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Knowledge Mining Based on the Generalized Modal Syllogism A□MI-1

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Abstract

This paper specifically focuses on the validity of generalized modal syllogism (i.e. A□MI-1) that contains the quantifier '*most*'. By making full use of generalized quantifier theory, modal logic and set theory, this paper derives 24 valid generalized modal syllogisms based on the validity of the syllogism A□MI-1. This method provides a concise mathematical framework that contributes to knowledge mining for generalized modal syllogism fragments.

Keywords: generalized modal syllogisms; validity; modality; reducibility

1. Introduction

Syllogistic reasoning plays an important role in both logic and natural language, as acknowledged by scholars such as Łukasiewicz (1957) and Moss (2008). There are various types of syllogisms, including Aristotelian syllogisms (Moss, 2010; Long, 2023; Haiping and

Xiaojun, 2024), Aristotelian modal syllogisms (Malink, 2006; Feifei, 2024), generalized syllogisms (Murinová and Novák, 2012; Baoxiang, 2024), and generalized modal syllogisms (Mingwei and Qing, 2024).

Due to the large number of generalized quantifiers in natural language (Xiaojun, 2018) and few studies on them, this paper specifically studies the generalized modal syllogisms, particularly those containing the common quantifier '*most*'.

2. Preliminaries

In the following, let *l*, *v* and *b* be lexical variables, and form sets *L, V*, and *B* using these variables. *D* denotes the domain of lexical variables. Let ξ, θ, ψ and φ be well-formed formulas (abbreviated as wff). The expression '$|L \cap B|$' denotes the cardinality of the intersection of the set *L* and *B*. '⊢ ' signifies that the wff is provable, and '$\xi =_{def} \varphi$' states that ξ can be defined by φ. '□' represents the necessary modality, and '◇' the possible one. The operators (such as ¬, →, ∧, ↔) in this paper are the common symbols in classical first-order logic (Barwise, 1977) and set theory (Kunen, 1980).

This paper only studies non-trivial generalized modal syllogisms involving the following 8 quantifiers: *all*, *no*, *some*, *not all, most, fewer than half of the, at most half of the, at least half of the*, which are respectively abbreviated as Proposition *A*, *E*, *I*, *O*, *M*, *F*, *H* and *S* (Jun and Mingwei, 2024). They can be respectively expressed as the following: *all(l, b)*, *no(l, b)*, *some(l, b)*, *not all(l, b)*, *most(l, b)*, *fewer than half of the(l, b)*, *at most half of the(l, b)*, and *at least half of the(l, b)*. Let *Q* be any of the above 8 quantifiers, ¬*Q* its outer quantifier and *Q*¬ its inner one.

A generalized modal syllogism is obtained by adding 'at least one and at most three non-trivial' necessary modality (□) or possible modality (◇) to a generalized syllogism (Liheng, 2024). Therefore, the generalized modal syllogisms in this paper involve the following 24 types of propositions: (1) *all(l, b)*, *no(l, b)*, *some(l, b)*, *not all(l, b)*, *most(l, b)*, *fewer than half of the(l, b)*, *at most half of the(l, b)*, and *at least half of the(l, b)*. (2) □*all(l, b)*, □*no(l, b)*, □*some(l, b)*, □*not all(l, b)*, □*most(l, b)*, □*fewer than half of the(l, b)*, □*at most half of the(l, b)*, and □*at least half of the(l, b)*. (3) ◇*all(l, b)*, ◇*no(l, b)*, ◇*some(l, b)*, ◇*not all(l, b)*, ◇*most(l, b)*, ◇*fewer than half of the(l, b)*, ◇*at most half of the(l, b)*, and ◇*at least half of the(l, b)*. The syllogism used as the basis for reasoning in this paper is the generalized modal syllogism A□MI-1. Its instance is as follows:

Major premise: All millionaires are wealthy.

Minor premise: Most NBA players are necessarily millionaires.

Conclusion: Some NBA players are wealthy.

Let *v* be a lexical variable that represents a millionaire, *b* be a lexical variable denoting people who are wealthy, and *l* be a lexical variable that stands for a NBA player. Then the above example can be formalized as $all(v, b) \wedge \Box most(l, v) \rightarrow some(l, b)$, which can be abbreviated as A□MI-1. The Others are similar. If not otherwise specified, the following syllogisms refer to non-trivial generalized modal syllogisms.

3. The Axiomatic System of Generalized Modal Syllogisms

This formalized axiom system is composed of the following: primitive symbols, formation rules and axioms, etc.

3.1 Primitive Symbols

(1) lexical variables: *l*, *v*, *b*

(2) quantifier: *all*, *most*

(3) modality: □

(4) operator: ¬, →

(5) brackets: (,)

3.2 Formation Rules

(1) If *Q* is a quantifier, *l* and *b* are lexical variables, then *Q(l, b)* is a wff.

(2) If ξ is a wff, then so are $\neg\xi$ and $\Box\xi$.

(3) If ξ and θ are wffs, then so is $\xi\rightarrow\theta$.

(4) The set of all wffs is generated by the above rules.

3.3 Basic Axioms

A1: If α is a valid formula in first-order logic, then $\vdash \alpha$.

A2: $\vdash all(v, b) \wedge \Box most(l, v) \rightarrow some(l, b)$(that is, the syllogism A□MI-1).

3.4 Rules of Deduction

Rule 1 (subsequent weakening): From ⊢ $(\xi\wedge\theta\rightarrow\psi)$ and ⊢ $(\psi\rightarrow\varphi)$ infer ⊢ $(\xi\wedge\theta\rightarrow\varphi)$.

Rule 2 (anti-syllogism): From ⊢ $(\xi\wedge\theta\rightarrow\psi)$ infer ⊢ $(\neg\psi\wedge\xi\rightarrow\neg\theta)$.

Rule 3 (anti-syllogism): From ⊢ $(\xi\wedge\theta\rightarrow\psi)$ infer ⊢ $(\neg\psi\wedge\theta\rightarrow\neg\xi)$.

3.5 Semantics

Let $\Omega=(D, \Re)$ be a model, in which $D\neq\varnothing$, and $\Re$ be an interpretation, where

$\Re(l)=L$, $L\subseteq D$ and $L\neq\varnothing$.

$\Re(v)=V$, $V\subseteq D$ and $V\neq\varnothing$.

$\Re(b)=B$, $B\subseteq D$ and $B\neq\varnothing$.

$\Re(d-x)=D-\Re(x)$, in which *x is l, v or b.*

If a wff ξ is true in Ω under an interpretation $\Re$, one can say that $\Omega, \Re\vDash\xi$.

(S1) $\Omega, \Re\vDash$ *all(l, b)*, just in case, $\Re(l)\subseteq\Re(b)$, that is $L\subseteq B$;

(S2) $\Omega, \Re\vDash$ *not all(l, b)*, just in case, $L\nsubseteq B$;

(S3) $\Omega, \Re\vDash$ *no(l, b)*, just in case, $L\cap B=\varnothing$;

(S4) $\Omega, \Re\vDash$ *some(l, b)*, just in case, $L\cap B\neq\varnothing$;

(S5) $\Omega, \Re\vDash$ *most(l, b)*, just in case, $|L\cap B|>0.5|L|$;

(S6) $\Omega, \Re\vDash$ *at most half of(l, b)*, just in case, $|L\cap B|\leq0.5|L|$;

(S7) $\Omega, \Re\vDash$ *few than half of(l, b)*, just in case, $|L\cap B|<0.5|L|$;

(S8) $\Omega, \Re\vDash$ *at least half of(l, b)*, just in case, $|L\cap B|\geq0.5|L|$.

If ξ is true under all interpretations in a model, one can say that ξ is valid in that model (that is, $\Omega\vDash\xi$). If ξ is valid in all models, one can say that ξ is valid (that is, $\vDash\xi$).

3.6 Relevant Definitions

D1: $(\xi\wedge\theta)=_{def}\neg(\xi\rightarrow\neg\theta)$;

D2: $(\xi\leftrightarrow\theta)=_{def}(\xi\rightarrow\theta)\wedge(\xi\rightarrow\theta)$;

D3: $(Q\neg)(l, b)=_{def}Q(l, D-b)$

D4: (¬*Q*)*(l, b)*=$_{\text{def}}$ It is not that *Q(l, b)*

D5: ◇*Q(l, b)*=$_{\text{def}}$ ¬□¬*Q(l, b)*

D6: *all(l, b)* is true iff $L \subseteq B$ is true in any real world;

D7: *some(l, b)* is true iff $L \cap B \neq \varnothing$ is true in any real world;

D8: *no(l, b)* is true iff $L \cap B = \varnothing$ is true in any real world;

D9: *not all(l, b)* is true iff $L \nsubseteq B$ is true in any real world;

D10: *most(l, b)* is true iff $|L \cap B| > 0.5|L|$ is true in any real world;

D11: □*most(l, b)* is true iff $|L \cap B| > 0.5|L|$ is true in any possible world;

The true value definitions of other quantifiers can be given similarly.

3.6 Relevant Facts

Fact 1(Inner Negation):

(1.1) ⊢ *all(l, b)*↔*no*¬*(l, b)*;

(1.2) ⊢ *no(l, b)*↔*all*¬*(l, b)*;

(1.3) ⊢ *some(l, b)*↔*not all*¬*(l, b)*;

(1.4) ⊢ *not all(l, b)*↔*some*¬*(l, b)*;

(1.5) ⊢ *most(l, b)*↔*fewer than half of the*¬*(l, b)*;

(1.6) ⊢ *fewer than half of the(l, b)*↔*most*¬*(l, b)*;

(1.7) ⊢ *at least half of the(l, b)*↔*at most half of the (l, b)*;

(1.8) ⊢ *at most half of the(l, b)*↔*at least half of the (l, b)*.

Fact 2(Outer Negation):

(2.1) ⊢ ¬*all(l, b)*↔*not all(l, b)*;

(2.2) ⊢ ¬*not all(l, b)*↔*all(l, b)*;

(2.3) ⊢ ¬*no(l, b)*↔*some(l, b)*;

(2.4) ⊢ ¬*some(l, b)*↔*no(l, b)*;

(2.5) ⊢ ¬*most(l, b)*↔*at most half of the(l, b)*;

(2.6) ⊢ ¬*at most half of the(l, b)*↔*most(l, b)*;

(2.7) ⊢ ¬*fewer than half of the(l, b)*↔*at least half of the(l, b)*;

(2.8) ⊢ ¬*at least half of the(l, b)*↔*fewer than half of the(l, b)*.

Fact 3(Symmetry):

(3.1) ⊢ *some(l, b)*↔*some(b, l)*;

(3.2) ⊢ *no(l, b)*↔*no(b, l)*.

Fact 4 (Dual):

(4.1) ⊢ ¬□*Q(l, b)*↔◇¬*Q(l, b)*;

(4.2) ⊢ ¬◇*Q(l, b)*↔□¬*Q(l, b)*.

Fact 5 (Subordination):

(5.1) ⊢ *all(l, b)*→*some(l, b)*;

(5.2) ⊢ *no(l, b)*→*not all(l, b)*;

(5.3) ⊢ □*Q(l, b)*→*Q(l, b)*;

(5.4) ⊢ □*Q(l, b)*→◇*Q(l, b)*;

(5.5) ⊢ *Q(l, b)*→◇*Q(l, b)*.

The above facts are elementary knowledge in generalized quantifier theory (Peters and Westerståhl, 2006) and modal logic (Chagrov and Zakharyaschev, 1997), and their proofs are omitted.

4. The Other Generalized Modal Syllogisms Derived from A□MI-1

The following Theorem 1 proves the validity of the syllogism A□MI-1. In Theorem 2, the expression '(1) A□MI-1→□MAI-4' asserts the validity of syllogism □MAI-4 deduced from the validity of syllogism A□MI-1, which means that there is reducibility between them. The others are similar.

Theorem 1 (A□MI-1): The generalized modal syllogism *all(v, b)*∧□*most(l, v)*→*some(l, b)* is valid.

Proof: Suppose that *all(v, b)* and □*most(l, v)* are true, then it is clear that $V \subseteq B$ is true in any real world according to Definition D6, and $|L\cap V|>0.5|L|$ is true in any possible world according to Definition D11. Due to the fact that a necessary world is also a real world, one can conclude that $|L\cap V|>0.5|L|$ is true in any real world. And it follows that $L\cap B\neq\varnothing$ is true in any real world. Thus, it can be seen that *some(l, b)* in the light of Definition D7, just as desired.

Theorem 2: The following 24 valid generalized modal syllogisms can be obtained from A□MI-1:

(1) A□MI-1→□MAI-4

(2) A□MI-1→AE◇H-2

(3) A□MI-1→AE◇H-2→AE◇H-4

(4) A□MI-1→AE◇H-2→EA◇H-2

(5) A□MI-1→E□MO-3

(6) A□MI-1→E□MO-3→E□MO-4

(7) A□MI-1→E□MO-3→A□MI-3

(8) A□MI-1→E□MO-3→A□MI-3→□MAI-3

(9) A□MI-1→E□MO-3→A□MI-3→□MAI-3→EA◇H-1

(10) A□MI-1→E□MO-3→A□MI-3→□MAI-3→E□MO-2

(11) A□MI-1→E□MO-3→A□MI-3→□MAI-3→□FAO-3

(12) A□MI-1→E□MO-3→A□MI-3→□MAI-3→□FAO-3→AA◇S-1

(13) A□MI-1→E□MO-3→A□MI-3→□MAI-3→□FAO-3→A□FO-2

(14) A□MI-1→E□MO-1

(15) A□MI-1→A□M◇I-1

(16) A□MI-1→□MAI-4→□MA◇I-4

(17) A□MI-1→E□MO-3→E□M◇O-3

(18) A□MI-1→E□MO-3→E□MO-4→E□M◇O-4

(19) A□MI-1→E□MO-3→A□MI-3→A□M◇I-3

(20) A□MI-1→E□MO-3→A□MI-3→□MAI-3→□MA◇I-3

(21) A□MI-1→E□MO-3→A□MI-3→□MAI-3→E□MO-2→E□M◇O-2

(22) A□MI-1→E□MO-3→A□MI-3→□MAI-3→□FAO-3→□FA◇O-3

(23) A□MI-1→E□MO-3→A□MI-3→□MAI-3→□FAO-3→A□FO-2→A□F◇O-2

(24) A□MI-1→E□MO-1→E□M◇O-1

Proof:

[1] ⊢*all(v, b)*∧□*most(l, v)*→*some(l, b)* (i.e. A□MI-1, Axiom A2)

[2] ⊢*all(v, b)*∧□*most(l, v)*→*some(b, l)* (i.e. □MAI-4, by [1] and Fact (3.1))

[3] $\vdash\neg some(l, b)\wedge all(v, b)\rightarrow\neg\Box most(l, v)$ (by [1] and Rule 2)

[4] $\vdash no(l, b)\wedge all(v, b)\rightarrow\Diamond\neg most(l, v)$ (by [3], Fact (2.4) and Fact (4.1))

[5] $\vdash no(l, b)\wedge all(v, b)\rightarrow\Diamond$ *at most half of the(l, v)* (i.e. AE◇H-2, by [4] and Fact (2.5))

[6] $\vdash no(b, l)\wedge all(v, b)\rightarrow\Diamond$ *at most half of the(l, v)* (i.e. AE◇H-4, by [5] and Fact (3.2))

[7] $\vdash all\neg(l, b)\wedge no\neg(v, b)\rightarrow\Diamond$ *at most half of the(l, v)* (by [5], Fact (1.1) and Fact (1.2))

[8] $\vdash all(l, D-b)\wedge no(v, D-b)\rightarrow\Diamond$ *at most half of the(l, v)*

(i.e. EA◇H-2, by [7] and Definition D3)

[9] $\vdash\neg some(l, b)\wedge\Box most(l, v)\rightarrow\neg all(v, b)$ (by [1] and Rule 3)

[10] $\vdash no(l, b)\wedge\Box most(l, v)\rightarrow$ *not all(v, b)* (i.e. E□MO-3, by [9], Fact (2.1) and Fact (2.4))

[11] $\vdash no(b, l)\wedge\Box most(l, v)\rightarrow$ *not all(v, b)* (i.e. E□MO-4, by [10] and Fact (3.2))

[12] $\vdash all\neg(l, b)\wedge\Box most(l, v)\rightarrow some\neg(v, b)$ (by [10], Fact (1.2) and Fact (1.4))

[13] $\vdash all(l, D-b)\wedge\Box most(l, v)\rightarrow some(v, D-b)$ (i.e. A□MI-3, by [12] and Definition D3)

[14] $\vdash all(l, D-b)\wedge\Box most(l, v)\rightarrow some(D-b, v)$ (i.e. □MAI-3, by [13] and Fact (3.1))

[15] $\vdash\neg some(D-b, v)\wedge all(l, D-b)\rightarrow\neg\Box most(l, v)$ (by [14] and Rule 2)

[16] $\vdash no(D-b, v)\wedge all(l, D-b)\rightarrow\Diamond\neg most(l, v)$ (by [15], Fact (2.4) and Fact (4.1))

[17] $\vdash no(D-b, v)\wedge all(l, D-b)\rightarrow\Diamond$ *at most half of the(l, v)*

(i.e. EA◇H-1, by [16] and Fact (2.5))

[18] $\vdash\neg some(D-b, v)\wedge\Box most(l, v)\rightarrow\neg all(l, D-b)$ (by [14] and Rule 3)

[19] $\vdash no(D-b, v)\wedge\Box most(l, v)\rightarrow$ *not all(l, D−b)*

(i.e. E□MO-2, by [18], Fact (2.1) and Fact (2.4))

[20] $\vdash all(l, D-b)\wedge\Box$ *fewer than half of the*$\neg(l, v)\rightarrow$ *not all*$\neg(D-b, v)$

(by [14], Fact (1.3) and Fact (1.5))

[21] $\vdash all(l, D-b)\wedge\Box$ *fewer than half of the(l, D−v)*$\rightarrow$ *not all(D−b, D−v)*

(i.e. □FAO-3, by [20] and Definition D3)

[22] $\vdash\neg$ *not all(D−b, D−v)*$\wedge all(l, D-b)\rightarrow\neg\Box$ *fewer than half of the(l, D−v)* (by [21] and Rule 2)

[23] $\vdash all(D-b, D-v)\wedge all(l, D-b)\rightarrow\Diamond\neg$ *fewer than half of the(l, D−v)*

(by [22], Fact (2.2) and Fact (4.1))

[24] $\vdash all(D-b, D-v)\wedge all(l, D-b)\rightarrow\Diamond$ *at least half of the(l, D−v)*

(i.e. AA◇S-1, by [23] and Fact (2.7))

[25] $\vdash\neg$ *not all(D−b, D−v)*$\wedge\Box$ *fewer than half of the(l, D−v)*$\rightarrow\neg all(l, D-b)$ (by [21] and Rule 3)

[26] $\vdash all(D-b, D-v)\wedge\Box$ *fewer than half of the(l, D−v)*$\rightarrow$ *not all(l, D−b)*

(i.e. A□FO-2, by [25], Fact (2.1) and Fact (2.2))

[27] $\vdash no\neg(v, b)\wedge\Box most(l, v)\rightarrow$ *not all*$\neg(l, b)$ (by [1], Fact (1.1) and Fact (1.3))

[28] $\vdash no(v, D-b)\wedge\Box most(l, v)\rightarrow$ *not all(l, D−b)* (i.e. E□MO-1, by [27] and Definition D3)

[29] $\vdash some(l, b)\rightarrow\Diamond some(l, b)$ (by Fact (5.5))

[30] $\vdash all(v, b)\wedge\Box most(l, v)\rightarrow\Diamond some(l, b)$ (i.e. A□M◇I-1, by[1], [29] and Rule 1)

[31] ⊢*some(b, l)*→◇*some(b, l)* (by Fact (5.5))

[32] ⊢*all(v, b)*∧□*most(l, v)*→◇*some(b, l)* (i.e. □MA◇I-4, by [2], [31] and Rule 1)

[33] ⊢*not all(v, b)*→◇*not all(v, b)* (by Fact (5.5))

[34] ⊢*no(l, b)*∧□*most(l, v)*→◇*not all(v, b)* (i.e. E□M◇O-3, by [10], [33] and Rule 1)

[35] ⊢*not all(v, b)*→◇*not all(v, b)* (by Fact (5.5))

[36] ⊢*no(b, l)*∧□*most(l, v)*→◇*not all(v, b)* (i.e. E□M◇O-4, by [11], [35] and Rule 1)

[37] ⊢*some(v, D−b)*→◇*some(v, D−b)* (by Fact (5.5))

[38] ⊢*all(l, D−b)*∧□*most(l, v)*→◇*some(v, D−b)* (i.e. A□M◇I-3, by [13], [37] and Rule 1)

[39] ⊢*some(D−b, v)*→◇*some(D−b, v)* (by Fact (5.5))

[40] ⊢*all(l, D−b)*∧□*most(l, v)*→◇*some(D−b, v)* (i.e. □MA◇I-3, by [14], [39] and Rule 1)

[41] ⊢*not all(l, D−b)*→◇*not all(l, D−b)* (by Fact (5.5))

[42] ⊢*no(D−b, v)*∧□*most(l, v)*→◇*not all(l, D−b)* (i.e. E□M◇O-2, by [19], [41] and Rule 1)

[43] ⊢*not all(D−b, D−v)*→◇*not all(D−b, D−v)* (by Fact (5.5))

[44] ⊢*all(l, D−b)*∧□*fewer than half of the(l, D−v)*→◇*not all(D−b, D−v)*
(i.e. □FA◇O-3, by [21], [43] and Rule 1)

[45] ⊢*not all(l, D−b)*→◇*not all(l, D−b)* (by Fact (5.5))

[46] ⊢*all(D−b, D−v)*∧□*fewer than half of the(l, D−v)*→◇*not all(l, D−b)*
(i.e. A□F◇O-2, by [26], [45] and Rule 1)

[47] ⊢*not all(l, D−b)*→◇*not all(l, D−b)* (by Fact (5.5))

[48] ⊢*no(v, D−b)*∧□*most(l, v)*→◇*not all(l, D−b)* (i.e. E□M◇O-1, by [28], [47] and Rule 1)

Theorem 2 denotes that the other 24 valid generalized modal syllogisms can be deduced from the validity of syllogism A□MI-1. Similarly, more valid syllogisms can be inferred from it. This indicates that there are reducible relations between/among these syllogisms. Their validity can be similarly proved as in Theorem 1.

5. Conclusion and Future Work

This paper firstly proves the validity of A□MI-1on the basis of generalized quantifier theory, modal logic, and set theory. Subsequently, 24 valid generalized modal syllogisms have been derived from the validity of A□MI-1 according to relevant facts and inference rules. This shows that there are reducible relationships between/among valid generalized modal syllogisms.

Undoubtedly, this approach presents a succinct and cohesive mathematical research framework for the study of other syllogisms. It is hoped that the work contributes to knowledge mining for generalized modal syllogism fragments.

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Reference

[1] Łukasiewicz, J. (1957) *Aristotle's Syllogistic: From the Standpoint of Modern Formal Logic*. second edition, Oxford: Clerndon Press.

[2] Moss, L. S. (2008) Completeness theorems for syllogistic fragments, in F. Hamm and S. Kepser (eds.), *Logics* for *Linguistic Structures*, Mouton de Gruyter, Berlin: 143-173.

[3] Moss, L. S. (2010) Syllogistic Logics with Verbs. Journal of Logic and Computation, 20(4): 947-967.

[4] Long W. (2023) Formal System of Categorical Syllogistic Logic Based on the Syllogism AEE-4, Open Journal of Philosophy, 13(1): 97-103.

[5] Haiping, W. and Xiaojun Z. (2024) Deductive Reasoning Based on the Valid Aristotelian Syllogism AEE-2. SumerianzJournal of Social Science, 7(1):1-4.

[6] Malink, M. (2006) A Reconstruction of Aristotle's Modal Syllogistic. History and Philosophy of Logic, (27): 95-141.

[7] Feifei, Y. and Xiaojun Z. (2024) The Deductibility of the Aristotelian Modal Syllogism E□I◇O-4 from the Perspective of Mathematical Structuralism, SCIREA Journal of Philosophy, 4(1): 23-33.

[8] Murinová, P. and Novák, V. (2012) A Formal Theory of Generalized Intermediate Syllogisms, Fuzzy Sets and Systems, 186: 47-80.

[9] Baoxiang, W. (2024) Knowledge Mining Based on the Valid Generalized Syllogism MMI-3 with the Quantifier 'Most', SCIREA Journal of Information Science, 8(2): 84-94.

[10] Mingwei, M. and Qing, C. (2024) Knowledge Mining about the Generalized Modal

Syllogism E□M◇F-2 with the Quantifiers in Square{fewer than half of the} and Square{no}, SCIREA Journal of Computer, 9(2): 37-46.

[11] Xiaojun, Z. (2018) Axiomatization of Aristotelian syllogistic logic based on generalized quantifier theory. Applied and Computational Mathematics, 7(3): 167-172.

[12] Barwise, J. (1977)An introduction to first-order logic, Studies in Logic and the Foundations of Mathematics, Elsevier, 90: 5-46.

[13] Kunen, K. (1980) Set theory: an introduction to independence proofs. The Netherlands: Elsevier Science Publishers B.V, 186-187.

[14] Jun, Q. and Mingwei, M. (2024) Knowledge Reasoning Based on the Reducibility of Valid Generalized Syllogisms. SCIREA Journal of Electrical Engineering, 9(1): 1-10.

[15] Liheng, H. (2024) The Validity of Generalized Modal Syllogisms Based on the Syllogism E□M◇O-1. SCIREA Journal of Mathematics, 9(1): 11-22.

[16] Peters, S. and Westerståhl, D. (2006) *Quantifiers in Language and Logic*, Claredon Press, Oxford.

[17] Chagrov, A, and Zakharyaschev, M. (1997) *Modal Logic*. Oxford: Clarendon Press.