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Capacitor Coupled Substation State Space Formulation for Power Tapping and Power Injection Application

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Abstract

This paper investigates the formulation of a State Space Model for a Capacitor Coupled Substation to facilitate power tapping and injection into an electrical transmission network. The primary objective is to establish a state space representation of the electrical system that can be used to simplify the modeling of complex multi-variable electrical circuits or systems. To achieve this, two equivalent circuits are developed: one for electrical power tapping and the other for electrical power injection into an electrical power transmission network. Typically employed for extracting power from high voltage transmission lines and converting it to medium voltage at the distribution level through coupling capacitors, Capacitor Coupled Substations can also be utilized for power injection from alternative electrical power sources, such as microgrids, with appropriate power flow control systems. Thus, the study develops generic state space matrices for both power tapping and injection scenarios. Overall, this research significantly contributes to the field of electrical engineering by providing valuable insights into the fundamental development of capacitor coupled substations state space system representation for electrical power tapping and significantly for electrical power injection into

the electrical power transmission network. The findings hold relevance for further modeling and analysis of micro-grid embedded electricity systems.

Keywords: Capacitor-Coupled Substation, Transmission Line, Simulation, State Space, Electrical Power Injection, Electrical Power Tapping

1. Introduction

Capacitor Coupled Substation (CCS) is a field that has recently been vastly studied. A well accepted definition of a CCS is given as a technology that taps electrical power from High Voltage (HV) electrical transmission lines and converts it to a distribution level voltage through the use of coupling capacitors [1]. Some studies focus on the feasibility of the CCS as an alternative technology for supplying electrical power to dedicated loads [2] . This technology is said to be one of the cost-effective technologies for supplying electrical power to sparsely populated areas [3]. The requirements for supplying power to sparsely populated areas has necessitated numerous studies focusing on different aspects of the CCS, such as its impact on the transmission network and the impact of multiple CCS on the transmission network [4] [5]. The design of a CCS can be bi-directional thus allowing for electrical power tapping and electrical power injection into the electrical power transmission network. With proper design, such as in a configuration that utilizes bi-direction power flow controllers, such as Controllable Network Transformers (CNT), a CCS can be used as a distribution mechanism to deliver electrical power from microgrids into the electrical power transmission network. This requires system modeling for effective studies. When a studied system has a number of variables, modeling becomes complex. Therefore, model of the physical system also becomes complex. However, a state space mathematical modeling can be adopted to model such systems.

A state space representation can be defined as a mathematical model used to describe the behaviour of electrical systems. For an electrical system, the state variables typically represent energy storage elements such as capacitors and inductors. The state space equations are usually formulated as a set of first-order differential equations or difference equations, which summarise the relationships between inputs, outputs, and the internal states of the system. This representation enables ease of analysing the system's behaviour, design control algorithms, and simulate its response to various inputs. State space formulation for electrical circuits is used to simplify multiple variable systems for modeling [6].

This article presents the State Space Formulation for a Capacitor Coupled Substation under Electrical Power Tapping and Electrical Power Injection conditions.

2. Background Theory

One of the recent instances of practical implementation of a CCS has clearly demonstrated the system's feasibility and effectiveness in supplying electrical power to dedicated loads [7] . Proposed studies indicate that a CCS can be used for both electrical power tapping and electrical power injection from and into the electrical power transmission network.

2.1. CCS Power Tapping

When CCS is used to for tapping electrical power from an HV network to supply dedicated loads, it is vital to ensure that other loads, either upstream or downstream of the tapping node, do not get negatively affected. This means that system voltage stability requires thorough consideration.

Different types of loads affect the voltage stability in their unique manner such as in the scenario of fast response loads where voltage stability would require comprehensive dynamic modeling analysis as the conventional power flow model is likely to lead to errors [8]. Prior to the development of passive damper-filters, the CCS is said not to be feasible due to the instability of the system under different operating conditions such as switching operations or load swings [1]. The primary components of a CCS are the capacitors. The CCS's capacitance characteristics can be used to control the voltage instabilities by introducing capacitive reactive power into the transmission network to compensate for voltage drops, thus maintaining voltage stability [9]. Capacitive reactive power compensation have been extensively studied as a form of correcting voltage instabilities in the distribution lines, especially when operating multiple photovoltaic power systems. In some studies, the capacitive reactive power is applied through the use of conventional capacitor banks connected at the load [10]. Thyristor controlled series capacitors which were developed in the 1980s have also been effectively used for transmission line power stability [11] . It, therefore, can be inferred that a CCS can also be used for both electrical power tapping from the HV lines and for voltage stability of the system it is connected to.

2.2. *CCS Power Injection*

The theory of electrical power injection into the electrical power transmission network using a CCS is a fairly new field under development. The available literature also considers a proposed Capacitor Coupled Substation incorporating Controllable Network Transformers as one of the configurations that can be used for effectively allowing the use of a CCS to inject electrical power into the electrical transmission network from microgrids [12]. Similarly to CCS power tapping, in the process of CCS power injection, any negative impact of the injected electrical power on the transmission network need to be prevented. The transmission line where the CCS power in injected should not result in any unwanted overloads or transients. Transmission networks where power is injected into it is said to be prone to transients. This phenomenon is also present when renewable power is injected into a transmission line [13]. Recent changes in the electricity generation sector with the increased usage of renewable energy sources has necessitated the need for voltage stability studies in order to assess and enhance power grid stability during increased energy penetration [14] . It is, therefore, vital to understand the impact of electrical power injection on the transmission network via a CCS from any source of electrical energy.

2.3. *State Space Formulation*

State space representation can be defined as a mathematical model of a physical system expressed as a function of input, output, and state variables related by first-order differential equations or difference equations [15] . Conventional studies of electrical systems use differential equations that relate the system inputs and outputs for analysis methods. The introduction of using state variables has been studied from as far back as the 1960s where Kuh and Rohrer introduced the state-variable approach to network analysis and proposed a systematic method of obtaining the state equation of a linear network based on topology theory [16] [6].

Recent studies have also proposed a combination of state space and nodal analysis for simulations of power systems [17]. At the centre of the variables in a state space formulation are the energy storage devices such as the inductors and capacitors [18]. Any electrical circuit with reactive elements can functionally be represented by its state variables in a state space using matrix format system equations [19].

3. Methods

The methodology outlines the sequential steps undertaken in a research study [20]. The aim of this study is to develop a repeatable state space model for CCS tapping and CCS injection application where a CCS can be used to tap electrical power from a transmission line or can be used for injection electrical power from a microgrid into a transmission line. This study is conducted using a proposed 400kV/11kV CCS with a CCS Load of 80kW and a micro-grip injection of a steady 80kW generated from a fixed generating source such as a photovoltaic plant.

3.1. CCS Power Tapping

A 400kV/11kV CCS is used for this study. The load tapped from the transmission network is a fixed resistive load of 80kW. An equivalent nominal-T medium transmission line representation is given by Figure 1 with a CCS tapping node.

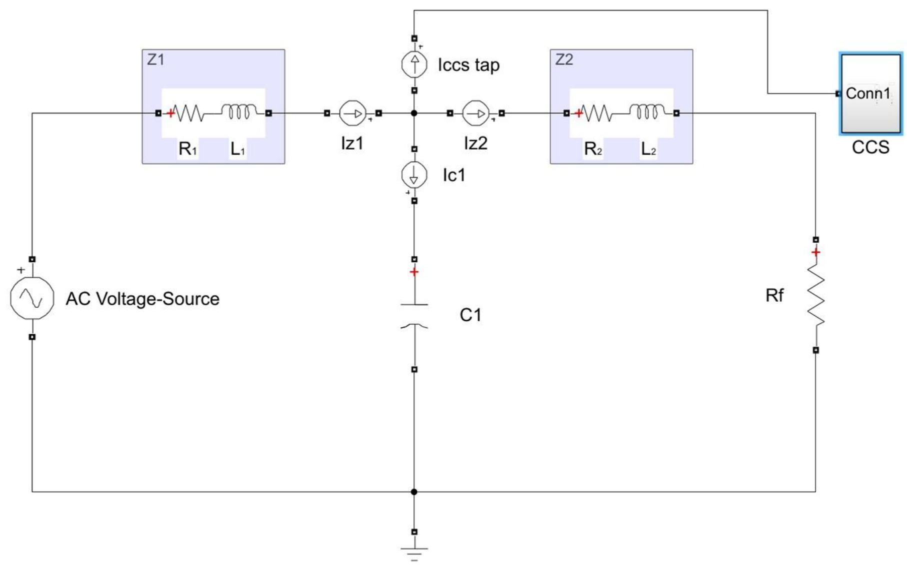

Figure 1: Nominal-T Transmission Line Equivalent Circuit with CCS Tap

Figure 1 presents an equivalent circuit utilizing a CCS to extract electrical power from an HV transmission line, with $I_{CCS_{TAP}}$ representing the tapped power from a transmission line into the CCS circuit. This Figure is used to develop state space variable equations when power is tapped by a CCS from the transmission network, and the resulting equations are outlined below using a loop method. The equivalent circuit is condensed into two loops.

Utilizing Kirchhoff's Laws [21], both the junction law for current and the loop rule for voltage yields the subsequent formulae.

Loop 1:

$$V_s = V_{Z_1} + V_{C_1}$$

$$V_s = V_{R_1} + V_{L_1} + V_{C_1} \quad (1)$$

and,

$$I_{Z_1} = I_{Z_2} + I_{C_1} + I_{CCS_{TAP}}$$

$$I_{C_1} = I_{Z_1} - I_{Z_2} - I_{CCS_{TAP}} \quad (2)$$

Loop 2:

$$-V_{C_1} + V_{Z_2} + V_{R_f} = 0$$

$$-V_{C_1} + (V_{R_2} + V_{L_2}) + V_{R_f} = 0$$

$$V_{L_2} = V_{C_1} - V_{R_2} - V_{R_f}$$

$$V_{L_2} = V_{C_1} - R_2 I_2 - V_{R_f} \quad (3)$$

And,

$$I_{Z_1} = I_{Z_2} + I_{C_1} + I_{CCS_{TAP}}$$

$$I_{Z_2} = I_{Z_1} - I_{C_1} + I_{CCS_{TAP}} \quad (4)$$

3.2. CCS Power Injection

Similar to the CCS power tapping configuration, the CCS power injection approach in this study entails reversing the direction of current flow at the tapping/injection node within the electrical transmission network.

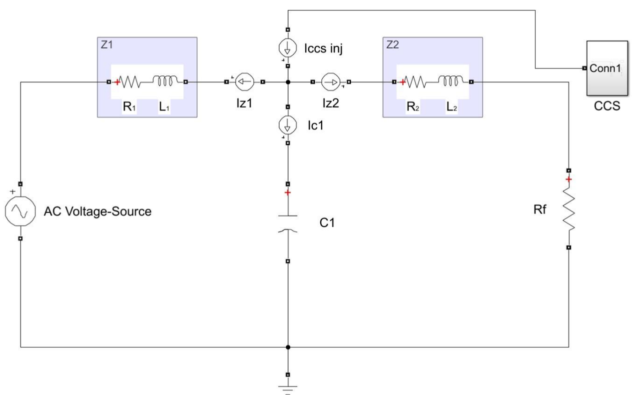

Figure 2: Nominal-T Transmission Line Equivalent Circuit with CCS Power Injection

Figure 2 presents an equivalent circuit where a CCS is used to inject electrical power from an electrical energy source into an HV transmission network, with $I_{CCS_{INJ}}$ representing the output from the CCS circuit into the transmission network. Figure 2 is used for deriving variable state equations for power injection into a transmission network from a CCS. The resulting equations are presented below using a loop method, similar to the methodology used in the power tapping circuit.

Loop 1:

$$\mathrm{V}_s = \mathrm{V}_{C_1} - \mathrm{V}_{Z_1}$$

$$\mathrm{V}_s = \mathrm{V}_{C_1} - \mathrm{V}_{R_1} - \mathrm{V}_{L_1} \tag{5}$$

and,

$$\mathrm{I}_{CCS_{INJ}} = \mathrm{I}_{Z_1} + \mathrm{I}_{Z_2} + \mathrm{I}_{C_1}$$

$$\mathrm{I}_{C_1} = \mathrm{I}_{CCS_{INJ}} - \mathrm{I}_{Z_1} - \mathrm{I}_{Z_2} \tag{6}$$

Loop 2:

$$\mathrm{V}_{C_1} + \mathrm{V}_{Z_2} + \mathrm{V}_{R_f} = 0$$

$$\mathrm{V}_{C_1} + (\mathrm{V}_{R_2} + \mathrm{V}_{L_2}) + \mathrm{V}_{R_f} = 0$$

$$\mathrm{V}_{L_2} = -\mathrm{V}_{C_1} - \mathrm{V}_{R_2} - \mathrm{V}_{R_f}$$

$$\mathrm{V}_{L_2} = -\mathrm{V}_{C_1} - \mathrm{R}_2\mathrm{I}_2 - \mathrm{V}_{R_f} \tag{7}$$

And,

$$\mathrm{I}_{CCS_{INJ}} = \mathrm{I}_{Z_1} + \mathrm{I}_{Z_2} + \mathrm{I}_{C_1}$$

$$\mathrm{I}_{Z_2} = \mathrm{I}_{CCS_{INJ}} - \mathrm{I}_{Z_1} - \mathrm{I}_{C_1} \tag{8}$$

3.3. State Space Formulation

The state space formulation is developed using a two loops in a nominal-T transmission line equivalent circuit as shown on the Figure 1 and Figure 2. The objective of the state space formulation is to provide a streamlined representation of the electrical system for modeling purposes [22]. State variables are chosen according to energy storage components within the system, namely, the voltage across the capacitors (V_C) and the current flowing through the inductors (I_L). Therefore, the selected state variables are: I_{L_1}, V_{C_1} and I_{L_2}. The state variables can be solved analytically within the time-domain [23].

Hence, from (1), (2), (3), (4), (5), (6), (7) and (8), the state space of the system can be given by the following equation [23] :

$$\left.\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned}\right\}$$

Where:

$\dot{x}(t) = Ax(t) + Bu(t)$ is the state equation,

$y(t) = Cx(t) + Du(t)$ is the output equation,

A = state matrix, B = input matrix, C = output matrix, and D = direct transmission matrix,

$u(t)$ = input vector, $y(t)$ = output vector, and $x(t)$ = state vector.

The state vector is determined through a matrix $[I_{L_1}, V_{C_1}, I_{L_2}]^T$

The selection of State Space variables adhered to Kirchhoff's Current Law (KCL) and Kirchhoff's Voltage Law (KVL), as outlined in Table 1:

Table 1: State Variables

Storage Device	Variable	Vector	Derivative	Derivative Representation
L_1	I_{L_1}	x_1	$L_1 \frac{di_{L_1}}{dt}$	$\dot{x}_1$
C_1	V_{C_1}	x_2	$C_1 \frac{dV_{C_1}}{dt}$	$\dot{x}_2$
L_2	I_{L_2}	x_3	$L_2 \frac{di_{L_2}}{dt}$	$\dot{x}_3$

4. Results and Discussion

The purpose of this article is to develop a state space representation of a CCS electrical power tapping system and a CCS electrical power injection system connected to a typical nominal-T electrical transmission line. The formulated state space representation is presented in this section.

4.1.1. *CCS Electrical Power Tapping*

The state space for power tapping is developed as detailed below.

Applying Kirchhoff's Laws, both the junction law for the current and the loop rule for the voltage, gives the loops as follows:

Loop 1:

$$\mathrm{V}_s = \mathrm{V}_{Z_1} + \mathrm{V}_{C_1} \Rightarrow \mathrm{V}_s = \mathrm{V}_{R_1} + \mathrm{V}_{L_1} + \mathrm{V}_{C_1}$$

Since the inductor's Ohm's law gives: $\mathrm{V} = \mathrm{L}\,\frac{\mathrm{di}}{\mathrm{dt}}$ and $\mathrm{V} = RI$ for a resistor, therefore:

$$\mathrm{V}_s = \mathrm{R}_1\mathrm{I}_1 + \mathrm{L}_1\frac{\mathrm{di}_{L_1}}{\mathrm{dt}} + \mathrm{V}_{C_1}$$

$$\frac{\mathrm{di}_{L_1}}{\mathrm{dt}} = \frac{1}{\mathrm{L}_1}\mathrm{V}_s - \frac{\mathrm{R}_1}{\mathrm{L}_1}x_1 + \frac{1}{\mathrm{L}_1}x_2$$

Thus, the first state variable ($\dot{\boldsymbol{x}}_{\mathbf{1}}$) is given by:

$$\dot{x}_1 = \frac{1}{\mathrm{L}_1}\mathrm{V}_s - \frac{\mathrm{R}_1}{\mathrm{L}_1}x_1 + \frac{1}{\mathrm{L}_1}x_2 \tag{9}$$

Similarly:

On the same Loop 1:

$$\mathrm{I}_{Z_1} = \mathrm{I}_{Z_2} + \mathrm{I}_{C_1} + \mathrm{I}_{CCS_{TAP}}$$

And the current across a capacitor is given by: $\mathrm{I}_c = \mathrm{C}\,\frac{\mathrm{dv}}{\mathrm{dt}}$

Therefore:

$$\mathrm{C}_1\frac{\mathrm{dV}_{C_1}}{\mathrm{dt}} = \mathrm{I}_{Z_1} - \mathrm{I}_{Z_2} - \mathrm{I}_{CCS_{TAP}}$$

$$\frac{\mathrm{dV}_{C_1}}{\mathrm{dt}} = \frac{1}{\mathrm{C}_1}x_1 - \frac{1}{\mathrm{C}_1}x_3 - \frac{1}{\mathrm{C}_1}\mathrm{I}_{CCS_{TAP}}$$

Thus, the second state variable ($\dot{\boldsymbol{x}}_{\mathbf{2}}$) is given by:

$$\dot{x}_2 = \frac{1}{\mathrm{C}_1}x_1 - \frac{1}{\mathrm{C}_1}x_3 - \frac{1}{\mathrm{C}_1}\mathrm{I}_{CCS_{TAP}} \tag{10}$$

Loop 2**: (**where R_f is the high impedance load)

$$-\mathrm{V}_{C_1} + \mathrm{V}_{Z_2} + \mathrm{V}_{R_f} = 0$$

$$-\mathrm{V}_{C_1} + (\mathrm{V}_{R_2} + \mathrm{V}_{L_2}) + \mathrm{V}_{R_f} = 0$$

$$V_{L_2} = V_{C_1} - R_2 I_2 - V_{R_f}$$

$$\frac{di_{L_2}}{dt} = \frac{1}{L_2} V_{C_1} - \frac{1}{L_2} I_2 (R_2 - R_f)$$

Therefore, the third state variable ($\dot{\boldsymbol{x}}_{\boldsymbol{2}}$) is given by:

$$\dot{x}_3 = \frac{1}{L_2} x_2 - \frac{R_2 + R_f}{L_2} x_3 \tag{11}$$

The final functions derived and that can be used for the state space modeling of the electrical power tapping application are given as (12), (13) and (14):

$$\dot{x}_1 = - \frac{R_1}{L_1} x_1 + \frac{1}{L_1} x_2 + \frac{1}{L_1} V_s \tag{12}$$

$$\dot{x}_2 = \frac{1}{C_1} x_1 - \frac{1}{C_1} x_3 - \frac{1}{C_1} I_{CCS_{TAP}} \tag{13}$$

$$\dot{x}_3 = \frac{1}{L_2} x_2 - \frac{R_2 + R_f}{L_2} x_3 \tag{14}$$

The following are thus the matrices developed:

$$A\ is\ the\ state\ matrix = \begin{bmatrix} -\frac{R_1}{L_1} & \frac{1}{L_1} & 0 \\ \frac{1}{C_1} & 0 & -\frac{1}{C_1} \\ 0 & \frac{1}{L_2} & -\frac{R_2 + R_f}{L_2} \end{bmatrix}$$

$$B\ is\ the\ input\ matrix = \begin{bmatrix} \frac{1}{L_1} & 0 & 0 \\ 0 & -\frac{1}{C_1} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$C\ is\ the\ output\ matrix = \begin{bmatrix} 0 & 0 & R_f \end{bmatrix}$$

$$D\ is\ the\ direct\ transmission\ matrix = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Therefore, the state equation $\dot{x}(t) = Ax(t) + Bu(t)$ is given by:

$$\dot{x}(t) = Ax(t) + Bu(t) =\gg \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -\frac{R_1}{L_1} & \frac{1}{L_1} & 0 \\ \frac{1}{C_1} & 0 & -\frac{1}{C_1} \\ 0 & \frac{1}{L_2} & -\frac{R_2+R_f}{L_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} \frac{1}{L_1} & 0 & 0 \\ 0 & -\frac{1}{C_1} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_s \\ I_{CCS_{TAP}} \\ 0 \end{bmatrix}$$

And the output equation $y(t) = Cx(t) + Du(t)$ is given by:

$$y(t) = Cx(t) + Du(t)$$

$$=\gg V_o = [0 \quad 0 \quad R_f] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} I_{CCS_{TAP}}$$

4.1.2. ***CCS Electrical Power Injection***

The state space for electrical power injection is developed by applying Kirchhoff's Laws, both the junction law for the current and the loop rule for the voltage, which gives the loops results as:

Loop 1:

$$V_s = V_{C_1} - V_{Z_1} \;=>\; V_s = V_{C_1} - V_{L_1} - V_{R_1}$$

Since the inductor's Ohm's law gives: $V = L\,\frac{di}{dt}$ and $V = RI$ for a resistor, therefore:

$$V_s = V_{C_1} - L_1 \frac{di_{L_1}}{dt} - R_1 I_1$$

$$\frac{di_{L_1}}{dt} = -\frac{R_1}{L_1}x_1 + \frac{1}{L_1}x_2 - \frac{1}{L_1}V_s$$

Thus, the first state variable ($\dot{\boldsymbol{x}}_\mathbf{1}$) is given by:

$$\dot{x}_1 = -\frac{R_1}{L_1}x_1 + \frac{1}{L_1}x_2 - \frac{1}{L_1}V_s \tag{15}$$

Similarly:

On the same Loop 1:

$$I_{CCS_{INJ}} = I_{Z_1} + I_{Z_2} + I_{C_1}$$

$$I_{C_1} = I_{CCS_{INJ}} - I_{Z_1} - I_{Z_2}$$

And current across a capacitor is given by: $\mathrm{I}_c = \mathrm{C}\frac{\mathrm{dv}}{\mathrm{dt}}$

Therefore:

$$\mathrm{C}_1\frac{\mathrm{dV}_{C_1}}{\mathrm{dt}} = \mathrm{I}_{CCS_{INJ}} - \mathrm{I}_{Z_1} - \mathrm{I}_{Z_2}$$

$$\frac{\mathrm{dV}_{C_1}}{\mathrm{dt}} = -\frac{1}{\mathrm{C}_1}x_1 - \frac{1}{\mathrm{C}_1}x_3 + \frac{1}{\mathrm{C}_1}\mathrm{I}_{CCS_{INJ}}$$

Thus, the second state variable ($\dot{\boldsymbol{x}}_{\mathbf{2}}$) is given by:

$$\dot{x}_2 = -\frac{1}{\mathrm{C}_1}x_1 - \frac{1}{\mathrm{C}_1}x_3 + \frac{1}{\mathrm{C}_1}\mathrm{I}_{CCS_{INJ}} \tag{16}$$

Loop 2**: (**where $\mathrm{R}_f$is the high impedance load)

$$\mathrm{V}_{L_2} =- \mathrm{V}_{C_1} - \mathrm{V}_{R_2} - \mathrm{V}_{R_f}$$

$$\mathrm{V}_{L_2} =- \mathrm{V}_{C_1} - \mathrm{R}_2\mathrm{I}_2 - \mathrm{V}_{R_f}$$

$$\frac{\mathrm{di}_{L_2}}{\mathrm{dt}} =- \frac{1}{\mathrm{L}_2}\mathrm{V}_{C_1} - \frac{1}{\mathrm{L}_2}\mathrm{I}_2(2\mathrm{R}_2 + \mathrm{R}_f)$$

Therefore, the third state variable ($\dot{\boldsymbol{x}}_{\mathbf{2}}$) is given by:

$$\dot{x}_3 =- \frac{1}{\mathrm{L}_2}x_2 - \frac{2\mathrm{R}_2 + \mathrm{R}_f}{\mathrm{L}_2}x_3 \tag{17}$$

The functions used for the state space modeling of the electrical power injection are derived as (18), (19) and (20):

$$\dot{x}_1 =- \frac{\mathrm{R}_1}{\mathrm{L}_1}x_1 + \frac{1}{\mathrm{L}_1}x_2 - \frac{1}{\mathrm{L}_1}\mathrm{V}_s \tag{18}$$

$$\dot{x}_2 = -\frac{1}{\mathrm{C}_1}x_1 - \frac{1}{\mathrm{C}_1}x_3 + \frac{1}{\mathrm{C}_1}\mathrm{I}_{CCS_{INJ}} \tag{19}$$

$$\dot{x}_3 =- \frac{1}{\mathrm{L}_2}x_2 - \frac{2\mathrm{R}_2 + \mathrm{R}_f}{\mathrm{L}_2}x_3 \tag{20}$$

The following are thus the matrices developed:

$$A \text{ is the state matrix} = \begin{bmatrix} -\frac{R_1}{L_1} & \frac{1}{L_1} & 0 \\ -\frac{1}{C_1} & 0 & -\frac{1}{C_1} \\ 0 & -\frac{1}{L_2} & -\frac{2\mathrm{R}_2 + \mathrm{R}_f}{\mathrm{L}_2} \end{bmatrix}$$

$$B \text{ is the input matrix} = \begin{bmatrix} \frac{1}{L_1} & 0 & 0 \\ 0 & -\frac{1}{\mathrm{C}_1} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$C \text{ is the output matrix} = \begin{bmatrix} 0 & 0 & \mathrm{R}_f \end{bmatrix}$$

$$D \text{ is the direct transmission matrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Therefore, the state equation $\dot{x}(t) = Ax(t) + Bu(t)$ is given by:

$$\dot{x}(t) = Ax(t) + Bu(t) \Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -\frac{R_1}{L_1} & \frac{1}{L_1} & 0 \\ -\frac{1}{C_1} & 0 & -\frac{1}{C_1} \\ 0 & -\frac{1}{L_2} & -\frac{2\mathrm{R}_2+\mathrm{R}_f}{\mathrm{L}_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} \frac{1}{L_1} & 0 & 0 \\ 0 & -\frac{1}{\mathrm{C}_1} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathrm{V}_s \\ \mathrm{I}_{CCS_{INJ}} \\ 0 \end{bmatrix}$$

And the output equation $y(t) = Cx(t) + Du(t)$ is given by:

$$y(t) = Cx(t) + Du(t)$$

$$\Rightarrow V_o = \begin{bmatrix} 0 & 0 & \mathrm{R}_f \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \mathrm{I}_{CCS_{INJ}}$$

4.2. State Space Solution

A state space solution refers to finding the mathematical description of how a system's variables change over time. This solution involves determining the values of the system's state variables at different points in time based on initial conditions and external inputs [24]. The initial condition response can be modelled using the standard MATLAB/Simulink initial condition response. The initial condition MATLAB code $sys, x(0)$ can be used to simulate the initial response of a state-space (*ss*) model (*sys)* with an initial condition of the states being the vector $x(0)$ where [25]:

$\dot{x} = Ax,\ where\ x(0) = x_0,$ and $y = Cx$

For this study, the response of the developed state-space model to initial condition can then be plotted based on the following matrices and taking the initial condition as:

$$x(0) = x_0 \Rightarrow x(0) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

With,

$$\dot{x} = Ax \Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -\frac{R_1}{L_1} & \frac{1}{L_1} & 0 \\ \frac{1}{C_1} & 0 & -\frac{1}{C_1} \\ 0 & \frac{1}{L_2} & -\frac{R_2+R_f}{L_2} \end{bmatrix},$$

$$y = Cx \Rightarrow y = \begin{bmatrix} 0 & 0 & R_f \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \text{ for power tapping, and}$$

$$\dot{x} = Ax \Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -\frac{R_1}{L_1} & \frac{1}{L_1} & 0 \\ -\frac{1}{C_1} & 0 & -\frac{1}{C_1} \\ 0 & \frac{1}{L_2} & -\frac{2R_2+R_f}{L_2} \end{bmatrix},$$

$$y = Cx \Rightarrow y = \begin{bmatrix} 0 & 0 & R_f \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \text{ for power injection.}$$

5. Conclusions

The aim of this study is limited to the development of a state space representation of the two CCS application that are for electrical power tapping and electrical power injection. State space formulation presents a unified and flexible approach to modeling electrical circuits, accommodating complex configurations with non-linear elements and time-varying parameters. This framework facilitates systematic analysis using linear algebra and control theory techniques, allowing for design feedback control strategies and assess circuit stability and performance. State space models are efficiently simulated using computational tools, aiding in thorough evaluation and validation of circuit designs. Additionally, they offer an intuitive representation of circuit dynamics, simplifying implementation and modification processes. Overall, state space formulation enhances understanding, analysis, and control of electrical circuits, making it a valuable tool in engineering practice.

Additionally, state space matrices enable efficient numerical simulation using computational tools, assisting in the validation and optimization of system performance. This approach enhances the understanding and design of dynamic systems across diverse engineering domains, ensuring robustness and efficiency in real-world applications.

This article presents state space matrices for a Capacitor Coupled Substation in the electrical power tapping and electrical power injection application. It, therefore, offers a state space representative for system modeling that can be implemented in system modeling for the CCS tapping and CCS injection application.

6. List of abbreviations

- CCS – Capacitor Coupled Substation
- HV – High Voltage
- CNT – Controllable Network Transformer
- kV – kilo-Volts
- kW – kilo-Watts
- KCL – Kirchhoff’s Current Law
- KVL - Kirchhoff's Voltage Law

7. Declarations

7.1. Availability of data and materials

- Not Applicable

7.2. Competing interests

- The authors declare that they have no competing interests

7.3. Funding

- Not Applicable

7.4. Authors' contributions

- SN drafted and wrote the article. All authors read and approved the final manuscript

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7.6. Authors' information (optional)

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