SCIREA Journal of Electrical Engineering
ISSN: 2995-7141
http://www.scirea.org/journal/DEE
May 14, 2024
Volume 9, Issue 1, February 2024
https://doi.org/10.54647/dee470357

Knowledge Reasoning Based on the Reducibility of Valid Generalized Syllogisms

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Abstract

This paper firstly proves that the generalized syllogism *HMO-3* is valid according to the relevant definitions, facts and rules, and then shows that at least the other 21 valid generalized syllogisms can be deduced from the syllogism *HMO-3* with the common generalized quantifiers '*most*' and '*at most half of the'*. The main conclusion of this paper is that there are reducible relationships between/among valid generalized syllogisms. Since all conclusions are obtained by means of deductive reasoning, therefore the results are consistent. The reason why valid generalized syllogisms can be mutually reduced is that: Aristotelian quantifiers can be mutually defined each other, and so can the four generalized quantifiers studied in this paper. This study provides the theoretical support for knowledge mining in artificial intelligence.

Keywords: generalized syllogisms; validity; knowledge mining; knowledge reasoning

1. Introduction

It is commonly known that syllogism reasoning has a long history and is widely applied in human life. There are many types of syllogisms in natural language, for instance, Aristotelian syllogisms (Moss, 2010; Xiaojun, 2018; Cheng, 2022; Yijiang, 2023), Aristotelian modal syllogisms (Johnson, 1989; Thom, 1996; Malink, 2006; Xiaojun, 2020; Cheng, 2023), generalized syllogisms (Endrullis and Moss, 2015; Liheng, 2024), and generalized modal syllogisms (Jing and Xiaojun, 2023), etc.

There is little literature on generalized syllogisms, and this paper mainly studies knowledge mining based on the validity of the generalized syllogisms with the generalized quantifiers '*most*' and '*at most half of the'*. More specifically, this paper demonstrates the reducible relationships between/among valid common generalized syllogisms, and reveals the process of knowledge representation and knowledge reasoning for this type syllogisms in natural language.

2. Knowledge Representation for Generalized Syllogisms

In this paper, let *k*, *r* and *v* be lexical variables, and their domain is denoted by *D*. The sets composed of *k*, *r* and *v* are respectively *K*, *R*, and *V*. '$|K\cap V|$' represents the cardinal of the intersection of sets *K* and *V*. Let ε, λ, π and ω be well-formed formulas (shorted as wff). *Q* stands for a quantifier, $\neg Q$ for its outer negative quantifier and $Q\neg$ for its inner one. '$\varepsilon =_{def} \lambda$' states that ε can be defined by λ. '$\vdash\varepsilon$' means that ε is provable. 'iff' represents if and only if. '$\neg$', '$\wedge$', '$\vee$', '$\rightarrow$', and '$\leftrightarrow$' are the common symbols in classical first-order logic (Hamilton, 1978).

The generalized syllogisms studied in this paper involves the four Aristotelian quantifiers: *no*, *some*, *all*, and *not all*, and the following four generalized quantifiers: *most*, *at most half of the*, *fewer than half of the,* and *at least half of the.* The eight propositions are composed of the above eight quantifiers as follows: *no*(*b*, *x*), *some*(*b*, *x*), *all*(*b*, *x*), *not all*(*b*, *x*), *most*(*b*, *x*), *at most half of the*(*b*, *x*), *fewer than half of the(b*, *x)* and *at least half of the(b*, *x).* The eight propositions are *shorted as* Proposition *E*, *I*, *A*, *O*, *M*, *H*, *F* and *S*, respectively. A non-trivial generalized syllogism includes at least one of Proposition *M*, *H*, *F,* and *S*.

The definitions of figures in generalized syllogisms are similar to those of ones in Aristotelian syllogisms (Bo, 2020). This paper provides a unified and consistent research paradigm for knowledge mining based on valid generalized syllogisms by studying the reducibility of the non-trivial generalized syllogism *HMO-3*. An instance of the syllogism *HMO-3* is as follows:

Major premise: At most half of dogs can catch rats.

Minor premise: Most dogs are domesticated pets.

Conclusion: Not all domesticated pets can catch rats.

Let *r* be a lexical variable that represents dogs, *v* be a lexical variable denoting things that catch rats, and *k* be a lexical variable that stands for domesticated pets. Then the above example can be formalized as *at* *most half of the*(r, v)∧*most*(r, k)→*not all*(k, v), and abbreviated as *HMO-3*. Others are similar to this.

3. Formal System of Generalized Syllogisms with the Generalized Quantifier '*most*'

This formal system includes the following relevant initial symbols, definitions, axioms, and rules.

3.1 Primitive Symbols

(1) lexical variables: k, r, v.

(2) quantifiers: *most*, *all*.

(3) operators: ¬, →, ∧.

(4) brackets: (,).

3.2 Formation Rules

(1) If Q is a quantifier, k and v are lexical variables, then $Q(k, v)$ is a wff.

(2) If ε is a wff, then so is ¬ε.

(3) If ε and λ are wffs, then so is ε→λ.

(4) The formulas formed only by above three rules are wffs.

3.3 Basic Axioms

A1: If ε is a valid formula in classical first-order logic, then ⊢ε.

A2: ⊢*at most half of the*(r, v)∧*most*(r, k)→*not all*(k, v) (that is, the syllogism *HMO-3*).

3.4 Deductive Rules

Rule 1(subsequent weakening): From ⊢(ε∧λ→π) and ⊢(π→ω) infer ⊢(ε∧λ→ω).

Rule 2(anti-syllogism): From ⊢(ε∧λ→π) infer ⊢(¬π∧ε→¬λ) or ⊢(¬π∧λ→¬ε).

3.5 Relevant Definitions

D1 (conjunction): $(\varepsilon\wedge\lambda)=_{def}\neg(\varepsilon\rightarrow\neg\lambda)$

D2 (bi-condition): $(\varepsilon\leftrightarrow\lambda)=_{def}(\varepsilon\rightarrow\lambda)\wedge(\lambda\rightarrow\varepsilon)$

D3 (inner negation): $(Q\neg)(k, v)=_{def}Q(k, D-v)$

D4 (outer negation): $(\neg Q)(k, v)=_{\text{def}}$ It is not that $Q(k, v)$

D5 (truth value): $all(k, v)=_{\text{def}} K\subseteq V$;

D6 (truth value): $some(k, v)=_{\text{def}} K\cap V\neq\varnothing$;

D7 (truth value): $no(k, v)=_{\text{def}} K\cap V=\varnothing$;

D8 (truth value): $not\ all(k, v)=_{\text{def}} K\not\subseteq V$.

D9 (truth value): $most(k, v)$ is true iff $|K\cap V|>0.5|K|$ is true;

D10 (truth value): $fewer\ than\ half\ of\ the(k, v)$ is true iff $|K\cap V|<0.5|K|$ is true;

D11 (truth value): $at\ most\ half\ of\ the(k, v)$ is true iff $|K\cap V|\leq 0.5|K|$ is true;

D12 (truth value): $at\ least\ half\ of\ the(k, v)$ is true iff $|K\cap V|\geq 0.5|K|$ is true.

3.6 Relevant Facts

On the basis of classical first-order logic, generalized quantifier theory (Peters & Westerståhl, 2006) and set theory (Halmos, 1974), it can be obtained the following relevant facts.

Fact 1 (Inner negation):

(1.1) $all(k, v)\leftrightarrow no\neg(k, v)$;

(1.2) $no(k, v)\leftrightarrow all\neg(k, v)$;

(1.3) $some(k, v)\leftrightarrow not\ all\neg(k, v)$;

(1.4) $not\ all(k, v)\leftrightarrow some\neg(k, v)$;

(1.5) $most(k, v)\leftrightarrow fewer\ than\ half\ of\ the\neg(k, v)$;

(1.6) $at\ least\ half\ of\ the(k, v)\leftrightarrow at\ most\ half\ of\ the\neg(k, v)$;

(1.7) $at\ most\ half\ of\ the(k, v)\leftrightarrow at\ least\ half\ of\ the\neg(k, v)$;

(1.8) $fewer\ than\ half\ of\ the(k, v)\leftrightarrow most\neg(k, v)$.

Fact 2 (Outer negation):

(2.1) $\neg all(k, v)\leftrightarrow not\ all(k, v)$;

(2.2) $\neg not\ all(k, v)\leftrightarrow all(k, v)$;

(2.3) $\neg no(k, v)\leftrightarrow some(k, v)$;

(2.4) $\neg$*some*(*k*, *v*)$\leftrightarrow$*no*(*k*, *v*);

(2.5) $\neg$*most*(*k*, *v*)$\leftrightarrow$*at most half of the*(*k*, *v*);

(2.6) $\neg$*at least half of the*(*k*, *v*)$\leftrightarrow$*fewer than half of the*(*k*, *v*);

(2.7) $\neg$*at most half of the*(*k*, *v*)$\leftrightarrow$*most*(*k*, *v*);

(2.8) $\neg$*fewer than half of the*(*k*, *v*)$\leftrightarrow$*at least half of the*(*k*, *v*).

Fact 3 (Symmetry):

(3.1) *some*(*k*, *v*)$\leftrightarrow$*some*(*v*, *k*);

(3.2) *no*(*k*, *v*)$\leftrightarrow$*no*(*v*, *k*).

Fact 4 (Subordination) :

(4.1) $\vdash$*no*(*k*, *v*)$\rightarrow$*not all*(*k*, *v*);

(4.2) $\vdash$*all*(*k*, *v*)$\rightarrow$*some*(*k*, *v*);

(4.3) $\vdash$*all*(*k*, *v*)$\rightarrow$*most*(*k*, *v*);

(4.4) $\vdash$*most*(*k*, *v*)$\rightarrow$*some*(*k*, *v*);

(4.5) $\vdash$*all*(*k*, *v*)$\rightarrow$*at least half of the*(*k*, *v*);

(4.6) $\vdash$*at least half of the*(*k*, *v*)$\rightarrow$*some*(*k*, *v*);

(4.7) $\vdash$*fewer than half of the*(*k*, *v*)$\rightarrow$*not all*(*k*, *v*);

(4.8) $\vdash$*at most half of the*(*k*, *v*)$\rightarrow$*not all*(*k*, *v*).

4. Knowledge Mining Based on the Reducibility of the Generalized Syllogism *HMO-3*

In the following, in order to prove the reduction relationships between/among different syllogisms, the strategy is firstly to prove the validity of syllogism *HMO-3* in Theorem 1, and then the other syllogisms can be derived from *HMO-3*. For example, '*HMO-3*$\rightarrow$*AMM-1*' in Theorem 2 says that the latter can be deduced from the former.

Theorem 1 (*HMO-3*): The Generalized Syllogism *at most half of the*(*r*, *v*)$\wedge$*most*(*r*, *k*)$\rightarrow$*not all*(*k*, *v*) is valid.

Proof: Assumed that *at most half of the*(r, v) and *most*(r, k) are true, then $|R\cap V|\leq 0.5|R|$ is true by Definition D11, and $|R\cap K|>0.5|R|$ is true by Definition D9. It can be easily concluded that $K\nsubseteq V$. Thus, *not all*(k, v) is true by Definition D8. This can be easily proved by reductio ad absurdum. Supposed that $K\nsubseteq V$ is not true, that is, $K\subseteq V$ is true. And it has been known that $|R\cap V|\leq 0.5|R|$ is true by Definition D11. Then, it can be easily obtained that $|R\cap K|\leq 0.5|R|$, which contradicts with $|R\cap K|>0.5|R|$ is true by Definition D9. So $K\subseteq V$ is not true, which means that $K\nsubseteq V$ is true. Therefore, it follows that *at most half of the*(r, v)$\wedge$*most*(r, k)$\rightarrow$*not all*(k, v) is valid, as expected.

Theorem 2: There are at least the following 21 valid generalized syllogisms can be inferred from the syllogism *HMO-3*:

(1) $\vdash$*HMO-3*$\rightarrow$*AMM-1*

(2) $\vdash$*HMO-3*$\rightarrow$*AMM-1*$\rightarrow$*AMI-1*

(3) $\vdash$*HMO-3*$\rightarrow$*AMM-1*$\rightarrow$*AMI-1*$\rightarrow$*MAI-4*

(4) $\vdash$*HMO-3*$\rightarrow$*AMM-1*$\rightarrow$*AMI-1*$\rightarrow$*EMO-3*

(5) $\vdash$*HMO-3*$\rightarrow$*AMM-1*$\rightarrow$*AMI-1*$\rightarrow$*EMO-3*$\rightarrow$*EMO-4*

(6) $\vdash$*HMO-3*$\rightarrow$*AMM-1*$\rightarrow$*AMI-1*$\rightarrow$*AEH-2*

(7) $\vdash$*HMO-3*$\rightarrow$*AMM-1*$\rightarrow$*AMI-1*$\rightarrow$*AEH-2*$\rightarrow$*AEH-4*

(8) $\vdash$*HMO-3*$\rightarrow$*AMM-1*$\rightarrow$*AMI-1*$\rightarrow$*AEH-2*$\rightarrow$*EAH-2*

(9) $\vdash$*HMO-3*$\rightarrow$*AMM-1*$\rightarrow$*AMI-1*$\rightarrow$*AEH-2*$\rightarrow$*EAH-2*$\rightarrow$*EAH-1*

(10) $\vdash$*HMO-3*$\rightarrow$*AHH-2*

(11) $\vdash$*HMO-3*$\rightarrow$*AHH-2*$\rightarrow$*AHO-2*

(12) $\vdash$*HMO-3*$\rightarrow$*AHH-2*$\rightarrow$*AHO-2*$\rightarrow$*HAO-3*

(13) $\vdash$*HMO-3*$\rightarrow$*AHH-2*$\rightarrow$*AHO-2*$\rightarrow$*AAM-1*

(14) $\vdash$*HMO-3*$\rightarrow$*AHH-2*$\rightarrow$*AHO-2*$\rightarrow$*AAM-1*$\rightarrow$*EAF-1*

(15) $\vdash$*HMO-3*$\rightarrow$*AHH-2*$\rightarrow$*AHO-2*$\rightarrow$*AAM-1*$\rightarrow$*EAF-1*$\rightarrow$*EAF-2*

(16) $\vdash$*HMO-3*$\rightarrow$*AHH-2*$\rightarrow$*AHO-2*$\rightarrow$*AAM-1*$\rightarrow$*EAF-1*$\rightarrow$*ESO-2*

(17) $\vdash$*HMO-3*$\rightarrow$*AHH-2*$\rightarrow$*AHO-2*$\rightarrow$*AAM-1*$\rightarrow$*EAF-1*$\rightarrow$*ESO-2*$\rightarrow$*ESO-1*

(18) $\vdash$*HMO-3*$\rightarrow$*SMI-3*

(19) $\vdash$*HMO-3*$\rightarrow$*SMI-3*$\rightarrow$*MSI-3*

(20) $\vdash$*HMO-3*$\rightarrow$*SMI-3*$\rightarrow$*EMF-1*

(21) $\vdash$*HMO-3*$\rightarrow$*SMI-3*$\rightarrow$*EMF-1*$\rightarrow$*EMF-2*

Proof:

[1] $\vdash at\ most\ half\ of\ the(r, v)\wedge most(r, k)\rightarrow not\ all(k, v)$ (i.e.*HMO-3*, Axiom A2)

[2] $\vdash\neg not\ all(k, v)\wedge most(r, k)\rightarrow\neg at\ most\ half\ of\ the(r, v)$ (by [1] and Rule 2)

[3] $\vdash all(k, v)\wedge most(r, k)\rightarrow most(r, v)$ (i.e.*AMM-1*, by [2], Fact (2.2) and (2.7))

[4] $\vdash all(k, v)\wedge most(r, k)\rightarrow some(r, v)$ (i.e.*AMI-1*, by [3], Fact (4.4))

[5] $\vdash all(k, v)\wedge most(r, k)\rightarrow some(v, r)$ (i.e.*MAI-4*, by [4], Fact (3.1))

[6] $\vdash\neg some(r, v)\wedge most(r, k)\rightarrow\neg all(k, v)$ (by [4] and Rule 2)

[7] $\vdash no(r, v)\wedge most(r, k)\rightarrow not\ all(k, v)$ (i.e.*EMO-3*, by [6], Fact (2.1) and (2.4))

[8] $\vdash no(v, r)\wedge most(r, k)\rightarrow not\ all(k, v)$ (i.e.*EMO-4*, by [7] and Fact (3.2))

[9] $\vdash\neg some(r, v)\wedge all(k, v)\rightarrow\neg most(r, k)$ (by [4] and Rule 2)

[10] $\vdash no(r, v)\wedge all(k, v)\rightarrow at\ most\ half\ of\ the(r, k)$ (i.e.*AEH-2*, by [9], Fact (2.4) and (2.5))

[11] $\vdash no(v, r)\wedge all(k, v)\rightarrow at\ most\ half\ of\ the(r, k)$ (i.e.*AEH-4*, by [10] and Fact (3.2))

[12] $\vdash all(r, D-v)\wedge no(k, D-v)\rightarrow at\ most\ half\ of\ the(r, k)$

(i.e.*EAH-2*, by [10], Fact (1.1) and (1.2), Definition D3)

[13] $\vdash all(r, D-v)\wedge no(D-v, k)\rightarrow at\ most\ half\ of\ the(r, k)$ (i.e.*EAH-1*, by [12] and Fact (3.2))

[14] $\vdash\neg not\ all(k, v)\wedge at\ most\ half\ of\ the(r, v)\rightarrow\neg most(r, k)$ (by [1] and Rule 2)

[15] $\vdash all(k, v)\wedge at\ most\ half\ of\ the(r, v)\rightarrow at\ most\ half\ of\ the(r, k)$

(i.e.*AHH-2*, by [14], Fact (2.2) and (2.5))

[16] $\vdash all(k, v)\wedge at\ most\ half\ of\ the(r, v)\rightarrow not\ all(r, k)$ (i.e.*AHO-2*, by[15] and Fact (4.8))

[17] $\vdash\neg not\ all(r, k)\wedge at\ most\ half\ of\ the(r, v)\rightarrow\neg all(k, v)$ (by [16] and Rule 2)

[18] $\vdash all(r, k)\wedge at\ most\ half\ of\ the(r, v)\rightarrow not\ all(k, v)$ (i.e.*HAO-3*, by [17] Fact (2.1) and (2.2))

[19] $\vdash\neg$*not all*$(r, k)\wedge$*all*$(k, v)\rightarrow\neg$*at most half of the*(r, v) (by [16] and Rule 2)

[20] $\vdash$*all*$(r, k)\wedge$*all*$(k, v)\rightarrow$*most*(r, v) (i.e.*AAM-1*, by [19], Fact (2.2) and (2.7))

[21] $\vdash$*all*$(r, k)\wedge$*no*$\neg(k, v)\rightarrow$*fewer than half of the*$\neg(r, v)$ (by [20], Fact (1.1) and (1.5))

[22] $\vdash$*all*$(r, k)\wedge$*no*$(k, D-v)\rightarrow$*fewer than half of the*$(r, D-v)$

(i.e.*EAF-1*,by [21] and Definition D3)

[23] $\vdash$*all*$(r, k)\wedge$*no*$(D-v, k)\rightarrow$*fewer than half of the*$(r, D-v)$ (i.e.*EAF-2*, by [22] and Fact (3.2))

[24] $\vdash\neg$*fewer than half of the*$(r, D-v)\wedge$*no*$(k, D-v)\rightarrow\neg$*all*(r, k) (by [22] and Rule 2)

[25] $\vdash$*at least half of the*$(r, D-v)\wedge$*no*$(k, D-v)\rightarrow$*not all*(r, k)

(i.e.*ESO-2*, by [24], Fact (2.1) and (2.8))

[26] $\vdash$*at least half of the*$(r, D-v)\wedge$*no*$(D-v, k)\rightarrow$*not all*(r, k) (i.e.*ESO-1*, by [25] and Fact (3.2))

[27] $\vdash$*at least half of the*$\neg(r, v)\wedge$*most*$(r, k)\rightarrow$*some*$\neg(k, v)$ (by [1], Fact (1.4) and (1.7))

[28] $\vdash$*at least half of the*$(r, D-v)\wedge$*most*$(r, k)\rightarrow$*some*$(k, D-v)$

(i.e.*SMI-3*, by [27] and Definition D3)

[29] $\vdash$*at least half of the*$(r, D-v)\wedge$*most*$(r, k)\rightarrow$*some*$(D-v, k)$(i.e.*MSI-3*, by [28] and Fact (3.1))

[30] $\vdash\neg$*some*$(k, D-v)\wedge$*most*$(r, k)\rightarrow\neg$*at least half of the*$(r, D-v)$ (by [28] and Rule 2)

[31] $\vdash$*no*$(k, D-v)\wedge$*most*$(r, k)\rightarrow$*fewer than half of the*$(r, D-v)$

(i.e.*EMF-1*, by [30], Fact (2.4) and (2.6))

[32] $\vdash$*no*$(D-v, k)\wedge$*most*$(r, k)\rightarrow$*fewer than half of the*$(r, D-v)$

(i.e.*EMF-2*, by [31] and Fact (3.2))

It has been proved that the above 21 valid generalized syllogisms can be obtained from the syllogism *HMO-3* through the above 32 reductive steps.

5. Conclusion and Future Work

Theorem 1 proves that the generalized syllogism *HMO-3* is valid according to the relevant definitions, facts and rules. Then Theorem 2 shows that at least the other 21 valid generalized syllogisms can be deduced from the syllogism *HMO-3* on the basis of classical first-order logic, set theory and generalized quantifier theory. The main conclusion of this paper is that there are reducible relationships between/among valid generalized syllogisms. Due to the fact that all conclusions are obtained by means of deductive reasoning, the results are consistent.

The reason why valid generalized syllogisms can be mutually reduced is that: Four Aristotelian quantifiers (that is, *some*, *not all*, *no* and *all*) can be mutually defined each other, and so can the four generalized quantifiers (that is, *most*, *at most half of the*, *fewer than half of the* and *at least half of the*). This study provides the theoretical support for knowledge mining in artificial intelligence.

How to establish a complete axiomatic system for the fragments of generalized syllogisms studied in this paper? This question is left to study in the future.

Acknowledgement

This work was supported by the National Social Science Fund of China under Grant No.21BZX100.

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