14.87) > 1$, according to degenerate conditions.

(ii) In the LD[a(d)- InP]BR-case and for $N_{a(d)} = 10^{17}(10^{16})\ cm^{-3}$, we respectively obtain: $\frac{-E_{Fp}}{k_B T}(\frac{E_{Fn}}{k_B T}) = -3.14\ (-1.45) < -1$, according to non-degenerate conditions. Thus, those limiting values of $N_{a(d)} = 10^{17}(10^{16})\ cm^{-3}$ can be used in the LD[a(d)-InP] BR-cases, respectively.

Appendix B. Approximate forms for band gap narrowing and apparent band gap narrowing

First of all, in the n(p)-type InP, we define the effective Wigner-Seitz radius r_s characteristic of the interactions by [1, 2]

$$r_{sn} \equiv r_s(N_d, r_d) = 1.1723 \times 10^8 \times \left(\frac{1}{N_d}\right)^{1/3} \times \frac{m_c}{\varepsilon(r_d)} \quad \text{(B1)}$$

and

$$r_{sp} \equiv r_s(N_a, r_a) = 1.1723 \times 10^8 \times \left(\frac{1}{N_a}\right)^{1/3} \times \frac{m_v}{\varepsilon(r_a)}, \quad \text{(B2)}$$

where m_c and m_v are given in (A2) and (A3). Therefore, the correlation energy of an effective electron gas, $E_{cn(cp)}\left(N_{d(a)}, r_{d(a)}\right)$, is given by [1, 2, 42]:

$$E_{cn(cp)}\left(N_{d(a)}, r_{d(a)}\right) = \frac{-0.87553}{0.0908+r_{sn(sp)}} + \frac{\frac{0.87553}{0.0908+r_{sn(sp)}}+\left(\frac{2[1-\ln(2)]}{\pi^2}\right)\times\ln\left(r_{sn(sp)}\right)-0.093288}{1+0.03847728\times r_{sn(sp)}^{1.67378876}}. \quad \text{(B3)}$$

Then, in the n-type heavily doped InP, the band gap narrowing is found to be given as [1, 2]:

$$\Delta E_{gn}(N_d, r_d) \simeq a_1 \times \frac{\varepsilon(r_P)}{\varepsilon(r_d)} \times N_r^{1/3} + a_2 \times \frac{\varepsilon(r_P)}{\varepsilon(r_d)} \times N_r^{\frac{1}{3}} \times (2.503 \times [-E_c(r_{sn}) \times r_{sn}]) + a_3 \times \left[\frac{\varepsilon(r_P)}{\varepsilon(r_d)}\right]^{5/4} \times \sqrt{\frac{m_v}{m_c}} \times N_r^{1/4} + a_4 \times \sqrt{\frac{\varepsilon(r_P)}{\varepsilon(r_d)}} \times N_r^{1/2} \times 2 + a_5 \times \left[\frac{\varepsilon(r_P)}{\varepsilon(r_d)}\right]^{\frac{3}{2}} \times N_r^{\frac{1}{6}},\ N_r \equiv \left(\frac{N_d}{9.999\times10^{17}\ cm^{-3}}\right), \quad \text{(B4)}$$

where $a_1 = 3.8 \times 10^{-3}(eV)$, $a_2 = 6.5 \times 10^{-4}(eV)$, $a_3 = 2.8 \times 10^{-3}(eV)$, $a_4 = 5.597 \times 10^{-3}(eV)$ and $a_5 = 8.1 \times 10^{-4}(eV)$, and in the p-type heavily doped InP, one has [1, 2]:

$$\Delta E_{gp}(N_a, r_a) \simeq a_1 \times \frac{\varepsilon(r_{In})}{\varepsilon(r_a)} \times N_r^{1/3} + a_2 \times \frac{\varepsilon(r_{In})}{\varepsilon(r_a)} \times N_r^{\frac{1}{3}} \times \left(2.503 \times [-E_c(r_{sp}) \times r_{sp}]\right) + a_3 \times \left[\frac{\varepsilon(r_{In})}{\varepsilon(r_a)}\right]^{5/4} \times \sqrt{\frac{m_c}{m_v}} \times N_r^{1/4} + 2a_4 \times \sqrt{\frac{\varepsilon(r_{In})}{\varepsilon(r_a)}} \times N_r^{1/2} + a_5 \times \left[\frac{\varepsilon(r_{In})}{\varepsilon(r_a)}\right]^{\frac{3}{2}} \times N_r^{\frac{1}{6}},\ N_r \equiv \left(\frac{N_a}{9.999\times10^{17}\ cm^{-3}}\right), \quad \text{(B5)}$$

where $a_1 = 3.15 \times 10^{-3}(eV)$, $a_2 = 5.41 \times 10^{-4}(eV)$, $a_3 = 2.32 \times 10^{-3}(eV)$, $a_4 = 4.12 \times 10^{-3}(eV)$ and $a_5 = 9.80 \times 10^{-5}(eV)$.

Further, in the donor (acceptor)- InP, we define the effective intrinsic carrier concentration $n_{ien(p)}$, by

$$n_{i\,en(p)}^2(N_{d(a)}, r_{d(a)}) \equiv N_{d(a)} \times p_o(n_o) \equiv n_{i\,n(p)}^2 \times \exp\left[\frac{\Delta E_{gan(p)}}{k_B T}\right], \quad \text{(B6)}$$

where we can define the "effective doping density" by: $N_{d(a)eff.} \equiv N_{d(a)}/\exp\left[\frac{\Delta E_{ga\,n(p)}}{k_B T}\right]$ so that $N_{d(a)eff.} \times p_o(n_o) \equiv n_{i\,n(p)}^2$ [8], and also the apparent band gap narrowing, $\Delta E_{ga\,n(p)}$, as

$$\Delta E_{ga\,n(p)} \equiv \Delta E_{g\,n(p)} + k_B T \times \ln\left(\frac{N_{d(a)}}{N_{c(v)}}\right) - E_{Fn}\left(\frac{N_d}{N_c}\right)\left[-E_{Fp}\left(\frac{N_a}{N_v}\right)\right], \quad \text{(B7)}$$

where $N_{c(v)}$ is defined in Eq. (A1), the Fermi energy is determined in Eq. (A6).

Appendix C. Minority-carrier transport parameters

Here, the minority-electron (hole) saturation current density injected into the LD[a(d)- InP]BR, with an acceptor density equal to $N_{a(d)}$, is given by [1, 2]:

$$J_{Bp(n)o}\left(N_{a(d)}, r_{a(d)}\right) = \frac{e \times n_i^2(r_{a(d)}) \times \sqrt{\frac{D_{e(h)}(N_{a(d)}, r_{a(d)})}{\tau_{e(h)B}(N_{a(d)})}}}{N_{a(d)}}, \quad \text{(C1)}$$

where $n_{i\,n(p)}^2(r_{d(a)})$ is determined in (A5), $D_{e(h)}(N_{a(d)}, r_{a(d)})$ is the minority-electron (hole) diffusion coefficient [8]:

$$D_e(N_a, r_a) = \frac{k_B T}{e} \times \left[200 + \frac{4500-200}{1+\left(\frac{N_a}{1.3\times10^{17}\,cm^{-3}}\right)^{0.91}}\right] \times \left(\frac{\varepsilon(r_a)}{12.85}\right)^2 (cm^2V^{-1}s^{-1}), \quad \text{(C2)}$$

$$D_h(N_d, r_d) = \frac{k_B T}{e} \times \left[80 + \frac{100-80}{1+\left(\frac{N_d}{8\times10^{17}\,cm^{-3}}\right)^{1.25}}\right] \times \left(\frac{\varepsilon(r_d)}{12.85}\right)^2 (cm^2V^{-1}s^{-1}), \quad \text{(C3)}$$

and $\tau_{e(h)B}(N_{d(a)})$ is the minority-electron (hole) lifetime in the base region:

$$\tau_{eB}(N_a)^{-1} = \frac{1}{10^{-7}} + 3\times10^{-13}\times N_a + 1.83\times10^{-31}\times N_a^2. \quad \text{(C4)}$$

$$\tau_{hB}(N_d)^{-1} = \frac{1}{10^{-7}} + 11.76\times10^{-13}\times N_d + 2.78\times10^{-31}\times N_d^2, \quad \text{(C5)}$$

Further, from (A6), (B4)-(B7)), in the HD[d(a)- InP]ER, we can define the following minority-hole(electron) transport parameter $F_{h(e)}$ as [8, 22, 25]:

$$F_{h(e)}\ (N_{d(a)}, r_{d(a)}) \equiv \frac{n_{i\,n(p)}^2(r_{d(a)})}{p_o(n_o)\times D_{h(e)}} = \frac{N_{d(a)eff.}}{D_{h(e)}} \equiv \frac{N_{d(a)}}{D_{h(e)}\times\exp\left[\frac{\Delta E_{g\,an(p)}}{k_B T}\right]}\ (cm^{-5}\times s), \quad \text{(C6)}$$

Furthermore, the minority-hole (electron) diffusion length, $L_{h(e)}\left(N_{d(a)}, r_{d(a)}\right)$ and the minority-hole(electron) lifetime $\tau_{h(e)E}$ in the HD[d(a)- InP]ER can be determined by

$$L_{h(e)}^{-2}\left(N_{d(a)}, r_{d(a)}\right) = \left[\tau_{h(e)E}\times D_{h(e)}\right]^{-1} = \left(C\times F_{n(p)}\right)^2 = \left(C\times\frac{N_{d(a)eff.}}{D_{h(e)}}\right)^2 = \left(C\times\frac{n_{i\,n(p)}^2(r_{d(a)})}{p_o(n_o)\times D_{h(e)}}\right)^2, \quad \text{(C7)}$$

where the constant C is chosen to be 2.0893×10^{-30} (cm^4/s), and then, $\tau_{h(e)E}$ can be computed by:

$$\tau_{h(e)E} = \frac{1}{D_{h(e)}\times\left(C\times F_{n(p)}\right)^2}. \quad \text{(C8)}$$

References

[1] H. Van Cong, K. C. Ho-Huynh Thi, P. Blaise, R. Brouzet, O. Henri-Rousseau, "31.474% (44.359%)- Limiting Highest Efficiencies obtained in $n^+(p^+) - p(n)$ Crystalline GaAs

Junction Solar Cells at 300K, Due to the Effects of Heavy (Low) Doping and Impurity Size, "SCIREA J. Phys., vol. 7, pp. 114-135, 2022.

[2] H. Van Cong, K. C. Ho-Huynh Thi, P. Blaise, O. Henri-Rousseau, R. Brouzet, J. Susian, and M. Cayrol, "31% (30.65%)- Limiting Highest Efficiencies obtained in $n^+(p^+) - p(n)$ Crystalline Silicon Junction Solar Cells at 300K, Due to the Effects of Heavy (Low) Doping and Impurity Size, "SCIREA J. Phys., vol. 7, pp. 80-103, 2022

[3] G. Otnes et al., "Understanding InP Nanowire Array Solar Cell performance by Nanoprobe-Enabled Single Nanowire measurements," Nano Lett., vol. 18, pp. 3038-046, 2018; D. A. Goldman, "Nanophotonic resonators for InP Solar Cells," Optics Express, vol. 24, pp. A925-A934, 2016.

[4] X. Yin et al., "19.2% Efficient InP Heterojunction Solar Cell with electron-selective TiO_2 *contact,*"ACS Photonics, vol. 1, pp.1245-1250; V. Raj et al., "Indium phosphide based solar cell using ultra-thin ZnO as an electron selective layer,"J. Phys. D: Appl. Phys., vol. 51, 395301, 2021.

[5] F. A. Lindholm, A. Neugroschel, C. T. Sah, M. P. Godlewski, H. W. Brandhorst, "A methodology for experimentally based determination of gap shrinkage and effective lifetimes in the emitter and base of p-n junction solar cells and other p-n junction devices, "IEEE Trans. Electron Devices ED, vol. 24, pp. 402-410, 1977.

[6] W.Shockley and H. J. Queisser, "Detailed balace limit of efficiency of p-n junction solar cells," J. Appl. Phys., vol. 32, pp. 510-519, 1961.

[7] M.A. Shibib, F.A. Lindholm, and F. Therez, "Heavily doped transparent-emitter region in junction solar cells, diodes, and transistors," IEEE Trans. Electron Devices 1979, vol. ED-26, pp. 959-965, 1979.

[8] C. Kittel, "Introduction to Solid State Physics, pp. 84-100. Wiley, New York (1976).

[9] R.A. Logan, J.F. Gilbert, and F.A. Trumbore, "Electron mobilities and tunneling currents in silicon," J. Appl. Phys., vol. 32, pp. 131-132, 1961.

[10] J. del Alamo, S. Swirhum, and R.M. Swanson, "Measuring and modeling minority carrier transport in heavily doped silicon," Solid-State Electron., vol. 28, pp. 47-54, 1985.

[11] D. Chattopadhyay, and H.J. Queisser, "Electron scattering by ionized impurities in semiconductors," Rev. Mod. Phys., vol. 53, pp. 745-768, 1981.

[12] J. del Alamo and R.M. Swanson, "Modeling of minority-carrier transport in heavily doped silicon emitters. Solid-State Electron., vol. 30, pp. 1127-1136, 1987.

[13] Z. Essa et al., "Doping profile measurement on textured silicon surface," EPJ Photovoltaics, vol. 9, p.5, 2018.

[14] S.C. Jain, E.L. Heasell, and D.J. Roulston, “Recent advances in the physics of silicon p-n junction solar cells including their transient response,” Prog. Quant. Electron., vol. 11, pp.105-204, 1987.

[15] S.C. Jain and D.J. Roulston,” A simple expression for band gap narrowing in heavily doped Si, Ge, GaAs and Ge_xSi_{1-x} strained layers. Solid-State Electron., vol. 34, pp. 453-465 (1991).

[16] D.B.M. Klaassen, J.W. Slotboom, and H.C. de Graaff, “Unified apparent band gap narrowing in n- and p-type silicon. Solid-State Electron. 1992, vol. 35, pp. 125-129, 1992.

[17] A. Zouari and A.B. Arab, “A simple formulation of the saturation current density in heavily doped emitters,” Can. J. Phys., vol. 81, pp. 1109-1120, 2003.

[18] J. W. Slotboom and H.C. de Graaff, “Measurements of band gap narrowing in Si bipolar transistors. Solid-State Electron,” vol. 19, pp. 857-862, 1976.

[19] M. A. Green, “Solar cell fill factors: general graph and empirical expressions. Solid-State Electron,” 1981, vol. 24, pp. 788-78, 1971.

[20] R.M. Swanson and R.A. Sinton, “Advances in Solar Energy,” edited by K. A. Bouer , American Solar Energy, Newark, Delaware, 1990.

[21] H. Van Cong, and S. Brunet, “Effective drift current densities in the n-type heavily doped emitter region of $p-n^+$ junction silicon solar cells. Solar Cells,” vol. 5, pp. 355-365, 1982.

[22] H. Van Cong, “A simple accurate solution to minority electron injection in the p-type heavily doped emitter region of silicon devices,” Physica Status Solidi A, vol. 149, pp. 619-628, 1995; H. Van Cong and G. Debiais, “About a conjunction between electrical and optical phenomena in p-type heavily doped silicon at room temperature,” Physica Status Solidi B, vol. 191, pp. 161-169, 1995.

[23] K. Masuko et al., “Achievement of more than 25% conversion efficiency with crystalline silicon heterojunction solar cell. IEEE J. Photovoltaic, vol. 4, pp. 1433-143, 2014.

[24] A. Fell, et al., “Input Parameters for the simulation of silicon solar cells in 2014,” IEEE J. Photovoltaics, vol. 5, pp. 1250-1263, 2015.

[25] H. Van Cong, and G. Debiais, “Energy band structure parameters and their data, derived from the measurements of minority carrier current density in heavily doped emitters of silicon devices,” Solar Ener. Mater. and Solar Cells, vol. 45, pp. 385-399, 1997; “Apparent band-gap narrowing and its data derived from the measurements of minority-carrier current density in heavily doped emitters of silicon devices,” Physica Status Solidi A, vol. 155, pp. 547-553, 1996; H. Van Cong, “ A new solution for minority-carrier injection into the heavily doped emitter of silicon devices,” Physica Status Solidi A, vol. 171, pp. 631-64, 1999.

[26] A. Richter, M. Hermle, and S.W. Glunz, “Reassessment of the limiting efficiency for crystalline silicon solar cells,” IEEE J. Photovoltaics, vol. 3, pp. 1184-1191, 2013.

[27] R.S. Davidsen, et al., “Black silicon laser-doped selective emitter solar cell with 18.1% efficiency. Sol. Energy Mater. Sol. Cells,” vol. 144, pp. 740-747, 2016.

[28] C. Battaglia, A. Cuevas, and S. de Wolf, “High-efficiency crystalline silicon solar cells: status and perspectives,” Energy Environ. Sci., vol. 9, pp. 1552-1576, 2016.

[29] M.A. Green, et al., “Solar cell efficiency tables (version 51),” Prog. Photovolt. Res. Appl., vol. 26, pp. 3-12, 2018.

[30] J.E. Lang, F.L. Madarasz, and P.M. Hemenger, “Temperature dependent density of states effective mass in non-parabolic p-type silicon,” J. Appl. Phys., vol. 54, pp. 3612-3612, 1983.

[31] M.A. Green, “Intrinsic concentration, effective densities of states, and effective mass in silicon,” J. Appl. Phys., vol. 67, pp. 2944-2954, 1990.

[32] H. Van Cong, “Band gap changes in excited intrinsic (heavily doped) Si and Ge semiconductors,” Physica B, vol. 405, pp. 1139-1149, 2010.

[33] R. Pässler, “Dispersion-related description of temperature dependencies of band gaps in semiconductors,” Phys. Rev. B, vol. 66, p. 085201, 2002.

[34] R. Pässler, “Semi-empirical descriptions of temperature dependences of band gaps in semiconductors,” Physica Status Solidi B, vol. 236, pp. 710-728, 2003.

[35] O. Henri-Rousseau, and P. Blaise, “Quantum Oscillators,” edited by John Wiley & Sons, Inc., Hoboken, New Jersey, 2011.

[36] A.B. Sproul, and M.A. Green, “Improved value for the silicon intrinsic carrier concentration from 275 to 375 K,” J. Appl. Phys., vol. 70, pp. 846-854, 1991.

[37] K. Misiakos, and D. Tsamakis, “Accurate measurements of the silicon intrinsic carrier density from 77 to 340 K,” J. Appl. Phys., vol. 74, pp. 3293-3297, 1993.

[38] R. Couderc, M. Amara, and M. Lemiti, “Reassessment of the intrinsic carrier density temperature dependence in crystalline silicon,” J. Appl. Phys., vol. 115, p. 093705, 2014.

[39] H. Van Cong, and G. Debiais, “ A simple accurate expression of the reduced Fermi energy for any reduced carrier density. J. Appl. Phys., vol. 73, pp. 1545-15463, 1993.

[40] H. Van Cong, and B. Doan Khanh, “Simple accurate general expression of the Fermi-Dirac integral $F_j(a)$ and for j> -1,” Solid-State Electron., vol. 35, pp. 949-951, 1992; H. Van Cong, “New series representation of Fermi-Dirac integral $F_j(-\infty < a < \infty)$ for arbitrary j> -1, and its effect on $F_j(a \geq 0_+)$ for integer j≥ 0,” Solid-State Electron., vol. 34, pp. 489-492, 1991.

[41] H. Van Cong, S. Abide, B. Zeghmati, and X. Chesneau, “Optical band gap in various impurity-Si systems from the metal-insulator transition study,” Physica B, vol. 436, pp. 130-139, 2014.

[42] H. Van Cong et al., “Size effect on different impurity levels in semiconductors,” Solid State Communications, vol. 49, pp. 697-699, 1984; H. Van Cong, “Effects of impurity size and heavy doping on energy-band-structure parameters of various impurity-Si systems,” Physica B, vol. 487, pp. 90-101, 2016.

[43] H. Van Cong, “Effects of donor size and heavy doping on optical, electrical and thermoelectric properties of various degenerate donor-silicon systems at low temperatures,” American Journal of Modern Physics, vol. 7, pp. 136-16, 2018.

[44] J. Wagner, and J.A. del Alamo, “Band-gap narrowing in heavily doped silicon: A comparison of optical and electrical data,” J. Appl. Phys., vol. 63, pp. 425-429, 1988.

[45] H. Van Cong, “Fermi energy and band-tail parameters in heavily doped semiconductors,” J. Phys. Chem. Solids, vol. 36, pp. 1237-1240, 1975.