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# Parameter Measurement of Four-circuit AC Lines on Same Tower 

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#### Abstract

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The paper presents a method to measure the parameters of four circuits AC lines on the same tower. At first, the positive sequence open circuit impedance and short circuit impedance are both measured for any one of four lines. Secondly, four cases of 2-phase systems are assembled by four lines on the same tower, and then the open circuit impedance and short circuit impedance of 2-phase systems are measured for each case. Based on open circuit impedance and short circuit impedance measurements above, the distributed impedance and admittance for each case are accordingly estimated. Finally, the mutual inductance and coupling capacitance, as well as self-parameters of each line can be calculated.


Key words: transmission line; parameter measurement of AC line, four circuit AC line on same tower, telegraph equation

## 0 Introduction

Four circuit AC transmission lines on same tower have an advantage to narrow corridors, and now are popular on the world. To ensure the normal operation of the lines, the calculation of protective settings, secondary arc current and over voltage is necessary, but the calculation needs accurate parameters. The parameters in four lines on same tower are quite complex. Despite of resistance, inductance and capacitance to ground in each phase of a 3-phase line, there are also mutual inductance and coupling capacitance between any two phases of a 3-phase line. Besides, there is also mutual inductance and coupling capacitance between any two of four lines, they are emerged during the zero sequence current through the lines.

There have been some references discussed the parameter measurement of multi-lines on same tower. Reference [1] developed the theory and method based on Laplace transform to measure the parameters in four AC lines on same tower, but the theory and the measurement procedure is too sophisticated. Reference [2] gave only a model for simulation. Reference [3] and [4] introduced the online measurement of zero sequence parameters among multi-lines. The key procedure was artificially to make an open phase operation for a short interval at one of the line. Meanwhile, the 3-phase voltage and current waveforms on each terminal of multi-lines were recorded, and then the zero sequence parameters were analyzed. In order to maintain
the safety of the system, the power dispatching center would not agree to do so.

Reference [5] was the primary discussion on the zero sequence parameter measurement of four AC lines by the author. It looked a 3-phase line as a bundled conductor, and then configured four lines as a type of zero sequence circuit, as well as three types of 2-phase circuits. By open circuit impedance and short circuit impedance measurements on above four types of circuits, all the mutual inductance and coupling capacitance between any two of four lines could be calculated, but the parameter measurement by this way could not distinguish mutual inductance and coupling capacitance between phases inside a 3-phase line.

As known to all, the power system fault analysis was mainly based on decoupled sequence components of 3-phase system. The typical and frequently occurred fault on four-circuits of lines was mainly on a single phase, and any fault on a single phase would induce a zero sequence voltage and current on its own 3-phase line, but also the zero sequence voltages and currents on other perfect 3-phase lines. The zero sequence current through a line might trig a protective relay and trip the corresponding circuit.

Based on the distributed model for single conductor ground return, the paper developed an algorithm and method to measure the parameters of four circuit AC lines on same tower, in-
cluding the mutual inductance and coupling capacitance between the phases inside a 3-phase line. By getting the open circuit impedance and short circuit impedance measurements of a 3-phase line in positive sequence configuration, then the open circuit impedance and short circuit impedance measurements from a zero sequence circuit configured by four 3-phase lines, also the positive sequence open circuit impedance and short circuit impedance measurements from three types of 2-phase systems configured by four 3-phase lines, all the parameters in four lines on same tower can be estimated

## 1 Telegraph equations for four circuit AC lines on same tower

Figure 1 is a typical arrangement for four circuit AC lines which are identified by I to IV and are set on same tower.


Fig. 1 Four circuit AC lines on same tower It is supposed that
(1)All the distributed self-impedance $z=r+\mathrm{j} \omega l$ and self-admittance $y=g+\mathrm{j} \omega c_{0}$ for each phase of conductors are the same;
(2)The 3 phase wires of each line are fully transposition, so the mutual inductance $m_{\mathrm{p}}$ and coupling capacitance $c_{\mathrm{p}}$ between phases inside a 3-phase line are same;
(3) The four lines are rectangle arranged and are symmetrical to the tower, and the space between lines I and II equals to the space between lines III and IV. So, the mutual inductance and couple capacitance between one phase of a 3-phase line and one phase of another 3-phase line meet following equations.

For the horizontal lines, there are

$$
\begin{align*}
& c_{\mathrm{IIII}}=c_{\mathrm{III-IV}}=c_{\mathrm{h}}  \tag{1}\\
& m_{\mathrm{IIII}}=m_{\mathrm{III-IV}}=m_{\mathrm{h}} \tag{2}
\end{align*}
$$

For the vertical lines, there are

$$
\begin{align*}
& c_{\mathrm{I}-\mathrm{III}}=c_{\mathrm{II} \mathrm{IV}}=c_{\mathrm{v}}  \tag{3}\\
& m_{\mathrm{I}-\mathrm{III}}=m_{\mathrm{II-VV}}=m_{\mathrm{v}} \tag{4}
\end{align*}
$$

For the lines between diagonal positions, there are

$$
\begin{align*}
& c_{\mathrm{IIV}}=c_{\mathrm{II}-\mathrm{III}}=c_{\mathrm{d}}  \tag{5}\\
& m_{\mathrm{IIVV}}=m_{\mathrm{II}-\mathrm{III}}=m_{\mathrm{d}} \tag{6}
\end{align*}
$$

Figure 2 illustrates the above parameters amongst phases of different lines. For example, the parameters which are relative to the phase ' a ' of line I are the self-impedance $z=r+\mathrm{j} \omega l$ and self-admittance $y=g+\mathrm{j} \omega c_{0}$, the mutual inductance $m_{\mathrm{h}}$ and coupling capacitance $c_{\mathrm{h}}$ with each phase ' $a$ ', ' $b$ ', ' $c$ ' of line II, the mutual inductance $m_{v}$ and coupling capacitance $c_{v}$ with each phase ' $a$ ', ' $b$ ', ' $c$ ' of line III, as well as the mutual inductance $m_{\mathrm{d}}$ and coupling capacitance $c_{\mathrm{d}}$ with each phase ' a ', ' b ', ' c ' of line IV. Besides, there are mutual inductance $m_{\mathrm{d}}$ and coupling capacitance $c_{\mathrm{d}}$ between phases inside line I.


Fig. 2 Coupling parameters among single phases of four lines

If the currents in each phase of four lines though a section are

$$
\dot{I}_{\mathrm{I}, \mathrm{a}}, \dot{I}_{\mathrm{I}, \mathrm{~b}}, \dot{\mathrm{I}}_{\mathrm{I}, \mathrm{c}} ; \dot{I}_{\mathrm{II} \mathrm{a}}, \dot{I}_{\mathrm{II}, \mathrm{~b}}, \dot{I}_{\mathrm{II}, \mathrm{c}} ; \dot{I}_{\mathrm{III}, \mathrm{a}}, \dot{I I I I L, ~}, \dot{I}_{\mathrm{III}, \mathrm{c}} ;
$$

$\dot{I}_{\mathrm{IV}, \mathrm{a}}, \dot{I}_{\mathrm{IV}, \mathrm{b}}, \dot{I}_{\mathrm{IV}, \mathrm{c}}$, respectively, then the ground return current is

$$
\begin{align*}
\dot{\mathrm{I}}_{g}= & \dot{I}_{\mathrm{I}, \mathrm{a}}+\dot{I}_{\mathrm{I}, \mathrm{~b}}+\dot{I}_{\mathrm{I}, \mathrm{c}}+\dot{I}_{\mathrm{II}, \mathrm{a}}+\dot{I}_{\mathrm{II}, \mathrm{~b}}+\dot{I}_{\mathrm{II}, \mathrm{c}} \\
& +\dot{I}_{\mathrm{III}, \mathrm{a}}+\dot{I}_{\mathrm{III}, \mathrm{~b}}+\dot{I}_{\mathrm{III}, \mathrm{c}}+\dot{I}_{\mathrm{IV}, \mathrm{a}}+\dot{I}_{\mathrm{IV}, \mathrm{~b}}+\dot{I}_{\mathrm{IV}, \mathrm{c}} \tag{7}
\end{align*}
$$

Referred to figure 2, the voltage increments on each 3-phase line can be written as
$\frac{\mathrm{d}}{\mathrm{d} x} \dot{\mathbf{U}}_{\mathrm{I}, \mathrm{abc}}=Z_{\mathrm{I}, \mathrm{abc}} \dot{\mathbf{I}}_{\mathrm{I}, \mathrm{abc}}+Z_{\mathrm{h}, \mathrm{abc}} \dot{\mathbf{I}}_{\mathrm{II}, \mathrm{abc}}+Z_{\mathrm{V}, \mathrm{abc}} \dot{\mathbf{I}}_{\mathrm{IIIabc}}+Z_{\mathrm{d}, \mathrm{abc}} \dot{\mathbf{I}}_{\mathrm{IV}, \mathrm{abc}}$ $\frac{\mathrm{d}}{\mathrm{d} x} \dot{\mathbf{U}}_{\mathrm{II}, \mathrm{abc}}=Z_{\mathrm{h}, \mathrm{ab}} \dot{\mathbf{I}}_{\mathrm{I}, \mathrm{abc}}+Z_{\mathrm{l}, \mathrm{ab}} \dot{\mathbf{c}}_{\mathrm{II}, \mathrm{abc}}+Z_{\mathrm{d}, \mathrm{abc}} \dot{\mathbf{I}}_{\mathrm{III}, \mathrm{abc}}+Z_{\mathrm{V}, \mathrm{abc}} \dot{\mathbf{I}}_{\mathrm{IV}, \mathrm{abc}}$ $\frac{\mathrm{d}}{\mathrm{d} x} \dot{\mathbf{U}}_{\text {III,abc }}=Z_{\mathrm{v}, \mathrm{abc}} \dot{\mathbf{I}}_{\text {labc }}+Z_{\mathrm{d}, \text { abc }} \dot{\mathbf{I}}_{\mathrm{II}, \mathrm{abc}}+Z_{\mathrm{labac}} \dot{\mathbf{I}}_{\text {III,abc }}+Z_{\mathrm{h}, \mathrm{abc}} \dot{\mathbf{I}}_{\mathrm{IV}, \mathrm{abc}}$ $\frac{\mathrm{d}}{\mathrm{d} x} \dot{\mathbf{U}}_{\mathrm{VV}, \mathrm{abc}}=Z_{\mathrm{d}, \mathrm{abc}} \dot{\mathrm{I}}_{\text {Iabc }}+Z_{\mathrm{v}, \mathrm{abc}} \dot{\mathbf{I}}_{\mathrm{II}, \mathrm{abc}}+Z_{\mathrm{h}, \mathrm{abc}} \dot{\mathbf{I}}_{\mathrm{IIIabc}}+Z_{\mathrm{l}, \mathrm{abc}} \dot{\mathbf{I}}_{\mathrm{IV}, \mathrm{abc}}$

And the current increments in each 3-phase line is
$\frac{\mathrm{d}}{\mathrm{d} x} \dot{\mathbf{I}}_{\mathrm{I}, \text { abc }}=\mathrm{Y}_{\mathrm{I}, \mathrm{abc}} \dot{\mathbf{U}}_{\mathrm{I}, \mathrm{abc}}-\mathrm{Y}_{\mathrm{h}, \text { abc }} \dot{\mathbf{U}}_{\mathrm{II}, \mathrm{abc}}-\mathrm{Y}_{\mathrm{v}, \text { abc }} \dot{\mathbf{U}}_{\mathrm{III}, \mathrm{abc}}-\mathrm{Y}_{\mathrm{d}, \mathrm{abc}} \dot{\mathbf{U}}_{\mathrm{IV}, \mathrm{abc}}$
$\frac{\mathrm{d}}{\mathrm{d} x} \dot{\mathrm{H}}_{\text {IIabc }}=-\mathrm{Y}_{\mathrm{h}, \text { abc }} \dot{\mathbf{U}}_{\mathrm{I}, \text { abc }}+\mathrm{Y}_{\mathrm{I}, \text { abc }} \dot{\mathbf{U}}_{\mathrm{II}, \text { abc }}-\mathrm{Y}_{\mathrm{d} \text { dabc }} \dot{\mathrm{U}}_{\text {III,abc }}-\mathrm{Y}_{\mathrm{V}, \text { abc }} \dot{\mathrm{U}}_{\mathrm{IV}, \mathrm{abc}}$
$\frac{\mathrm{d}}{\mathrm{d} x} \dot{\mathrm{I}}_{\mathrm{II}, \mathrm{abc}}=-\mathrm{Y}_{\mathrm{V}, \mathrm{abc}} \dot{\mathbf{U}}_{\mathrm{I}, \mathrm{abc}}-\mathrm{Y}_{\mathrm{d}, \mathrm{abc}} \dot{\mathbf{U}}_{\mathrm{I}, \text { abac }}+\mathrm{Y}_{\mathrm{I}, \mathrm{abc}} \dot{\mathbf{U}}_{\mathrm{III}, \mathrm{abc}}-\mathrm{Y}_{\mathrm{D}, \mathrm{abc}} \dot{\mathbf{U}}_{\mathrm{IV}, \mathrm{abc}}$
$\frac{\mathrm{d}}{\mathrm{d} x} \dot{\mathrm{I}}_{\mathrm{IV}, \mathrm{abc}}=-\mathrm{Y}_{\mathrm{d}, \mathrm{abc}} \dot{\mathbf{U}}_{\mathrm{I}, \mathrm{abc}}-\mathrm{Y}_{\mathrm{V}, \mathrm{abc}} \dot{\mathbf{U}}_{\mathrm{II}, \mathrm{abc})}-\mathrm{Y}_{\mathrm{h}, \mathrm{abc}} \dot{\mathbf{U}}_{\mathrm{III}, \mathrm{abc}}+\mathrm{Y}_{\mathrm{I}, \mathrm{abc}} \dot{\mathbf{U}}_{\mathrm{IV}, \mathrm{abc}}$

The voltage and current vectors in above equations are respectively as
$\mathbf{U}_{\mathrm{I}, \mathrm{abc}}=\left[\begin{array}{lll}\dot{U}_{\mathrm{I}, \mathrm{a}} & \dot{U}_{\mathrm{I}, \mathrm{b}} & \dot{U}_{\mathrm{I}, \mathrm{c}}\end{array}\right]^{\mathrm{T}}, \mathbf{U}_{\mathrm{II}, \mathrm{abc}}=\left[\begin{array}{lll}\dot{U}_{\mathrm{II}, \mathrm{a}} & \dot{U}_{\mathrm{II}, \mathrm{b}} & \dot{U}_{\mathrm{II}, \mathrm{c}}\end{array}\right]^{\mathrm{T}}$
$\mathbf{U}_{\mathrm{III}, \mathrm{abc}}=\left[\begin{array}{lll}\dot{U}_{\mathrm{III}, \mathrm{a}} & \dot{U}_{\mathrm{III}, \mathrm{b}} & \dot{U}_{\mathrm{III}, \mathrm{c}}\end{array}\right]^{\mathrm{T}}$
$\mathbf{U}_{\mathrm{IV}, \mathrm{abc}}=\left[\begin{array}{lll}\dot{U}_{\mathrm{IV}, \mathrm{a}} & \dot{U}_{\mathrm{IV}, \mathrm{b}} & \dot{U}_{\mathrm{IV}, \mathrm{c}}\end{array}\right]^{\mathrm{T}}$
$\mathbf{I}_{\mathrm{I}, \mathrm{abc}}=\left[\begin{array}{lll}\dot{I}_{\mathrm{I}, \mathrm{a}} & \dot{I}_{\mathrm{I}, \mathrm{b}} & \dot{I}_{\mathrm{I}, \mathrm{c}}\end{array}\right]^{\mathrm{T}}, \quad \mathbf{I}_{\mathrm{II}, \mathrm{abc}}=\left[\begin{array}{lll}\dot{I}_{\mathrm{II}, \mathrm{a}} & \dot{I}_{\mathrm{II}, \mathrm{b}} & \dot{I}_{\mathrm{II}, \mathrm{c}}\end{array}\right]^{\mathrm{T}}$,
$\mathbf{I}_{\mathrm{III}, \mathrm{abc}}=\left[\begin{array}{lll}\dot{I}_{\mathrm{III}, \mathrm{a}} & \dot{I}_{\mathrm{III}, \mathrm{b}} & \dot{I}_{\mathrm{III}, \mathrm{c}}\end{array}\right]^{\mathrm{T}} \quad \mathbf{I}_{\mathrm{IV}, \mathrm{abc}}=\left[\begin{array}{lll}\dot{I}_{\mathrm{IV}, \mathrm{a}} & \dot{I}_{\mathrm{IV}, \mathrm{b}} & \dot{I}_{\mathrm{IV}, \mathrm{c}}\end{array}\right]^{\mathrm{T}}$

The self-impedance matrix for each line is

$$
Z_{\mathrm{l}, \mathrm{abc}}=\left[\begin{array}{ccc}
r_{\mathrm{g}}+z & r_{\mathrm{g}}+\mathrm{j} \omega m_{\mathrm{p}} & r_{\mathrm{g}}+\mathrm{j} \omega m_{\mathrm{p}}  \tag{11}\\
r_{\mathrm{g}}+\mathrm{j} \omega m_{\mathrm{p}} & r_{\mathrm{g}}+z & r_{\mathrm{g}}+\mathrm{j} \omega m_{\mathrm{p}} \\
r_{\mathrm{g}}+\mathrm{j} \omega m_{\mathrm{p}} & r_{\mathrm{g}}+\mathrm{j} \omega m_{\mathrm{p}} & r_{\mathrm{g}}+z
\end{array}\right]
$$

The mutual impedance matrix between horizontal lines is

$$
Z_{\mathrm{h}, \mathrm{abc}}=\left(r_{g}+\mathrm{j} \omega m_{\mathrm{h}}\right)\left[\begin{array}{lll}
1 & 1 & 1  \tag{12}\\
1 & 1 & 1 \\
1 & 1 & 1
\end{array}\right]
$$

The mutual impedance matrix between vertical lines is

$$
Z_{\mathrm{v}, \mathrm{abc}}=\left(r_{g}+\mathrm{j} \omega m_{\mathrm{v}}\right)\left[\begin{array}{lll}
1 & 1 & 1  \tag{13}\\
1 & 1 & 1 \\
1 & 1 & 1
\end{array}\right]
$$

The mutual impedance matrix between diagonal lines is

$$
Z_{\mathrm{dabc}}=\left(r_{g}+\mathrm{j} \omega m_{\mathrm{d}}\right)\left[\begin{array}{lll}
1 & 1 & 1  \tag{14}\\
1 & 1 & 1 \\
1 & 1 & 1
\end{array}\right]
$$

The voltage increment (8) which described in phases can be transformed to symmetrical vectors
$\frac{\mathrm{d}}{\mathrm{d} x} \dot{\mathbf{U}}_{\mathrm{l}, 120}=\mathrm{Z}_{\mathrm{l}, 120} \dot{\mathbf{I}}_{\mathrm{I}, 120}+\mathrm{Z}_{\mathrm{h}, 120} \dot{\mathbf{I}}_{\mathrm{I}, 120}+\mathrm{Z}_{\mathrm{v}, 120} \dot{\mathbf{I}}_{\mathrm{II}, 120}+\mathrm{Z}_{\mathrm{d}, 120} \dot{\mathbf{I}}_{\mathrm{V}, 120}$
$\frac{\mathrm{d}}{\mathrm{d} x} \dot{\mathbf{U}}_{\mathrm{II}, 120}=\mathrm{Z}_{\mathrm{h}, 120} \dot{\mathrm{I}}_{\mathrm{I}, 120}+\mathrm{Z}_{\mathrm{l}, 120} \dot{\mathbf{I}}_{\mathrm{II}, 120}+\mathrm{Z}_{\mathrm{d}, 120} \dot{\mathbf{I}}_{\mathrm{III}, 120}+\mathrm{Z}_{\mathrm{v}, 120} \dot{\mathbf{I}}_{\mathrm{IV}, 120}$ $\frac{\mathrm{d}}{\mathrm{d} x} \dot{\mathbf{U}}_{\mathrm{II}, 120}=\mathrm{Z}_{\mathrm{v}, 120} \dot{\mathbf{I}}_{\mathrm{I}, 120}+\mathrm{Z}_{\mathrm{d}, 120} \dot{\mathbf{I}}_{\mathrm{II}, 120}+\mathrm{Z}_{\mathrm{l}, 120} \dot{\mathbf{I}}_{\mathrm{II}, 120}+\mathrm{Z}_{\mathrm{h}, 120} \dot{\mathbf{I}}_{\mathrm{IV}, 120}$ $\frac{\mathrm{d}}{\mathrm{d} x} \dot{\mathbf{U}}_{\mathrm{IV}, 120}=\mathrm{Z}_{\mathrm{d}, 120} \dot{\mathbf{I}}_{\mathrm{I}, 120}+\mathrm{Z}_{\mathrm{v}, 120} \dot{\mathbf{I}}_{\mathrm{II}, 120}+\mathrm{Z}_{\mathrm{h}, 120} \dot{\mathbf{I}}_{\mathrm{III}, 120}+\mathrm{Z}_{\mathrm{l}, 12 \mathbf{I}} \dot{\mathbf{I}}_{\mathrm{V}, 120}$

And the current increment (9) can also be transformed to symmetrical vectors
$\frac{\mathrm{d}}{\mathrm{d} x} \dot{I}_{\mathrm{I}, 120}=\mathrm{Y}_{\mathrm{l}, 120} \dot{\mathrm{U}}_{\mathrm{I}, 120}-\mathrm{Y}_{\mathrm{h}, 120} \dot{\mathbf{U}}_{\mathrm{II}, 120}-\mathrm{Y}_{\mathrm{v}, 120} \dot{\mathbf{U}}_{\mathrm{III}, 120}-\mathrm{Y}_{\mathrm{d}, 120} \dot{\mathrm{U}}_{\mathrm{V}, 120}$
$\frac{\mathrm{d}}{\mathrm{d} x} \dot{\mathbf{I}}_{\mathrm{II}, 120}=-\mathrm{Y}_{\mathrm{h}, 120} \dot{\mathrm{U}}_{\mathrm{I}, 120}+\mathrm{Y}_{\mathrm{I}, 120} \dot{\mathbf{U}}_{\mathrm{II}, 120}-\mathrm{Y}_{\mathrm{d}, 120} \dot{\mathbf{U}}_{\mathrm{III}, 120}-\mathrm{Y}_{\mathrm{v}, 120} \dot{\mathrm{U}}_{\mathrm{IV}, 120}$
$\frac{\mathrm{d}}{\mathrm{d} x} \dot{\mathbf{I}}_{\mathrm{II}, 120}=-\mathrm{Y}_{\mathrm{v}, 120} \dot{\mathrm{U}}_{\mathrm{I}, 120}-\mathrm{Y}_{\mathrm{d}, 120} \dot{\mathbf{U}}_{\mathrm{H}, 120}+\mathrm{Y}_{\mathrm{l}, 120} \dot{\mathbf{U}}_{\mathrm{II}, 120}-\mathrm{Y}_{\mathrm{h}, 120} \dot{\mathrm{U}}_{\mathrm{V}, 120}$
$\frac{\mathrm{d}}{\mathrm{d} \mathrm{I}} \dot{\mathrm{I}}_{\mathrm{V}, 120}=-\mathrm{Y}_{\mathrm{d}, 120} \dot{\mathrm{U}}_{\mathrm{I}, 120}-\mathrm{Y}_{\mathrm{v}, 120} \dot{\mathbf{U}}_{\mathrm{H}, 120}-\mathrm{Y}_{\mathrm{h}, 120} \dot{\mathbf{U}}_{\mathrm{HII}, 120}+\mathrm{Y}_{\mathrm{l}, 120} \dot{\mathbf{U}}_{\mathrm{V}, 120}$

The voltage and current vectors of each line expressed by symmetrical components are
$\mathbf{U}_{\mathrm{I}, 120}=\left[\begin{array}{lll}\dot{U}_{\mathrm{I}, 1} & \dot{U}_{\mathrm{I}, 2} & \dot{U}_{\mathrm{I}, 0}\end{array}\right]^{\mathrm{T}}=\mathrm{T}^{-1} \mathbf{U}_{\mathrm{I}, \text { abc }}$,
$\mathbf{U}_{\mathrm{II}, 120}=\left[\begin{array}{lll}\dot{U}_{\mathrm{II}, 1} & \dot{U}_{\mathrm{II}, 2} & \dot{U}_{\mathrm{II}, 0} \mathrm{~T}^{\mathrm{T}}=\mathrm{T}^{-1} \mathbf{U}_{\mathrm{II}, \mathrm{abc}},\end{array}\right.$
$\mathbf{U}_{\mathrm{III}, 120}=\left[\begin{array}{lll}\dot{U}_{\mathrm{III}, 1} & \dot{U}_{\mathrm{III}, 2} & \left.\dot{U}_{\mathrm{II}, 0}\right]^{\mathrm{T}}=\mathrm{T}^{-1} \mathbf{U}_{\mathrm{III}, \mathrm{abc}},\end{array}\right.$
$\mathbf{U}_{\mathrm{IV}, 120}=\left[\begin{array}{lll}\dot{U}_{\mathrm{IV}, 1} & \dot{U}_{\mathrm{IV}, 2} & \dot{U}_{\mathrm{IV}, 0}\end{array}\right]^{\mathrm{T}}=\mathrm{T}^{-1} \mathbf{U}_{\mathrm{VV}, \mathrm{abc}}$,
$\mathbf{I}_{\mathrm{I}, 120}=\left[\begin{array}{lll}\dot{I}_{\mathrm{I}, 1} & \dot{I}_{\mathrm{I}, 2} & \left.\dot{I}_{\mathrm{I}, 0}\right]^{\mathrm{T}}=\mathrm{T}^{-1} \mathbf{I}_{\mathrm{I}, \mathrm{abc}},\end{array}\right.$
$\mathbf{I}_{\mathrm{II}, 120}=\left[\begin{array}{lll}\dot{I}_{\mathrm{II}, 1} & \dot{I}_{\mathrm{II}, 2} & \dot{I}_{\mathrm{II}, 0}\end{array}\right]^{\mathrm{T}}=\mathrm{T}^{-1} \mathbf{I}_{\mathrm{II}, \mathrm{abc}}$,
$\mathbf{I}_{\text {III, } 120}=\left[\begin{array}{lll}\dot{I}_{\text {III, }} & \dot{I}_{\text {III } 2} & \dot{I}_{\text {III }, 0}\end{array}\right]^{\mathrm{T}}=\mathrm{T}^{-1} \mathbf{I}_{\mathrm{II}, \text { abc }}$,
$\mathbf{I}_{\mathrm{VV}, 120}=\left[\begin{array}{lll}\dot{I}_{\mathrm{VV}, 1} & \dot{I}_{\mathrm{IV}, 2} & \dot{I}_{\mathrm{IV}, 0}\end{array}\right]^{\mathrm{T}}=\mathrm{T}^{\prime} \mathbf{I}_{\mathrm{IV}, \mathrm{abc}}$
The impedance and admittance matrices in equations (20) and (21) are in the form of symmetrical components, they are respectively as following.

The self-impedance of each 3-phase line is

$$
\mathrm{Z}_{1,120}=\mathrm{T}^{-1} \mathrm{Z}_{\mathrm{I}, \mathrm{abc}} \mathrm{~T}=\left[\begin{array}{ccc}
z-\mathrm{j} \omega m_{\mathrm{p}} & 0 & 0  \tag{23}\\
0 & z-\mathrm{j} \omega m_{\mathrm{p}} & 0 \\
0 & 0 & 3 r_{\mathrm{g}}+z+\mathrm{j} 2 \omega m_{\mathrm{p}}
\end{array}\right]
$$

The mutual impedance between horizontal lines is

$$
\mathrm{Z}_{\mathrm{h}, 120}=\mathrm{T}^{-1} \mathrm{Z}_{\mathrm{h}, \mathrm{abc}} \mathrm{~T}=3\left(r_{\mathrm{g}}+\mathrm{j} \omega m_{\mathrm{h}}\right)\left[\begin{array}{lll}
0 & 0 & 0  \tag{24}\\
0 & 0 & 0 \\
0 & 0 & 1
\end{array}\right]
$$

The mutual impedance between vertical lines is

$$
\mathrm{Z}_{\mathrm{v}, 120}=\mathrm{T}^{-1} \mathrm{Z}_{\mathrm{v}, \mathrm{abc}} \mathrm{~T}=3\left(r_{\mathrm{g}}+\mathrm{j} \omega m_{\mathrm{v}}\right)\left[\begin{array}{lll}
0 & 0 & 0  \tag{25}\\
0 & 0 & 0 \\
0 & 0 & 1
\end{array}\right]
$$

The mutual impedance between diagonal lines is

$$
Z_{\mathrm{d}, 120}=\mathrm{T}^{-1} \mathrm{Z}_{\mathrm{d}, \mathrm{abc}} \mathrm{~T}=3\left(r_{g}+\mathrm{j} \omega m_{\mathrm{d}}\right)\left[\begin{array}{lll}
0 & 0 & 0  \tag{26}\\
0 & 0 & 0 \\
0 & 0 & 1
\end{array}\right]
$$

The self-admittance matrix for each 3-phase line is

$$
\begin{gather*}
\mathrm{Y}_{\mathrm{l}, 120}=\mathrm{T}^{-1} \mathrm{Y}_{1, \mathrm{abc}} \mathrm{~T}=\left[\begin{array}{ccc}
y_{\mathrm{s} 1} & 0 & 0 \\
0 & y_{\mathrm{s} 1} & 0 \\
0 & 0 & y_{\mathrm{s} 0}
\end{array}\right] ; \\
y_{\mathrm{s} 1}=y+\mathrm{j} 3 \omega\left(c_{\mathrm{p}}+c_{\mathrm{h}}+c_{\mathrm{v}}+c_{\mathrm{d}}\right)  \tag{27}\\
y_{\mathrm{s} 0}=y+\mathrm{j} 3 \omega\left(c_{\mathrm{h}}+c_{\mathrm{v}}+c_{\mathrm{d}}\right)
\end{gather*}
$$

The coupling admittance between horizontal lines is

$$
\mathrm{Y}_{\mathrm{h}, 120}=\mathrm{T}^{-1} \mathrm{Y}_{\mathrm{h}, \mathrm{abc}} \mathrm{~T}=\mathrm{j} 3 \omega c_{\mathrm{h}}\left[\begin{array}{lll}
0 & 0 & 0  \tag{28}\\
0 & 0 & 0 \\
0 & 0 & 1
\end{array}\right],
$$

The coupling admittance between vertical lines
is

$$
\mathrm{Y}_{\mathrm{v}, 120}=\mathrm{T}^{-1} \mathrm{Y}_{\mathrm{v}, \mathrm{abc}} \mathrm{~T}=\mathrm{j} 3 \omega c_{\mathrm{v}}\left[\begin{array}{lll}
0 & 0 & 0  \tag{29}\\
0 & 0 & 0 \\
0 & 0 & 1
\end{array}\right],
$$

The coupling admittance between diagonal lines is

$$
\mathrm{Y}_{\mathrm{d}, 120}=\mathrm{T}^{-1} \mathrm{Y}_{\mathrm{d}, \mathrm{abc}} \mathrm{~T}=\mathrm{j} 3 \omega c_{\mathrm{d}}\left[\begin{array}{lll}
0 & 0 & 0  \tag{30}\\
0 & 0 & 0 \\
0 & 0 & 1
\end{array}\right],
$$

After each equation which describes the 3-phase line transformed to symmetrical components, it can be found that the positive sequence component is equal to the negative sequence component, while the coupling admittance and mutual impedance left among 4 lines are the zero sequence component only.

The positive sequence component of each 3-phase line is in fact the form of telegraph equation [5] as

$$
\begin{align*}
& \frac{\mathrm{d}}{\mathrm{~d} x} \dot{U}_{1}=z_{1} \dot{I}_{1}  \tag{31}\\
& \frac{\mathrm{~d}}{\mathrm{~d} x} \dot{I}_{1}=y_{1} \dot{U}_{1} \tag{32}
\end{align*}
$$

Where the positive sequence component of the distributed impedance and admittance is

$$
\begin{align*}
& z_{1}=r+\mathrm{j} \omega\left(l-m_{\mathrm{p}}\right)  \tag{33}\\
& y_{1}=\mathrm{g}+\mathrm{j} \omega\left[c_{0}+3\left(c_{\mathrm{p}}+c_{\mathrm{h}}+c_{\mathrm{v}}+c_{\mathrm{d}}\right)\right] \tag{34}
\end{align*}
$$

The zero sequence component of voltage increment for each 3-phase line is

$$
\begin{align*}
\frac{\mathrm{d}}{\mathrm{dx}} \dot{U}_{\mathrm{I}, 0}= & \left(3 r_{\mathrm{g}}+z+\mathrm{j} 2 \omega m_{\mathrm{p}}\right) \dot{I}_{\mathrm{I}, 0}+3\left(r_{\mathrm{g}}+\mathrm{j} \omega m_{\mathrm{h}}\right) \dot{I}_{\mathrm{II}, 0}  \tag{35}\\
& +3\left(r_{\mathrm{g}}+\mathrm{j} \omega m_{\mathrm{v}}\right) \dot{I}_{\mathrm{III}, 0}+3\left(r_{\mathrm{g}}+\mathrm{j} \omega m_{\mathrm{d}}\right) \dot{I}_{\mathrm{IV}, 0} \\
\frac{\mathrm{~d}}{\mathrm{dx}} \dot{U}_{\mathrm{II}, 0}= & 3\left(r_{\mathrm{g}}+\mathrm{j} \omega m_{\mathrm{h}}\right) \dot{I}_{\mathrm{I}, 0}+\left(3 r_{\mathrm{g}}+z+\mathrm{j} 2 \omega m_{\mathrm{p}}\right) \dot{I}_{\mathrm{II}, 0}  \tag{36}\\
& +3\left(r_{\mathrm{g}}+\mathrm{j} \omega m_{\mathrm{d}}\right) \dot{I}_{\mathrm{III}, 0}+3\left(r_{\mathrm{g}}+\mathrm{j} \omega m_{\mathrm{v}}\right) \dot{I}_{\mathrm{IV}, 0} \\
\frac{\mathrm{~d}}{\mathrm{dx}} \dot{U}_{\mathrm{III}, 0}= & 3\left(r_{\mathrm{g}}+\mathrm{j} \omega m_{\mathrm{v}}\right) \dot{I}_{\mathrm{I}, 0}+3\left(r_{\mathrm{g}}+\mathrm{j} \omega m_{\mathrm{d}}\right) \dot{I}_{\mathrm{II}, 0}  \tag{37}\\
& +\left(3 r_{\mathrm{g}}+z+\mathrm{j} 2 \omega m_{\mathrm{p}}\right) \dot{I}_{\mathrm{III}, 0}+3\left(r_{\mathrm{g}}+\mathrm{j} \omega m_{\mathrm{h}}\right) \dot{I}_{\mathrm{IV}, 0} \\
\frac{\mathrm{~d}}{\mathrm{dx}} \dot{U}_{\mathrm{IV}, 0} & =3\left(r_{\mathrm{g}}+\mathrm{j} \omega m_{\mathrm{d}}\right) \dot{I}_{\mathrm{I}, 0}+3\left(r_{\mathrm{g}}+\mathrm{j} \omega m_{\mathrm{v}}\right) \dot{I I I}, 0  \tag{38}\\
& +3\left(r_{\mathrm{g}}+\mathrm{j} \omega m_{\mathrm{h}}\right) \dot{I}_{\mathrm{III}, 0}+\left(3 r_{\mathrm{g}}+z+\mathrm{j} 2 \omega m_{\mathrm{p}}\right) \dot{I}_{\mathrm{IV}, 0}
\end{align*}
$$

The zero sequence component of current increment for each 3-phase line is

$$
\begin{align*}
\frac{\mathrm{d}}{\mathrm{dx}} \dot{I}_{\mathrm{I}, 0}= & {\left[y+\mathrm{j} \omega\left(3 \mathrm{c}_{\mathrm{h}}+3 c_{\mathrm{v}}+3 c_{\mathrm{d}}\right)\right] \dot{U}_{\mathrm{I}, 0} }  \tag{39}\\
& -\mathrm{j} 3 \omega c_{\mathrm{h}} \dot{U}_{\mathrm{II}, 0}-\mathrm{j} 3 \omega c_{\mathrm{v}} \dot{U}_{\mathrm{III}, 0}-\mathrm{j} 3 \omega c_{\mathrm{d}} \dot{U}_{\mathrm{IV}, 0}
\end{align*}
$$

$$
\begin{align*}
\frac{\mathrm{d}}{\mathrm{dx}} \dot{I I I}, 0= & {\left[y+\mathrm{j} \omega\left(3 c_{\mathrm{h}}+3 c_{\mathrm{v}}+3 c_{\mathrm{d}}\right)\right] \dot{U}_{\mathrm{II}, 0} }  \tag{40}\\
& -\mathrm{j} 3 \omega c_{\mathrm{h}} \dot{U}_{\mathrm{I}, 0}-\mathrm{j} 3 \omega c_{\mathrm{d}} \dot{U}_{\mathrm{II}, 0}-\mathrm{j} 3 \omega c_{\mathrm{v}} \dot{U}_{\mathrm{IV}, 0}
\end{align*}
$$

$$
\begin{align*}
\frac{\mathrm{d}}{\mathrm{dx}} \dot{I I I I}, 0= & {\left[y+\mathrm{j} \omega\left(3 c_{\mathrm{h}}+3 c_{\mathrm{v}}+3 c_{\mathrm{d}}\right)\right] \dot{U}_{\mathrm{III}, 0} }  \tag{41}\\
& -\mathrm{j} 3 \omega c_{\mathrm{v}} \dot{U}_{\mathrm{I}, 0}-\mathrm{j} 3 \omega c_{\mathrm{d}} \dot{U}_{\mathrm{II}, 0}-\mathrm{j} 3 \omega c_{\mathrm{h}} \dot{U}_{\mathrm{IV}, 0} \\
\frac{\mathrm{~d}}{\mathrm{dx}} \dot{\mathrm{IVV}}_{\mathrm{IV}(0)} & =\left[y+\mathrm{j} \omega\left(3 c_{\mathrm{h}}+3 c_{\mathrm{v}}+3 c_{\mathrm{d}}\right)\right] \dot{\mathrm{IV}}, 0  \tag{42}\\
& -\mathrm{j} 3 \omega c_{\mathrm{d}} \dot{U}_{\mathrm{I}, 0}-\mathrm{j} 3 \omega c_{\mathrm{v}} \dot{U}_{\mathrm{II}, 0}-\mathrm{j} 3 \omega c_{\mathrm{h}} \dot{U}_{\mathrm{III}, 0}
\end{align*}
$$

If the voltage and current increments (35)-(42) can be transformed into as the form of telegraph equation, there will be the possibility to measure and calculate the parameters of illustrated on figure 2 .

## 2 The method to measure the parameters of four circuit 3-phase AC lines

After deriving the theoretical models as above section, the detail steps to measure the parameters can be set up. Herein the 5 types of circuits described by telegraph equations are constituted by the combination of the four 3-phase lines, then the open circuit impedance and short circuit impedance are measured on each type of circuits.

### 2.1 Case 1, positive sequence impedance measurement on a 3-phase line

The positive sequence component of voltage and current increments (31)-(34) which describe the single 3-phase line are of the form telegraph equation. This means the positive sequence components in four 3-phase lines are decoupled each other. So, a positive sequence short circuit impedance and open circuit impedance measurement can be done on any of a 3-phase line.


Fig. 3 Positive sequence short circuit impedance measurement of a line


Fig. 4 Positive sequence open circuit impedance measurement of a line

Referred to figure 3, let one of the 3-phase lines shorted at receiving terminals, and a symmetrical 3- phase source applied on sending terminal. After reading the voltage $\mathbf{U}_{\mathrm{S} 1}=\left[\begin{array}{llll}\dot{U}_{\mathrm{A}, \mathrm{S} 1} & \dot{U}_{\mathrm{B}, \mathrm{S} 1} & \dot{U}_{\mathrm{C}, \mathrm{S} 1}\end{array}\right]^{\mathrm{T}}$ and current $\mathbf{I}_{\mathrm{S} 1}=$ $\left[\begin{array}{lll}\dot{I}_{\mathrm{A}, \mathrm{S} 1} & \dot{I}_{\mathrm{B}, \mathrm{S} 1} & \left.\dot{I}_{\mathrm{C}, \mathrm{SI}}\right]^{\mathrm{T}} \text { from the source, the short }\end{array}\right.$ circuit impedance $Z_{\mathrm{SI}}$ of the 3-phase line can be calculated as

$$
Z_{\mathrm{S} 1}=\frac{\left[\begin{array}{lll}
1 & a & a^{2}
\end{array}\right] \mathbf{U}_{\mathrm{S} 1}}{\left[\begin{array}{lll}
1 & a & \left.a^{2}\right] \mathbf{I}_{\mathrm{S} 1} \tag{43}
\end{array}\right.}
$$

Referred to figure 4, let the corresponding 3-phase line opened at the receiving terminals. After reading the voltage $\mathbf{U}_{\mathrm{OI}}=\left[\begin{array}{lll}\dot{U}_{\mathrm{A}, \mathrm{Ol}} & \dot{U}_{\mathrm{B}, \mathrm{O} 1} & \dot{U}_{\mathrm{C}, \mathrm{OI}}\end{array}\right]^{\mathrm{T}}$ and current $\mathbf{I}_{\mathrm{OI}}=$ $\left[\begin{array}{lll}\dot{I}_{\mathrm{A}, \mathrm{Ol}} & \dot{I}_{\mathrm{B}, 01} & \dot{I}_{\mathrm{C}, \mathrm{OI}}\end{array}\right]^{\mathrm{T}}$ from the source, the open circuit impedance $Z_{\mathrm{O} 1}$ of the 3-phase line can be calculated as

$$
\left.Z_{\mathrm{O} 1}=\frac{\left[\begin{array}{lll}
1 & a & a^{2} \tag{44}
\end{array}\right] \mathbf{U}_{\mathrm{O} 1}}{[1} a a^{2}\right] \mathbf{I}_{\mathrm{O} 1}
$$

### 2.2 Case 2, zero sequence impedance measurement on all of four lines

Algebraically sum the equations (35) to (38), and equations (39) to (42) respectively, and merge similar items, the following equations are finally derived.

$$
\begin{align*}
& \frac{\mathrm{d}}{\mathrm{~d} x}(4 \dot{U})=z_{2}\left(\frac{\dot{I}}{3}\right)  \tag{45}\\
& \frac{\mathrm{d}}{\mathrm{~d} x}(\dot{I})=y_{2}(4 \dot{U}) \tag{46}
\end{align*}
$$

Where

$$
\begin{gather*}
\dot{U}=\dot{U}_{\mathrm{I}, 0}=\dot{U}_{\mathrm{II}, 0}=\dot{U}_{\mathrm{III}, 0}=\dot{U}_{\mathrm{IV}, 0}  \tag{47}\\
\dot{I}=3\left(\dot{I}_{\mathrm{I}, 0}+\dot{I}_{\mathrm{II}, 0}+\dot{I}_{\mathrm{II}, 0}+\dot{I}_{\mathrm{IV}, 0}\right)  \tag{48}\\
z_{2}=12 r_{\mathrm{g}}+r+\mathrm{j} \omega\left(l+2 m_{\mathrm{p}}+3 m_{\mathrm{h}}+3 m_{\mathrm{v}}+3 m_{\mathrm{d}}\right) \tag{49}
\end{gather*}
$$

$$
\begin{equation*}
y_{2}=g+\mathrm{j} \omega c_{0} \tag{50}
\end{equation*}
$$



Fig. 5 Zero sequence open circuit impedance measurement of four lines


Fig. 6 Zero sequence short circuit impedance measurement of four lines

To implement the above equations, the 12 phases of four lines are parallel connected together at sending terminals, and applied a single phase source to ground. Referred to figure 5, the voltage $\dot{U}_{\mathrm{O} 2}$ and current $\dot{I}_{\mathrm{O} 2}$ from the source are read while the receiving terminals opened, and the zero sequence open circuit impedance $Z_{02}$ can be calculated as

$$
\begin{equation*}
Z_{\mathrm{O} 2}=\frac{12 \times \dot{U}_{02}}{\dot{I}_{02}} \tag{51}
\end{equation*}
$$

Then referred to figure 6 , the receiving terminals are grounded; the voltage $\dot{U}_{\mathrm{S} 2}$ and current $\dot{I}_{\mathrm{S} 2}$ of the source can be used to calculate the zero sequence short circuit impedance $Z_{\mathrm{s} 2}$ as

$$
\begin{equation*}
Z_{\mathrm{s} 2}=\frac{12 \times \dot{U}_{\mathrm{s} 2}}{\dot{I}_{\mathrm{s} 2}} \tag{52}
\end{equation*}
$$

2.3 Case 3, a 2-phase system configured by connecting sending terminals of 3-phase lines
I and II, also connecting the 3-phase lines III and IV

Equations (35) to (42) are algebraically summed as (35)+(36)-(37)-(38) and (39)+(40)-(41)-(42) respectively. After merging similar items, result in the following equations.

$$
\begin{align*}
& \frac{\mathrm{d}}{\mathrm{~d} x}\left[2\left(\dot{U}_{\mathrm{A}}-\dot{U}_{\mathrm{B}}\right)\right]=z_{3}\left(\frac{\dot{I}_{\mathrm{A}}-\dot{I}_{\mathrm{B}}}{3}\right)  \tag{53}\\
& \frac{\mathrm{d}}{\mathrm{~d} x}\left(\frac{\dot{I}_{\mathrm{A}}-\dot{I}_{\mathrm{B}}}{3}\right)=y_{3}\left[2\left(\dot{U}_{\mathrm{A}}-\dot{U}_{\mathrm{B}}\right)\right] \tag{54}
\end{align*}
$$



Fig. 7 Positive sequence open circuit impedance measurement of 3- phase system for Case 3


Fig. 8 Positive sequence short circuit impedance measurement of 2-phase system for Case 3

Where

$$
\begin{align*}
& 2\left(\dot{U}_{\mathrm{A}}-\dot{U}_{\mathrm{B}}\right)=\left(\dot{U}_{\mathrm{I}, 0}+\dot{U}_{\mathrm{II}, 0}\right)-\left(\dot{U}_{\mathrm{III}, 0}+\dot{U}_{\mathrm{IV}, 0}\right)  \tag{55}\\
& \dot{I}_{\mathrm{A}}-\dot{I}_{\mathrm{B}}=3\left[\left(\dot{I}_{\mathrm{I}, 0}+\dot{I}_{\mathrm{II}, 0}\right)-\left(\dot{I}_{\mathrm{III}, 0}+\dot{I}_{\mathrm{IV}, 0}\right)\right] \tag{56}
\end{align*}
$$

$$
\begin{align*}
& z_{3}=r+\mathrm{j} \omega\left(l+2 m_{\mathrm{p}}+3 m_{\mathrm{h}}-3 m_{\mathrm{v}}-3 m_{\mathrm{d}}\right)  \tag{57}\\
& y_{3}=g+\mathrm{j} \omega\left(c_{0}+6 c_{\mathrm{v}}+6 c_{\mathrm{d}}\right) \tag{58}
\end{align*}
$$

So, referred to equations (53)-(54), connect the sending terminals of 3-phase lines I and II together, also connect the sending terminals of 3-phase lines III and IV together, and a 2-phase source is applied across the lines I (II) and III (IV) as figure 7. By reading 2-phase positive sequence voltages $\dot{U}_{\mathrm{A}, 03}, \quad \dot{U}_{\mathrm{B}, 03}$ and currents $\dot{I}_{\mathrm{A}, 03}, \dot{I}_{\mathrm{B}, 03}$ while the receiving terminals are opened, the 2-phase positive sequence open circuit impedance $Z_{03}$ in such a case can be calculated as

$$
\begin{equation*}
Z_{\mathrm{O} 3}=\frac{6\left(\dot{U}_{\mathrm{A}, 03}-\dot{U}_{\mathrm{B}, 03}\right)}{\dot{I}_{\mathrm{A}, \mathrm{O} 3}-\dot{I}_{\mathrm{B}, 03}} \tag{59}
\end{equation*}
$$

Then referred to figure 8 , the receiving terminals are shorted, after reading the voltages $\dot{U}_{\mathrm{A}, 53}, \dot{U}_{\mathrm{B}, \mathrm{S} 3}$ and currents $I_{\mathrm{A}, 53}, \dot{B}_{\mathrm{B}, \mathrm{S} 3}$, the 2-phase positive sequence short circuit impedance $Z_{\mathrm{S} 3}$ in such a case can be calculated by

$$
\begin{equation*}
Z_{\mathrm{S} 3}=\frac{6\left(\dot{U}_{\mathrm{A}, \mathrm{~S} 3}-\dot{U}_{\mathrm{B}, \mathrm{~S} 3}\right)}{\dot{I}_{\mathrm{A}, \mathrm{~S} 3}-\dot{I}_{\mathrm{B}, \mathrm{~S} 3}} \tag{60}
\end{equation*}
$$

2.4 Case 4, a 2-phase system configured by connecting the sending terminals of 3-phae lines I and III, also connecting the 3-phase lines II and IV

Equations (35) to (42) are algebraically summed as (35)-(36)+(37)-(38) and (39)-(40)+(41)-(42) respectively, after merging similar items, the following equations are fi-
nally derived.

$$
\begin{align*}
& \frac{\mathrm{d}}{\mathrm{~d} x}\left[2\left(\dot{U}_{\mathrm{A}}-\dot{U}_{\mathrm{B}}\right)\right]=z_{4}\left(\frac{\dot{I}_{\mathrm{A}}-\dot{I}_{\mathrm{B}}}{3}\right)  \tag{61}\\
& \frac{\mathrm{d}}{\mathrm{~d} x}\left(\frac{\dot{I}_{\mathrm{A}}-\dot{I}_{\mathrm{B}}}{3}\right)=y_{4} \cdot\left[2\left(\dot{U}_{\mathrm{A}}-\dot{U}_{\mathrm{B}}\right)\right] \tag{62}
\end{align*}
$$

Where

$$
\begin{align*}
& 2\left(\dot{U}_{\mathrm{A}}-\dot{U}_{\mathrm{B}}\right)=\left(\dot{U}_{\mathrm{I}, 0}+\dot{U}_{\mathrm{II}, 0}\right)-\left(\dot{U}_{\mathrm{III}, 0}+\dot{U}_{\mathrm{IV}, 0}\right)  \tag{63}\\
& \dot{I}_{\mathrm{A}}-\dot{I}_{\mathrm{B}}=3\left[\left(\dot{I}_{\mathrm{I}, 0}+\dot{I}_{\mathrm{II}, 0}\right)-\left(\dot{I}_{\mathrm{III}, 0}+\dot{I}_{\mathrm{IV} .0}\right)\right]  \tag{64}\\
& z_{4}=r+\mathrm{j} \omega\left(l+2 m_{\mathrm{p}}-3 m_{\mathrm{h}}+3 m_{\mathrm{v}}-3 m_{\mathrm{d}}\right)  \tag{65}\\
& y_{4}=g+\mathrm{j} \omega\left(c_{0}+6 c_{\mathrm{h}}+6 c_{\mathrm{d}}\right) \tag{66}
\end{align*}
$$



Fig. 9 Positive sequence open circuit impedance measurement of 2-phase system for Case 4


Fig. 10 Positive sequence short circuit impedance measurement of 2-phase system for Case 4

Referred to equations (61) - (66), connect the sending terminals of 3-phase lines I and III together, and connect the sending terminals of 3-phase lines II and IV together. A 2-phase source is applied across the sending terminals of line I (III) and II (IV) with the receiving terminals opened as figure 9. After reading the voltages $\dot{U}_{\mathrm{A}, 04}, \dot{U}_{\mathrm{B}, 04}$ and currents $\dot{I}_{\mathrm{A}, 04}, \dot{I}_{\mathrm{B}, 04}$,
the 2-phase positive sequence open circuit impedance $Z_{04}$ in such a case can be calculated by

$$
\begin{equation*}
Z_{\mathrm{O} 4}=\frac{6\left(\dot{U}_{\mathrm{A}, 04}-\dot{U}_{\mathrm{B}, 04}\right)}{\dot{I}_{\mathrm{A}, 04}-\dot{I}_{\mathrm{B}, 04}} \tag{67}
\end{equation*}
$$

Then the receiving terminals are shorted as figure 10. After reading the voltages $\dot{U}_{\mathrm{A}, 54}, \dot{U}_{\mathrm{B}, 54}$ and currents $I_{\mathrm{A}, 54}, \dot{I}_{\mathrm{B}, 54}$, the 2 -phase positive sequence short circuit impedance $Z_{\text {S4 }}$ is calculated as

$$
\begin{equation*}
Z_{\mathrm{S} 4}=\frac{6\left(\dot{U}_{\mathrm{A}, \mathrm{St}}-\dot{U}_{\mathrm{B}, 54}\right)}{\dot{I}_{\mathrm{A}, \mathrm{~S} 4}-\dot{I}_{\mathrm{B}, \mathrm{~S} 4}} \tag{68}
\end{equation*}
$$

2.5 Case 5, a 2-phase system configured by connecting sending terminals of the 3-phase lines $I$ and $I V$, also connecting the 3-phase lines II and III

Equations (35) to (42) are algebraically summed as (35)-(36)-(37)+(38) and (39)-(40)-(41)+(42) respectively, after merging similar items, the following equations are finally derived.

$$
\begin{align*}
& \frac{\mathrm{d}}{\mathrm{~d} x}\left[2\left(\dot{U}_{\mathrm{A}}+\dot{U}_{\mathrm{B}}\right)\right]=z_{5}\left(\frac{\dot{I}_{\mathrm{A}}-\dot{I}_{\mathrm{B}}}{3}\right)  \tag{69}\\
& \frac{\mathrm{d}}{\mathrm{~d} x}\left(\frac{\dot{I}_{\mathrm{A}}-\dot{I}_{\mathrm{B}}}{3}\right)=y_{5}\left[2\left(\dot{U}_{\mathrm{A}}+\dot{U}_{\mathrm{B}}\right)\right] \tag{70}
\end{align*}
$$

where

$$
\begin{align*}
& 2\left(\dot{U}_{\mathrm{A}}+\dot{U}_{\mathrm{B}}\right)=\left(\dot{U}_{\mathrm{I}, 0}+\dot{U}_{\mathrm{IV}, 0}\right)-\left(\dot{U}_{\mathrm{II}, 0}+\dot{U}_{\mathrm{II}, 0}\right)  \tag{71}\\
& \dot{I}_{\mathrm{A}}+\dot{I}_{\mathrm{B}}=3\left[\left(\dot{I}_{\mathrm{I}, 0}+\dot{I}_{\mathrm{IV}, 0}\right)-\left(\dot{I}_{\mathrm{I}, 0}+\dot{I}_{\mathrm{II}, 0}\right)\right]  \tag{72}\\
& z_{5}=r+\mathrm{j} \omega\left(l+2 m_{\mathrm{p}}-3 m_{\mathrm{h}}-3 m_{\mathrm{v}}+3 m_{\mathrm{d}}\right)  \tag{73}\\
& y_{5}=g+\mathrm{j} \omega\left(c_{0}+6 c_{\mathrm{h}}+6 c_{\mathrm{v}}\right) \tag{74}
\end{align*}
$$



Fig. 11 Positive sequence open circuit impedance measurement of 2-phase system for Case 5


Fig. 12 Positive sequence short circuit impedance measurement of 2-phase system for Case 5

Referred to equations (69)-(74), the 3-phase lines I and IV are connected together, and the 3-phase lines II and III are connected together. A 2-phase source is applied across terminals I (IV) and II (III) with receiving terminals opened as figure 11. After reading the voltages $\dot{U}_{\mathrm{A}, 05}, \dot{U}_{\mathrm{B}, 05}$ and currents $\dot{I}_{\mathrm{A}, 05}, \dot{I}_{\mathrm{B}, 05}$, the 2-phase positive sequence open circuit impedance $Z_{05}$ in such a case is calculated as

$$
\begin{equation*}
Z_{05}=\frac{6\left(\dot{U}_{\mathrm{A}, 5}-\dot{U}_{\mathrm{B}, 05}\right)}{\dot{I}_{\mathrm{A}, 05}-\dot{I}_{\mathrm{B}, 05}} \tag{75}
\end{equation*}
$$

Then the receiving terminals are shorted together as figure 12. After reading voltages $\dot{U}_{\mathrm{A}, 55}, \dot{U}_{\mathrm{B}, 55}$ and currents $I_{\mathrm{A}, 55}, \dot{I}_{\mathrm{B}, \mathrm{S5}}$, the positive sequence short circuit impedance $Z_{\mathrm{S} 5}$ in such a case is calculated as

$$
\begin{equation*}
Z_{\mathrm{S} 5}=\frac{6\left(\dot{U}_{\mathrm{A}, \mathrm{~S}}-\dot{U}_{\mathrm{B}, \mathrm{SS}}\right)}{\dot{I}_{\mathrm{A}, \mathrm{~S}}-\dot{I}_{\mathrm{B}, \mathrm{S5}}} \tag{76}
\end{equation*}
$$

## 3 Parameters calculation

When the short circuit impedance and open circuit impedance measurements $Z_{\mathrm{S} 1}$ and $Z_{\mathrm{O} 1}, Z_{\mathrm{S} 2}$ and $Z_{\mathrm{O} 2}$, $Z_{\mathrm{S} 3}$ and $Z_{\mathrm{O} 3}, Z_{\mathrm{S4} 4}$ and $Z_{\mathrm{O} 4}, Z_{\mathrm{S} 5}$ and $Z_{\mathrm{O} 5}$ in above cases are finished, the characteristic ance $z_{c, \mathrm{i}}$ and propagation constant $\gamma_{\mathrm{i}}$ in each case $i$ can be calculated as following

$$
\begin{array}{ll}
z_{\mathrm{c}, i}=\sqrt{Z_{\mathrm{si}} Z_{\mathrm{o} i}} & (i=1,2,3,4,5) \\
\gamma_{\mathrm{i}}=\frac{\operatorname{coth}^{-1} \sqrt{\frac{Z_{0} i}{Z_{\mathrm{s} i}}}}{D} \quad(i=1,2,3,4,5) \tag{78}
\end{array}
$$

Where $D$ is the distance of four lines on same tower. Then the distributed impedance $z_{\mathrm{i}}$ and admittance $y_{\mathrm{i}}$ for each case are calculated as

$$
\begin{array}{ll}
z_{i}=z_{\mathrm{c}, \mathrm{i}} \gamma_{i} & (i=1,2,3,4,5) \\
y_{i}=\gamma_{i} / z_{\mathrm{c}, i} & (i=1,2,3,4,5) \tag{80}
\end{array}
$$

Subscript $i=1,2, \ldots, 5$ refer to the measurement case above. Then the distributed parameters illustrated in figure 2 can be calculated as

$$
\begin{align*}
& r=\frac{1}{4} \operatorname{real}\left(z_{1}+z_{3}+z_{4}+z_{5}\right),  \tag{81}\\
& g=\frac{1}{5} \operatorname{real}\left(y_{1}+y_{2}+y_{3}+y_{4}+y_{5}\right)  \tag{82}\\
& r_{g}=\frac{\operatorname{real}\left(z_{2}\right)-r}{12} \tag{83}
\end{align*}
$$

$$
\omega\left[\begin{array}{ccccc}
1 & -1 & 0 & 0 & 0  \tag{84}\\
1 & 2 & 3 & 3 & 3 \\
1 & 2 & 3 & -3 & -3 \\
1 & 2 & -3 & 3 & -3 \\
1 & 2 & -3 & -3 & 3
\end{array}\right]\left[\begin{array}{c}
l \\
m_{\mathrm{p}} \\
m_{\mathrm{h}} \\
m_{\mathrm{v}} \\
m_{\mathrm{d}}
\end{array}\right]=\operatorname{imag}\left[\begin{array}{c}
z_{1} \\
z_{2} \\
z_{3} \\
z_{4} \\
z_{5}
\end{array}\right]
$$

$$
\omega\left[\begin{array}{ccccc}
1 & 3 & 3 & 3 & 3  \tag{85}\\
1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 6 & 6 \\
1 & 0 & 6 & 0 & 6 \\
1 & 0 & 6 & 6 & 0
\end{array}\right]\left[\begin{array}{c}
c_{0} \\
c_{\mathrm{p}} \\
c_{\mathrm{h}} \\
c_{\mathrm{v}} \\
c_{\mathrm{d}}
\end{array}\right]=\operatorname{imag}\left[\begin{array}{l}
y_{1} \\
y_{2} \\
y_{3} \\
y_{4} \\
y_{5}
\end{array}\right]
$$

In equations (81)-(85), real( $\cdot$ ) is the real part and $\operatorname{imag}(\cdot)$ is the imaginary part of a complex.

## 4 Discussion

All the impedance measurements should be at a given frequency. To avoid the interference from power frequency, it is suggested that the measurement are done around the power frequency $f_{\mathrm{S}}$, say at the frequencies $f_{s}-\Delta f, f_{s}+\Delta f$. Where $\Delta f$ may be $2.5 \mathrm{~Hz}, 5 \mathrm{~Hz}$ or 10 Hz . The measurement and calculation are done for each frequency. Finally, the parameters at power frequency can be calculated by interpolation.

Due to the induced voltage and current on the line to be measured may be great, it should be very careful in the measurement to avoid the damage to the instrument and hurt to the workers.

## 5. Conclusion

In general, multi-circuit AC transmission lines
might be decoupled if they are symmetrically arranged in cross section geometry and are full transposition. The measurement steps are: (i) take the positive sequence open circuit impedance and short circuit impedance measurement on a 3-phase line; (ii) the zero sequence open circuit impedance and short circuit impedance measurement of all the ac lines connected at sending terminals; (iii) all the 3-phase lines are assembled as several 2-phase systems or 3-phase systems, and take the positive open circuit impedance and short circuit impedance measurement on the 2-phase systems or 3-phase systems. The pre-condition is that by merging the similar items of the zero sequence voltage and current increments for each line, and left only the form of telegraph equation.

After getting the open circuit impedance and short circuit impedance measurement, then characteristic impedance and propagation coefficient for each type of systems can be calculated, further the distributed impedance and admittance can be resulted. Finally, the self-impedance and ground capacitance for each phase, the mutual inductance and coupling capacitance between phases inside a single 3-phase line, mutual impedance and coupling capacitance amongst the lines can all be solved.

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## 改进的同塔四回交流输电线路参数测量方法

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摘要：对同塔四回交流输电线路参数的测量方法做了改进。首先是测量单回线路的正序开路阻抗和短路阻抗；然后将四回线路以两两组合的方式，组成四组两相系统，并对四组两相系统的开路阻抗和短路阻抗进行测量；最后分别计算各电路组合下的分布阻抗和分布导纳，进而精确地计算出各回线路之间的零序耦合电感和耦合电容，以及零序 $r, ~ l, ~ c$ 自参数。

关键词：输电线路；阻抗测量；测量标准；线路参数测量，电报方

