



## Investigation Of Sub harmonic Oscillations In Ferroresonance Chains With Obnem Magnetopodes

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### Abstract

This article considers the mode of excitation of sub harmonic oscillations of the third order in a three-phase circuit consisting of active, capacitive and inductive elements having a common magnetic bond, which are analogous to the power line "line - unloaded transformer". Equations of motion are derived from the method of averaging with the corresponding phases. From the condition of the existence of the periodic solution, phase relations are determined that are different from the phase ratios for three-phase circuits with group Ferro magnetic elements. In the stationary mode, the conditions of excitation, the scope of existence, the dependence of the output values on the parameters of the circuit and the applied effect are determined.

**Keywords:** ferroresonance, self-oscillation, subharmonic, approximation, lowest harmonic, small parameters, ferromagnetic element

## Introduction

Currently, in various branches of the electric power industry: automation devices, telemechanics, information and measuring equipment, pre-transcending equipment and others, various kinds of energy converters, phase numbers and frequencies based on self-oscillating phenomena in three-phase nonlinear circuits with concentrated parameters are widely used.

These devices, unlike existing energy converters, made mainly on the basis of single-phase nonlinear circuits, having phase-discrete features, are reliable and easy to operate. These qualities are especially clearly revealed when using three-phase circuits in the mode of excitation of subharmonic oscillations in combination with semiconductors incompletely controlled devices. The excitation of subharmonic oscillations and the control of the phases of the resulting oscillations with the help of these devices make it possible to create various kinds of alternating current switching devices that differ fundamentally from the existing ones by the phase-sensitivity of the equilibrium state [1,2,3].

On the other hand, three-phase nonlinear systems are to some extent schemes for replacing a power line, the main elements of which are: capacitors of longitudinal compensation, transverse compensation reactors, increasing and lowering transformers with nonlinear characteristics. In fact, three-phase self-oscillating circuits are physical models of power lines. [1,4,6].

The study of physical models, the development of methods for calculating internal overvoltages make it possible to determine the basic patterns of overvoltages of power lines of partial and full compensation.

One of the special overvoltages is the excitation of subharmonic oscillations in the power line that occur after a short-circuit shutdown behind the longitudinal compensation capacitor. Exceeding several times the nominal values of the voltage of the equipment, the resulting fluctuations lead to various kinds of accidents of consumers and false operation of relay protection and automation [1,2,4,6,7,8,9]

Analysis of the conditions of excitation of subharmonic modes of three-phase nonlinear systems, depending on the parameters of the circuit, and the applied effect, allows to identify the main patterns of overvoltages in power lines and possible measures to prevent or reduce them to permissible values.

## 1. Conclusion at the alignment of motion

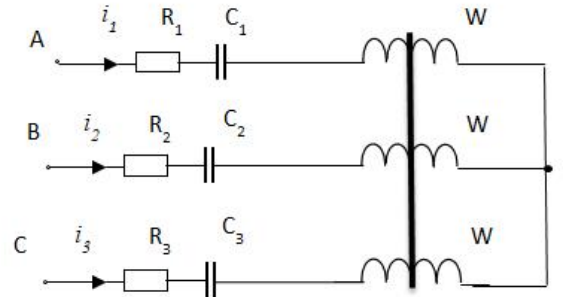
The three-phase circuit to be observed, each phase of which consists of a series-connected three-phase ferromagnetic element, capacitance and active resistance, is presented in Fig. 1. The processes in such a system are described by the following Integra-differential equations in matrix form.

$$U = Ri + D^* \int i dt + WSA^* \dot{B} \quad (1)$$

Where:

$$R = \begin{bmatrix} R_1 & -R_2 & 0 \\ 0 & R_2 & -R_3 \\ -R_1 & 0 & R_3 \end{bmatrix}; \quad D^* = \begin{bmatrix} C_1^{-1} & -C_2^{-1} & 0 \\ 0 & C_2^{-1} & -C_3^{-1} \\ C_1^{-1} & 0 & C_3^{-1} \end{bmatrix};$$

$$A^* = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} -$$



**Fig.1.** Three-phase ferroresonance circuits with common magnetic inputs

-square special matrices of the strand  $\lambda$  ( $\lambda$ -number of phases);

$u, i, \int i dt, \dot{B}$  -columnmatrices of instantaneous values of linear stresses of symmetric source of three-phase voltage, currents of all branches, integrals of current of all phases and derivative of magnetic induction of each rod of three-phase ferromagnetic element;

$R_1, R_2, R_3, C_1, C_2, C_3$  -Active resistances and capacitances according to the phases.

Since a three-phase unloaded transformer is considered as a ferromagnetic element, the current of each of its windings of high voltage and magnetic strength about the field of the rods are connected by the following relations:

$$A^*Wi = A^*\ell h \quad (2)$$

where,  $h, \ell$  -lies the column matrix of instantaneous values of the magnetic field strength of the rods of the three-phase Ferro of the magnetic element and its diagonal matrix of the corresponding average lengths of the rods;  $W$ - is the number of turns of the windings of each rod.

By introducing generally accepted restrictions, i.e., taking the core material of the ferromagnetic elements isotropic; neglecting the active resistance of the windings, the dependence of the magnetic field strength on induction is approximated by an incomplete polynomial.

$$H = \alpha B + \beta B^3 \quad (3)$$

Where:  $\alpha = \text{diag}[\alpha_\lambda]$ ,  $\beta = \text{diag}[\beta_\lambda]$  – diagonal matrices of coefficients of the approximating function,  $B^3$  – column matrix of induction to the cubic degree.

Expressing the linear voltages of the power supply through the phase ones and introducing the notation  $R_1 = nR_2 = mR_3$ ,  $C_1 = pC_2 = qC_3$ , and then performing elementary transformations over the parameter matrices ( $r$ ,  $D^*$ ) of the original equation (1), similarly to the transformation of the matrix  $A^*$ , we write the equation of motion (1) as

$$-3u = R_1(A + D)\dot{i} + \frac{1}{C_1}(A + E) \int i dt + w S(A + B^*)\dot{B} \quad (4)$$

Where,  $u$  is the column matrix of instantaneous values of phase stresses of a symmetric three-phase voltage source;

$D, E$  – Square matrices, the elements of which are coefficients that take into account the ratios of the electrical parameters of the system:

$$D = \begin{bmatrix} -3 & (p-1) & (q-1) \\ 0 & -(2p+1) & (q-1) \\ 0 & (p-1) & -(2q+1) \end{bmatrix}; \quad E = \begin{bmatrix} -3 & -(n-1)/n & -(m-1)/m \\ 0 & -(n+2)/n & -(m-1)/m \\ 0 & -(n-1)/n & -(m+2)/m \end{bmatrix}$$

The expression (3) is substituted in (4) and after some mathematical transformations passing to the new time  $\tau = \frac{\omega t}{\chi}$ , where,  $\chi$  is the order of sub harmonic oscillations, we obtain a system of nonlinear differential equations in the matrix form of the following form:

$$\ddot{B} + \frac{\chi^2 \alpha \alpha}{w^2 C_1} D' B + \frac{\chi^2 \beta \alpha}{w^2 C_1} (D' + D'') B^3 + \frac{\chi r \alpha \alpha}{w} E' \dot{B} + \frac{\chi r \beta \alpha}{w} (3E' + E'') B^2 \dot{B} = d \chi \dot{u} \quad (5)$$

where:

$D' = B^{*-1} D C$ ;  $D'' = -\frac{1}{3} B^{*-1} D A$ ;  $E' = B^{*-1} E C$ ;  $E'' = -B^{*-1} E A$  – square matrices of order  $\lambda$ .

$$\alpha = l_1 / W^2 S; d = 1 / W S \omega$$

The last equation is the equation of motion of the three-phase system, presented in Fig. 1. In the special case, this equation easily turns into the equation of a symmetric three-phase system with group ferromagnetic elements  $K = n = m = q = p = 1$  [1,2]. In addition, the resulting equation (5) allows you to write the equations for each phase separately, containing terms with second derivatives of the induction of the corresponding phases and the first derivatives of the induction of all phases.

The above algorithm for deriving the equation of motion makes it possible to obtain equations of a kind in the "standard" form, which are not subject to any preliminary transformation when applying the averaging method.

## 2. Analysis of stationary mode and conditions of existence of sub harmonic oscillations

The solution of the equation of motion (5) of the system, in the mode of excitation of sub-harmonic oscillations of the third order ( $\chi = 3$ ), we will look for in the form of:

$$B_\lambda = B_{\lambda m} \cos[3\tau - \frac{2\pi}{3}(\lambda - 1)] + B_{\lambda 3m} \cos(\tau + \varphi_{\lambda 3}) + \mu w W_\lambda(\tau) \quad (6)$$

Where:  $B_{\lambda m} B_{\lambda 3m} \varphi_{\lambda 3}$  – slow-changing amplitudes and phases of induction of the main and sub-harmonic components of oscillations;  $\mu W_\lambda(\tau)$  – amendment to decisions.

The desired solution will obviously be of the form (6) in the event that the function remains small for a sufficiently long period of time. The periodic decision condition for an amendment imposes the following restrictions on it  $\omega_\lambda(\tau) \mu W_\lambda(\tau)$

$$\frac{1}{2\pi} \int_0^{2\pi} H_\lambda(\tau) \rightarrow \cos 3\tau d\tau = 0, \sin 3\tau d\tau = 0 \quad (7)$$

$$\frac{1}{2\pi} \int_0^{2\pi} H_\lambda(\tau) \rightarrow \cos \tau d\tau = 0, \sin \tau d\tau = 0 \quad (8)$$

Integrating equations (5) taking into account the ratios (6) can be brought to the form:

$$\dot{\varphi}_{\lambda 3} = \frac{1}{2} \varepsilon_{0\lambda} + \frac{9\beta\alpha}{2w^2C} [Q_\lambda B_{\lambda m}^2 + P_\lambda B_{\lambda 3m}^2 - (M_\lambda \cos 3\alpha_{\lambda 3} - \Pi_\lambda \sin 3\alpha_{\lambda 3}) B_{\lambda m} B_{\lambda 3m}] \quad (9)$$

$$\dot{B}_{\lambda 3} = -\frac{1}{2} \delta_{0\lambda} B_{\lambda 3m} - \frac{9\beta\alpha}{2w^2C} (M_\lambda \sin 3\alpha_{\lambda 3} + \Pi_\lambda \cos 3\alpha_{\lambda 3}) B_{\lambda m} B_{\lambda 3m},$$

Where,  $\varepsilon_{0\lambda} \delta_{0\lambda}$  – coefficients that take into account the races and losses in the system.

Equating the left parts of equations (9) to zero and squared, we obtain the following algebraic equations that allow us to determine in stationary mode the amplitude-frequency, input-output characteristics of the sub harmonic mode of the third order of the original nonlinear system (5).

$$Q_\lambda^2 X_\lambda^2 + P_\lambda^2 Y_\lambda^2 + [2Q_\lambda P_\lambda - (M_\lambda^2 \Pi_\lambda^2)] X_\lambda Y_\lambda + 2Q_\lambda \varepsilon_{0\lambda} \frac{w^2 C_1}{9\beta\alpha} X_\lambda + 2P_\lambda \varepsilon_{0\lambda} \frac{w^2 C_1}{9\beta\alpha} Y_\lambda + (\frac{w^2 C_1}{9\beta\alpha})^2 (\varepsilon_{0\lambda}^2 + \delta_{0\lambda}^2) = 0, \quad (10)$$

$$\text{Where: } X_\lambda = B_{\lambda}^2; Y_\lambda = B_{\lambda 3}^2$$

These equations describe second-order curves [1,2,10], whose invariants are defined by the following expressions:

$$\delta_\lambda = (M_\lambda^2 + \Pi_\lambda^2) Q_\lambda P_\lambda - (\frac{M_\lambda^2 + \Pi_\lambda^2}{2})^2;$$

$$N_\lambda = Q_\lambda^2 P_\lambda^2 ; \quad (11)$$

$$\Delta_\lambda = \left(\frac{w^2 C_1}{9\beta\alpha}\right)^2 [\delta_\lambda \delta_{0\lambda}^2 - \left(\frac{M_\lambda^2 + \Pi_\lambda^2}{2}\right)^2 \varepsilon_{0\lambda}^2]$$

The variables X and Y are squares of the amplitudes of the main and sub harmonic components. For the existence of a sub harmonic mode in the system, it is necessary that the curve describing the mode in question be in the first square of the X0U plane. If the conditions are met

$$\Delta_\lambda < 0 \quad (12)$$

Equation (11) describes the real ellipses (Fig. 2) located in the first square, the coordinates of the centers and lengths of the semi-axes of which are determined by the expressions:

$$X_{0\lambda} = -\frac{m_\lambda w^2 C_1}{2\delta_\lambda 9\beta\alpha} Q_\lambda(k) \varepsilon_{0\lambda} > 0; Y_{0\lambda} = -\frac{m_\lambda w^2 C_1}{2\delta_\lambda 9\beta\alpha} P_\lambda(k) \varepsilon_{0\lambda} > 0 ;$$

$$\alpha_\lambda = \frac{w^2 C_1}{9\beta\alpha} \sqrt{\frac{m_\lambda^2 + \varepsilon_\lambda^2}{2\delta_\lambda(N_\lambda + \sqrt{N_\lambda^2 - 4\delta_\lambda})}} > 0; b_\lambda = \frac{w^2 C_1}{9\beta\alpha} \sqrt{\frac{m_\lambda^2 - 4\delta_\lambda \delta_\lambda^2}{2\delta_\lambda(N_\lambda - \sqrt{N_\lambda^2 - 4\delta_\lambda})}} > 0; \quad (13)$$

Where:  $m_\lambda = M_\lambda^2 + \Pi_\lambda^2$

The semi-axes of the ellipses are turned relative to the axes of the coordinates at the appropriate angles. These angles depend only on the ratios of the average lengths (k) of the magnetic cores of the ferromagnetic element.

It follows from (13) that for the existence of a sub harmonic mode in the system, it is necessary that the coordinates of the centers and the length of the semi-axes of the ellipses be greater than zero [10], sub harmonic oscillations will exist under the condition:

a) Mode "A".

$$\varepsilon_{01} = -1 + \frac{9\alpha a}{w^2 C_1} \left(\frac{13+3.5k}{18}\right) < 0;$$

$$\varepsilon_{02} = -1 + \frac{9\alpha a}{w^2 C_1} \left(\frac{5+5.5k}{9}\right) < 0;$$

$$\varepsilon_{03} = -1 + \frac{9\alpha a}{w^2 C_1} \left(\frac{8-2k}{9}\right) < 0;$$

b) Mode "B".

$$\varepsilon_{01} = \varepsilon_{02} = \varepsilon_{03} = -1 + \frac{9\alpha a}{w^2 C_1} \left(\frac{1+2k}{3}\right) < 0; \quad (14)$$

c) Mode "C".

$$\varepsilon_{01} = -1 + \frac{9\alpha a}{w^2 C_1} \left(\frac{16-4k}{9}\right) < 0;$$

$$\varepsilon_{02} = -1 + \frac{9\alpha a}{w^2 C_1} \left(\frac{10+11k}{9}\right) < 0;$$

$$\varepsilon_{03} = -1 + \frac{9\alpha a}{w^2 C_1} \left(\frac{26-7k}{18}\right) < 0;$$

And for all modes and expressions (13, 14.) you can see:

$$\delta_{0\lambda} < \frac{m_\lambda}{2\sqrt{\delta_\lambda}} |\varepsilon_{0\lambda}| \quad (15)$$

From where it should come from:

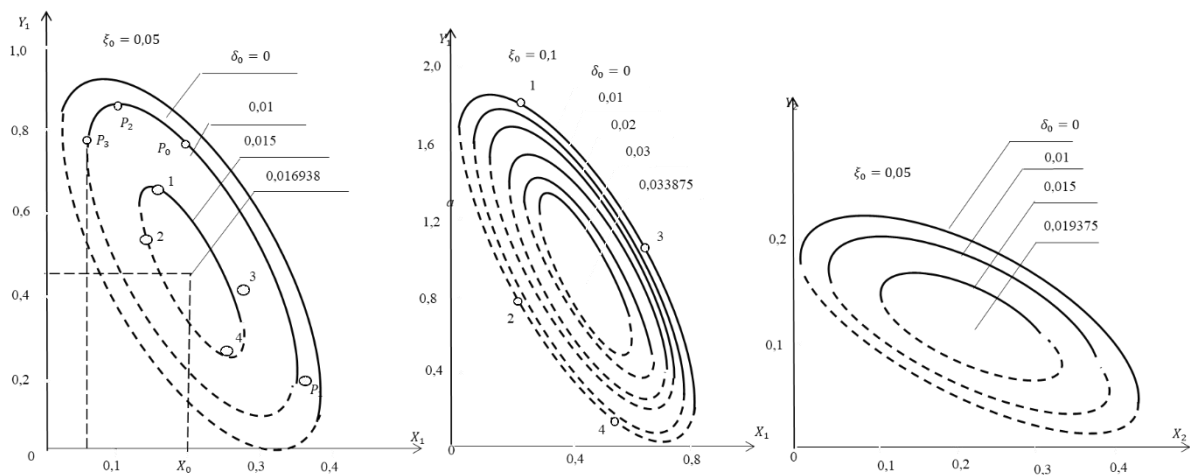
-sub harmonic oscillations exist at negative values of the resistance, i.e., provided that the characteristics of the linear capacitive resistances in phases at the sub harmony frequency cross the volt-ampere characteristic of the ferromagnetic element at the same frequency;

-Sub harmonic oscillations exist only at certain dissipation values;

-At dissipation values above critical, defined expressions (14), there are no sub harmonic fluctuations for any applied effect.

On Fig. 2 shows in general the calculated ratios of solutions (10), in particular, for mode "A" at the values of the system parameters corresponding to the conditions of existence (14), (15) of sub harmonic oscillations.

In all three phases, sub harmonic oscillations are excited simultaneously at a certain value of the input effect. With an increase in the input effect, the amplitudes of the oscillations that have arisen decrease, and at a certain value there is a failure of oscillations.



**Fig.2.** Input-output characteristics of the third order sub harmonic oscillations depending on the change in tuning at  $U=80B$ ,  $C=25 \mu f$ ,  $R=50m$ ,  $\alpha = 1.2$  and  $\beta = 0.4$

The capture area of sub harmonic oscillations at different values of the circuit parameters is different. In particular, with an increase in the upset area increases, and with an increase in dissipation, it decreases (Fig. 2)

## Conclusion

1. The proposed algorithm for the formation of the equation of motion of the process in three-phase nonlinear circuits with common ferromagnetic elements, in contrast to the forming the equation using only independent equations, allows us to obtain equations of motion in standard form without any preliminary transformations.

2. Sub harmonic oscillations  $\frac{\omega}{2}$  in three-phase nonlinear circuits with three-phase ferromagnetic elements, as in other circuits with energy-intensive elements, are excited "rigidly", but unlike three-phase circuits with group ferromagnetic elements, oscillations in all phases are excited simultaneously.

3. With the same values of capacitance and active resistance in phases, the area of existence of sub harmonic oscillations on the input effect will be greater in circuits with common ferromagnetic elements than in three-phase circuits with group ones.

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