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Generalized Z.T. Gao Method for Estimating Three Parameters of Weibull Distribution

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Abstract

In the process of fatigue research, it is found that most of the fatigue life data of structures conform to Weibull distribution rather than Gaussian distribution, and Weibull distribution is in a sense more general distribution than Gaussian distribution. But the biggest obstacle to the application of Weibull distribution is the complexity of Weibull distribution, especially the estimation of its three parameters is difficult. This is because the correlation coefficient estimation, MLE and other methods proposed by people have a common characteristic that the mathematical derivation is complicated and the calculation is complex. Based on the estimation of the correlation coefficients, author proposed Z.T. Gao method which can avoid these difficulties and can easily estimate the three parameters of Weibull distribution. Further study found that the idea of Z.T. Gao method can be used to avoid the difficulty of MLE, author call it generalized Z.T. Gao method can also conveniently get more ideal results.

Keywords: Three Parameter Weibull Distribution, Correlation Coefficient Estimation, Z.T. Gao (Gao Zhentong) Method, Maximum Likelihood Estimation(MLE), Generalized Z.T. Gao(G- Z.T. Gao) Method

1. Introduction

W. Weibull, a Swedish engineer, scientist and mathematician, made the Weibull distribution famous with a famous paper[1] in 1951, although the probability distribution he proposed in the paper was not yet named Weibull distribution. Since then, people started to study the Weibull distribution and its applications in depth[2]. At first, it was applied more in the field of fatigue, because the fatigue life of a structure is mostly in accordance with the Weibull distribution. It was pointed out[3] that the Weibull distribution is a full state distribution, i.e., it can depict not only left-skewed and right-skewed data but also, to a certain extent, symmetric and data satisfying a power law. However, the complexity in the Weibull distribution itself makes the proposed methods to estimate its three parameters, such as linear correlation estimation[4-5], and MLE[6-10], quite tedious both in terms of mathematical derivation and computation. In [3] author avoided the complicated derivation and computation of the correlation coefficient estimation and used the feature of Python called Z.T. Gao method to directly derive the estimation of the corresponding maximum correlation coefficient for the three parameters of the Weibull distribution, which proved to be quite effective. Further, even for the more complicated MLE the idea of Z.T. Gao method can be used to avoid more complicated derivation and computation to estimate the three parameters of Weibull distribution directly, and the result is also very good. Author call this method the generalized Z.T. Gao(G-Z.T. Gao) method. In this paper, we focus on this method.

2. Characteristics of Weibull Distribution

Weibull distribution can be expressed in a variety of ways, and a relatively general form is adopted here[3]. Its PDF is,

$$f(x)=(b/\lambda)[(x-x_0)/\lambda)]^{b-1}*$$

$$\exp\{-[(x-x_0)/\lambda]^{b}\} \quad (1)$$

Where b is the shape parameter, λ is the scale or proportional parameter, and x_0 is called the position parameter, and it is customary in the field of fatigue to use fatigue life N instead of x, N_0 instead of x_0 and call it safe life. In a non-strict sense[3], "when $0 < b < 1$ resembles a power-law function, while $1 < b < 3$ is a left-skewed distribution,

$3 \le b \le 4$ approximates a Gaussian distribution, and b > 4 is a right-skewed distribution". This is the reason why the Weibull distribution is called the "full state distribution". The PDF of the representative Weibull distribution is shown in Fig. 1[11].

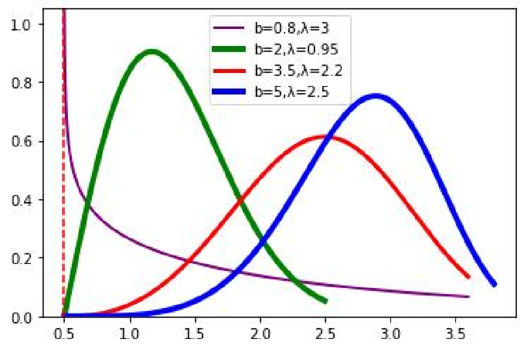

Fig.1. PDF of various Three-Parameter Weibull distributions when x_0=0.5

It is easy to prove that the corresponding reliability[3] with a life of x_i is,

$$p_i=\exp\{-[(x_i-x_0)/\lambda]^b\} \quad (2)$$

So when x is equal to x_0, p_0 is equal to 100%. That's where the 100% safe life comes in. If p_{50}=50%, then it means that the corresponding x is called the median x_m of X, that is,

$$50\%=\exp\{-[(x_m-x_0)/\lambda]^b\} \quad (3)$$

By definition, it is not difficult to obtain the expectation and variance of a three-parameter Weibull distribution[12],

$$E(X)=x_0+\lambda\Gamma(1+1/b) \quad (4)$$

$$Var(X)=\lambda^2[\Gamma(1+2/b)-\Gamma^2(1+1/b)] \quad (5)$$

In this way, the fatigue life data are given and the three parameters of the Weibull distribution can be derived by (3), (4) and (5), which is the analytical method[12]. In addition to the analytical method recently used more is the correlation coefficient estimation, MLE and some methods derived from it[6-7], but all of them have problems such as cumbersome derivation and inconvenient calculation. The following two section will point out how to overcome these difficulties by using Z.T. Gao method.

3. Correlation Coefficient Estimation and Z.T. Gao Method

Theoretically, if a set of fatigue life data N is given, then using the median (N_m), mean (N_{av}) and mean squared deviation (s) of the set, then using the three equations (3), (4) and (5) it is possible to solve for the estimated values of the three parameters of the Weibull distribution. However, for convenience (5), (6) and (7) can be reduced to a transcendental equation with respect to b[3]：

$$(N_{av}-N_m)[\Gamma(1+2/b)-\Gamma^2(1+1/b)]+$$

$$s[D^{1/b}-\Gamma(1+1/b)]^{1/2}=0 \qquad (6)$$

Where D=ln2. This equation is solvable by Newton's method, and after obtaining b, λ and N_0 can be found by (5), (4). but[3],[11]it is not difficult to find that sometimes the N_0 derived by the analytical method is greater than the minimum value of the fatigue life of this group. And this is in contradiction with the definition of safe life N_0. That is, the problem of non-consistence occurs. Another question is what happens if the normal distribution is used to fit this data set? Which is the more appropriate distribution to fit?

The second question can be judged by the magnitude of R^2 with the so-called "average rank"[12] as the coefficient of determination[13] for the ideal reliability fit, which is the following equation independent of the specific distribution [12],

$$p_i=1-i/(n+1) \qquad (7)$$

Where i is the ordinal number of data (observations) arranged from the smallest to the largest, and n is the number of data.

And the first problem is solved by Z.T. Gao method[3]. The following is a brief description of Z.T. Gao method induced by the correlation coefficient estimation[4]. Taking the logarithm of both sides of (2) twice yields that,

$$\ln(\ln(1/p_i))=b\ln(N_i-N_0)-b\ln(\lambda) \qquad (8)$$

$$\text{if set, } Y_i=\ln(\ln(1/p_i)),\ X_i=\ln(N_i-N_0) \qquad (9)$$

$$d=-b\ln(\lambda),\ \lambda=\exp(-d/b) \qquad (10)$$

So (10) became，

$$Y_i=bX_i+d \qquad (11)$$

This is a system of linear regression equations that can be derived by the least squares method for the coefficients *b* and *d*. However, it is important to note that here X_i is related not only to the given data *N*, but also to the required safety lifetime N_0 of the Weibull distribution. This problem can be solved by determining the extreme value of the absolute value of the relative coefficient r of the regression line to determine the corresponding N_0, but the mathematical derivation of this method is complex and error-prone[4]. It is better to use a different idea to use Python to find the series of r about N_0 directly in the interval $0 \le N_0 < N_{min}$ (here N_{min} is taken as the minimum value of the given data). Python then intelligently finds the N_0 of *r* with the largest correlation coefficient, and at the same time determines *b* and λ. This is known as Z.T. Gao algorithm. For simplify it is called by Z.T. Gao(Gao Zhentong) method [3].

Example 1. Using the data on P136 in [12], Z.T. Gao method is used to determine the three parameters of Weibull distribution and compare the results with the normal distribution. The results are as follows:

N= [350, 380, 400, 430, 450, 470, 480, 500, 520, 540, 550, 570, 600, 610, 630, 650, 670, 730, 770, 840]

N_{av}=, 557.0 ,*s*= 132.152 , N_m= 545.0

r= 0.99922 ,*b*= 2.040 ,λ= 320.98 ,N_0= 276.60

Relative to idea reliability:

Gaussian fitting: *r*=0.99675 ,*R*^2=0.98948

Weibull fitting: *r*=0.99914 ,*R*^2=0.99824

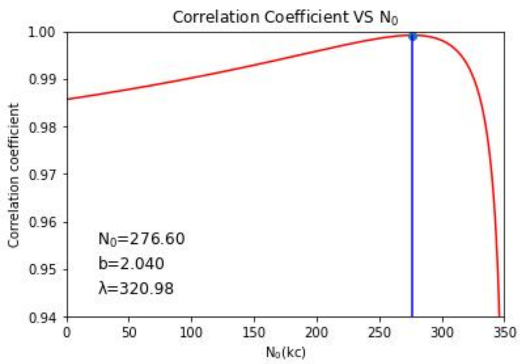

Fig.2. Schematic graph of Z.T. Gao method

This fig.2 graphically shows how the Z.T. Gao method finds the corresponding safe life that maximizes the correlation coefficient. Because in this process in the beginning has been clear about the N_0 is impossible is greater than the data of the minimum life, so not possible non-consistence situation. For another comparison, the results obtained through tedious derivation and calculation in [4] are as follows:

b=2.041;λ=321.14;N_0=276.47;r=-0.9993

It can be said that there is almost no difference between the results calculated by the Z.T. Gao method, but the Z.T. Gao method requires no derivation and the code is simple. Of course, the sign of the correlation coefficient seems to be the opposite, because in [4], Y= -ln(ln(1/p)), while in this paper, author take the positive sign.

The advantage of Z.T. Gao method is that the physical meaning is very obvious and there is no problem of "non-consistence". This method is not only convenient for solving the estimation of the three parameters of the Weibull distribution, but also easy to determine whether the original data are better fitted with the Weibull distribution or the Gaussian distribution. It is also easy to extend to solve similar problems, such as fitting fatigue performance curves with three parameters[3], and the confidence intervals of these three parameters will be discussed in separate papers [14-15].

4. Maximum likelihood Estimation (MLE) and Generalized Z.T. Gao (G-Z.T. Gao) Methods

For MLE is mentioned in general textbooks of mathematical statistics[12]. However, if the Gaussian distribution is taken as an example, it is very simple to estimate the two parameters of the Gaussian distribution by using MLE when a certain data set is assumed to meet the Gaussian distribution. The only problem is that the variance with MLE is biased, although there is not much difference between biased and unbiased estimation when n is relatively large.

However, almost no mathematical statistics textbook uses MLE the three parameters of the Weibull distribution, because this method is really tedious, as shown in the mathematical derivation is more troublesome and the calculation is quite complicated[6-8].

It is not difficult to obtain the likelihood function of the Weibull distribution after taking the logarithm according to (1),

$$LL=\ln L=n\ln(b/\lambda)+(b-1)\,\Sigma_{i=1}^{n}\ln[(x_i-x_0)/\lambda]-$$

$$\Sigma_{i=1}^{n}[(x_i-x_0)/\lambda]^{b} \qquad (12)$$

The so-called MLE involves determining the appropriate parameters b, λ and x_0 to make LL maximum. It is customary to take the partial derivatives of the above equations for b, λ, and x_0 respectively and make them zero, and then solve for the values of these three parameters:

$$\partial LL/\partial b=n/b+\Sigma_{i=1}^{n}\ln[(x_i-x_0)/\lambda]-$$

$$\Sigma_{i=1}^{n}[(x_i-x_0)/\lambda]^{b}\ln[(x_i-x_0)/\lambda]=0 \quad (13)$$

$$\partial LL/\partial\lambda=-n/\lambda-n(b-1)/\lambda+$$

$$(b/\lambda)\,\Sigma_{i=1}^{n}[(x_i-x_0)/\lambda]^{b}=0 \qquad (14)$$

$$\partial LL/\partial x_0=-(b-1)\,\Sigma_{i=1}^{n}1/(x_i-x_0)+$$

$$(b/\lambda)\,\Sigma_{i=1}^{n}[(x_i-x_0)/\lambda]^{b-1}=0 \qquad (15)$$

This is probably the main reason why it is impossible to use MLE the three parameters of the Weibull distribution in the days when computers were not widely available. But even using a computer to solve this system of nonlinear equations is definitely not an easy task. This is because one cannot guarantee that there is a solution, and even if there is a solution, there is no guarantee that it is unique. What is even more fatal is that if the initial values are not chosen properly, then it is likely that the problem that had a solution becomes unsolvable [7].

So can Z.T. Gao method be used to solve this problem? Obviously it is not possible to use Z.T. Gao method directly, because the objects are different. However, the idea of using Z.T. Gao method is feasible. This is because the essence of Z.T. Gao method is to use the brute force method to find the extreme value. MLE is also to find the maximum value, except that there are not only one but three parameters, but there is no fundamental difference from the mathematical point of view. As for the initial value and the variation interval, the results obtained by Z.T. Gao method can also be used as a reference, and then an appropriate choice can be made.

According to this idea, author simply ignore the system of equations (13), (14) and (15), and directly calculate the values of LL within the "reasonable range" of the three parameters b,λ,x_0 by the brute force method, find the maximum values of LL, and then in turn give the corresponding values of the three parameters b,λ,x_0, and These values are the results of MLE. Author call this method the generalized Z.T. Gao (G-Z.T. Gao) method, because it is in the same line of thought as Z.T. Gao method, and it also requires Z.T. Gao method to provide the estimates of the three parameters in order to obtain their "reasonable ranges". The Python code for this method is also relatively easy to write. Here is an example to illustrate how to use this method.

Example 2. Using 50 randomly generated numbers from the b=2.5, λ=30, x_0=20 Weibull distribution provided in [10] to perform the test and compare with the results obtained by MLE.

Solution: 1). Determine the range of the three parameters by Z.T. Gao method.

The original data are X= [25.6, 28.0, 29.7, 30.6, 31.5, 32.7, 33.4, 34.5, 35.3, 36.0, 36.7, 37.3, 37.9, 38.6, 39.2, 39.8, 40.4, 40.9, 41.7, 42.4, 43.2, 43.7, 44.3, 44.9, 45.4, 45.9, 46.5, 47.1, 47.7, 48.2, 48.8, 49.5, 50.3, 51.1, 51.9, 52.6, 53.4, 54.2, 55.0, 55.7, 56.4, 57.4, 58.5, 59.6, 60.8, 62.4, 64.5, 66.4, 69.9, 75.0]

The values of the three parameters are estimated by Z.T. Gao method as follows:

b= 2.411 ,λ= 30.09 ,x_0= 19.88

The range of variation of b can be initially determined as, [1.5, 3.0]; and the range of variation of λ as, [25, 35]; such determination is somewhat subjective, but it can still be adjusted by the calculation results. As for the variation range of x_0, it should be the same as Z.T. Gao method, i.e., [0, 25.6).

2). Determine the step of variation of the three parameters, i.e., the accuracy. For example, the step size of b can be initially determined as 0.015, while the step size of λ is 0.2, and the step size of x_0 is 0.2*25.6; the reason for this determination is to take into account the balance between calculation volume and accuracy. Of course, it can be adjusted according to the calculation results.

3). It is easy to write the Python code to calculate the LL values of different three parameters on the basis of 1) and 2), and it is not difficult to find the extreme value of LL after the calculation, and then the computer can invert the values of the three parameters b, λ, and x_0 that produce this extreme value. The calculation results are as follows:

b= 2.205 ,λ= 26.4 ,x_0= 23.04

4). The three parameters obtained by Z.T. Gao、G-Z.T. Gao methods and MLE can be compared:

Table 1. Comparison of results obtained by various estimation methods

	b	λ	x_0	R^2	LL
Z.T. Gao	2.411	30.09	19.88	0.99977	-190.654
G-Z.T. Gao	2.205	26.4	23.04	0.99850	-190.275
MLE	2.226	26.64	22.85	0.99860	**-190.273**
Original	2.5	30	20	**0.99982**	-190.503

Where the data for MLE are from [10] and Original denotes the actual values of the population parameters. From the data in Table 1, it is easy to see that the *LL* values obtained by G-Z.T. GAO method are almost indistinguishable from the results obtained by MLE, but the estimates of the three parameters still differ slightly, i.e., indicating that the MLE probably does not have a unique solution, and even if the *LL* achieves the same extreme value does not mean that the corresponding three estimates must be the same. This indicates that one should not be too superstitious about MLE, but still needs to look at other fit indicators, such as the coefficient of determination of the fit to the ideal reliability. From Table 1, it is not difficult to find that the *LL* values obtained with the population original parameters, not yet as large as those obtained with MLE (although the difference is not large), still have the largest coefficients of determination, while those obtained with Z.T. Gao method are the second largest. And it is also not difficult to find that the estimates of the three parameter values obtained directly with Z.T. Gao method are the closest to the original values. In this sense, does it become "redundant" to use MLE? The author believes that this is not an appropriate statement, because the method cannot be dismissed with a single example. The next example will illustrate this point.

5). Finally, the histogram of the data can be compared with the PDF of the Weibull distribution obtained by each method by drawing a visual graph.

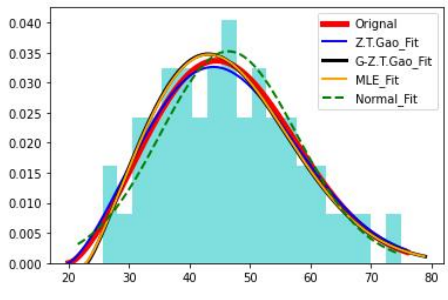

Fig.3 Fitting diagram of data histogram and different Weibull Distribution PDF

As shown in Figure 3, it is easy to see that even the PDF obtained from the original parameters cannot be a perfect first fit to the histogram of the original data, after all, there are only 50 samples. At the same time, it can also be seen that the three parameters estimated by different methods, although there are considerable differences, but the resulting PDF is very close, which also confirms that a single maximum cannot guarantee that the three parameters have a unique estimate. The deeper reason is that these three parameters are not completely independent, there is a rather complex relationship between them, a little change in one parameter will affect the other two parameters change, and finally still make *LL* obtain an extremely value. Because of this, it is not possible to believe in the estimated value obtained by a certain method, and it is still necessary to look at several fitting indicators. But in any case, there is really no big difference between the estimates obtained by G-Z.T. GAO method and the estimates obtained by MLE, especially under the condition that the *LL* is extremely large, and in this sense the G-Z.T. GAO method can completely replace MLE.

Example 3. The data in this example is from [7]. Unlike in Ex. 2, the data is not "artificial" but from actual measurements.

Solution: Similar to Ex.2, but simplified: Given the data, *X*= [3956.42, 4004.18, 4091.61, 4355.05, 4355.4, 4376.01, 4391.79, 4487.68, 4487.68, 4736.67, 4736.67, 4939.85, 4963.62, 5220.19, 5353.41, 5372.72, 5418.04, 5444.11, 5603.17, 5698.1, 5746.17, 5843.52, 6175.14, 6197.41, 6249.69, 6279.76, 6279.76, 6572.74, 6740.48, 6887.65, 7183.09, 7209, 7209, 7209, 7209, 7366.4, 7581.64, 7581.64, 7581.64, 7645.59, 8246, 8599.7, 8713.97, 8936.34, 9044.22, 9197.45, 9511.73, 9754.47, 9967.45, 10136.31, 10172.88, 10172.88, 10308.04, 10395, 10609.23, 10609.23, 10788.97, 10879.97, 10971.75, 11594.41, 11990.59, 12237.31, 12400.31, 12400.31, 12550.01, 13198.73, 13947.78, 15557.12, 17646.12, 19848.23, 23199.07]

The estimated values of the corresponding three parameters were obtained by Z.T. Gao method, from which the range of variation and the step of variation of these three parameters were determined, and then the corresponding estimated values were obtained by the generalized Z.T. Gao method, and then compared with the estimated values of MLE to obtain the following Table 2.

Table 2. Comparison of results obtained by various estimation methods

	b	λ	x_0	R^2	LL
Z.T. Gao	1.146	4957.7	3813.0	0.99391	-667.728
G-Z.T. Gao	1.190	4800.0	3679.3	0.99688	**-677.054**
MLE	1.139	4936.1	3821.5	**0.99860**	-667.704

The estimates of the three parameters of the MLE in the table are from [7]. From the data in this table, it can be found that all three methods are quite close to each other in terms of both *LL* values and estimates of the three parameters, and it is difficult to judge which method obtains better estimates, perhaps with other fitting criteria. In any case, it can still be said that the results obtained by the simple Z.T. Gao method are still satisfactory. Also the results obtained by G-Z.T. Gao method are almost the same as MLE, and its corresponding *LL* value is still the greatest. It is proved again that G-Z.T. Gao method is effective. Finally the histogram of the data and the fitted plots of each Weibull distribution PDF are still available.

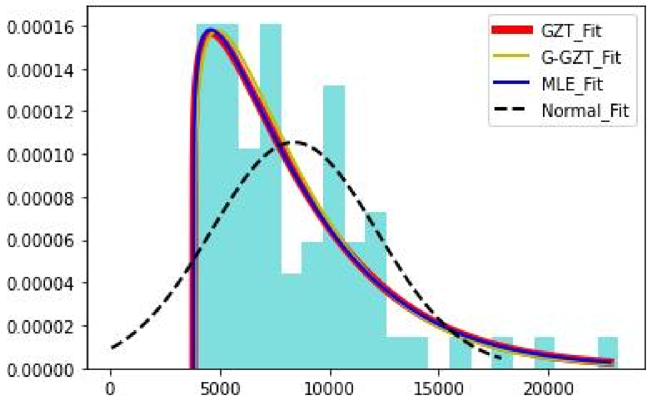

Fig.4 Fitting diagram of data histogram and different Weibull Distribution PDF

The graphs of the PDFs of the Weibull distribution estimated by the three different methods are almost indistinguishable as seen in Fig. 4 This also shows that in a sense, the same Weibull distribution can correspond to almost infinite combinations of three different parameters within a certain error range. Of course, no matter how the combination of its location parameters is not likely to be greater than the minimum value of the given data.

5. conclusion

1. There are conditions for the validity of Z.T. Gao method, i.e. not only the brute

force method is used to find the extreme value of the correlation coefficient corresponding to the location parameter, but also because the range of this location parameter is determined. That is, the location parameter in the fatigue life of the structure is precisely the minimum safe life, is not possible to exceed the minimum value of the given data. Without this condition brute force method is also very difficult to work.

2. The G-Z.T. Gao method is also effective because Z.T. Gao method provides a range of variations of the three parameters. Of course the range of the location parameters is the same as Z.T. Gao method.

3. For the three parameters of the Weibull distribution, different estimation methods will naturally give different results. But even using the same method, but by different algorithms, it is possible to get different estimates because of computational errors or the existence of complex correlations of these three parameters. Therefore, multiple fitting criteria are needed, but the problem is that it is difficult to have a set of parameters that are the best among the criteria. This is because each fitting criterion is only a measure of the fit from a certain perspective. Therefore, which set of parameters to adopt needs to be analyzed in detail and withstand the "test" of the actual situation.

4. Whether Z.T. Gao method or G-Z.T. Gao method is very intuitive and easy to learn, can greatly reduce the burden of mathematical derivation and the difficulty of writing code, which is undoubtedly a good thing for the relevant scientific and engineering personnel, easy to grasp and will be used in their own work.

5. Just as Weibull distribution can not only be used in structural fatigue life, Z.T. Gao method can not only be used to simplify the relative coefficient estimation, but also to simplify the MLE. For example, the birth of the G-Z.T. Gao method is a good example, which can be extended to a variety of extreme value fields.

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