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How to Obtain Other Valid Generalized Modal Syllogisms from the Syllogism □EF◇O-1

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Abstract

For the sake of obtaining valid generalized modal syllogisms, the article first proves the validity of the generalized modal syllogism □EF◊O-1 by means of set theory and modal logic, and then deduces the other 22 valid generalized modal syllogisms from the syllogism □EF◊O-1 in accordance with modern modal logic, generalized quantifier theory, and so on. The reason why there are reducibilities between different generalized modal syllogisms is that: (1) any of the Aristotelian quantifiers is definable by the other three Aristotelian quantifiers; (2) any of the four generalized quantifiers mentioned in this article is definable by the other three generalized quantifiers; (3) the transformation relationship between necessity and possibility; (4) the symmetry of *some* and *no*. The article presents a formal research method for generalized modal syllogistic, which not only provides a unified mathematical research paradigm for other generalized modal syllogisms and even other kinds of syllogisms, but also meet with the demands for formalization transformation of modern logic in the era of artificial intelligence. Therefore, this study has considerable theoretical and practical values.

Keywords: Generalized Modal Syllogism, Reducibility, Validity, Generalized Quantifier

1. Introduction

Syllogistic reasoning is one of the important forms of reasoning in natural language and logic. There are various forms of syllogisms, such as Aristotelian syllogisms, Aristotelian modal syllogisms, generalized syllogisms, and generalized modal syllogisms. There are reducibilities between syllogisms, that is, the validity of other syllogisms can be obtained from the validity of one syllogism.

The Aristotelian school believes that 22 other valid Aristotelian syllogisms can be derived from the two syllogisms AAA-1 and EAE-1 (Westerståhl, 2007). Xiaojun and Sheng (2016) demonstrated this in a formal way by generalized quantifier theory. Xiaojun et al. (2022) took only the syllogism OAO-3 as a basic axiom and deduced the remaining 23 valid Aristotelian syllogisms. Hui (2023) completed the same task only from the syllogism EIO-2. Cheng (2023) derived 91 other Aristotelian modal syllogisms just from the syllogism □I□A□I-3. Jing and Xiaojun (2023a) discussed how to obtain valid generalized modal syllogisms from valid generalized syllogisms. Jing and Xiaojun (2023b) studied the reducibility of generalized modal syllogisms based on □AM◇I-1.

On the basis of the research results above, this article attempts to deduce other valid generalized modal syllogisms from the syllogism □EF◇O-1 in virtue of classical propositional logic, modal logic, set theory and generalized quantifier theory.

2. Preliminaries

For simplicity we shall use the letters *B*, *D* and *G* as lexical variables in categorical propositions, and *U* the universe of lexical variables. The categorical propositions involved in the generalized modal syllogisms are of the following forms: 'All *B*s are *G*', 'No *B*s are *G*', 'Some *B*s are *G*', 'Not all *B*s are *G*', 'Fewer than half of the *B*s are *G*', 'At least half of the *B*s are *G*', 'Most *B*s are *G*', 'At most half of the *B*s are *G*'. These eight propositions are commonly abbreviated as *all*(*B*, *G*), *no*(*B*, *G*), *some*(*B*, *G*), *not all*(*B*, *G*), *fewer than half of the*(*B*, *G*), *at least half of the*(*B*, *G*), *most*(*B*, *G*), *at most half of the*(*B*, *G*), and denoted by the proposition A, E, I, O, F, L, M, and H, respectively.

Generalized modal syllogisms are the syllogisms which include generalized quantifiers and modalities (that is, the necessary modality $\Box$ and/or the possible modality $\Diamond$). So generalized modal syllogisms characterize the semantic and inferential properties of generalized quantifiers and generalized modal quantifiers.

Example 1:

Major premise: No goats are necessarily carnivorous animals.

Minor premise: Fewer than half of animals on this farm are goats.

Conclusion: Not all animals on this farm are possibly carnivorous animals.

Let B be the set of all animals in the universe, D the set of all goats in the universe, and G the set of all carnivorous animals in the universe. The generalized modal syllogism of Example 1 can be formalized as $\Box\, no(D, G) \wedge \textit{fewer than half of the}(B, D) \rightarrow \Diamond\, \textit{not all}(B, G)$. The definitions of figures of generalized modal syllogisms are similar to those of Aristotelian syllogisms. This syllogism is the first figure, so it can be abbreviated as $\Box$EF$\Diamond$O-1.

The following definitions can be obtained in the light of set theory (Halmos, 1974), generalized quantifier theory (Peters and Westerståhl, 2006; Xiaojun, 2014) and possible world semantics (Chellas, 1980), in which ‘iff’ means ‘if and only if’, Q is a generalized quantifier.

Definition 1 (truth value definitions):

(1) $all(B, G)$ is true iff $B \subseteq G$ is true;

(2) $some(B, G)$ is true iff $B \cap G \neq \varnothing$ is true;

(3) $no(B, G)$ is true iff $B \cap G = \varnothing$ is true;

(4) $\textit{not all}(B, G)$ is true iff $B \nsubseteq G$ is true;

(5) $\textit{fewer than half of the}(B, G)$ is true iff $|B \cap G| < 0.5|B|$ is true;

(6) $\Box all(B, G)$ is true iff $B \subseteq G$ is true in any possible world;

(7) $\Diamond all(B, G)$ is true iff $B \subseteq G$ is true in at least one possible world;

(8) $\Box some(B, G)$ is true iff $B \cap G \neq \varnothing$ is true in any possible world;

(9) $\Diamond some(B, G)$ is true iff $B \cap G \neq \varnothing$ is true in at least one possible world;

(10) $\Box no(B, G)$ is true iff $B \cap G = \varnothing$ is true in any possible world;

(11) $\Diamond no(B, G)$ is true iff $B \cap G = \varnothing$ is true in at least one possible world;

(12) □*not all*(*B*, *G*) is true iff *B*⊄ *G* is true in any possible world;

(13) ◇*not all*(*B*, *G*) is true iff *B*⊄ *G* is true in at least one possible world;

(14) □*fewer than half of the*(*B*, *G*) is true iff |*B*∩*G*|<0.5|*B*| is true in any possible world;

(15) ◇*fewer than half of the*(*B*, *G*) is true iff |*B*∩*G*|<0.5|*B*| is true in at least one possible world.

Definition 2 (inner negation): *Q*¬(*B*, *G*) $=_{\mathrm{def}}$ *Q*(*B*, *U*−*G*).

Definition 3 (outer negation): ¬*Q*(*B*, *G*) $=_{\mathrm{def}}$ It is not that *Q*(*B*, *G*).

On the basis of Definition 2, what can be obtained is Fact 1 (Cheng, 2022).

Fact 1:

(1) *all*(*B*, *G*)=*no*¬(*B*, *G*); (2) *no*(*B*, *G*)=*all*¬(*B*, *G*);

(3) *some*(*B*, *G*)=*not all*¬(*B*, *G*); (4) *not all*(*B*, *G*)=*some*¬(*B*, *G*);

(5) *fewer than half of the*(*B*, *G*)=*most*¬(*B*, *G*); (6) *most*(*B*, *G*)=*fewer than half of the*¬(*B*, *G*);

(7) *at least half of the*(*B*, *G*)=*at most half of the* (*B*, *G*);

(8) *at most half of the*(*B*, *G*)=*at least half of the* (*B*, *G*).

By Definition 3, Fact 2 can be obtained as follows (Cheng, 2022).

Fact 2 :

(1) ¬*not all*(*B*, *G*)=*all*(*B*, *G*); (2) ¬*all*(*B*, *G*)=*not all*(*B*, *G*);

(3) ¬*no*(*B*, *G*)=*some*(*B*, *G*); (4) ¬*some*(*B*, *G*)=*no*(*B*, *G*);

(5) ¬*fewer than half of the*(*B*, *G*)=*at least half of the*(*B*, *G*);

(6) ¬*at least half of the*(*B*, *G*)=*fewer than half of the*(*B*, *G*) ;

(7) ¬*most*(*B*, *G*)=*at most half of the*(*B*, *G*); (8) ¬*at most half of the*(*B*, *G*)=*most*(*B*, *G*).

The following Fact 3 to 8 are the basic knowledge in classical modal logic (Chagrov and Zakharyaschev, 1997) or generalized quantifier theory (Peters and Westerståhl, 2006), in which *Q*(*B*, *G*) is a categorical proposition.

Fact 3: (1) ¬□*Q*(*B*, *G*)=◇¬*Q*(*B*, *G*); (2) ¬◇*Q*(*B*, *G*)=□¬*Q*(*B*, *G*).

Fact 4: $\vdash \Box Q(B, G) \rightarrow Q(B, G)$.

Fact 5: $\vdash \Box Q(B, G) \rightarrow \Diamond Q(B, G)$.

Fact 6: $\vdash Q(B, G) \rightarrow \Diamond Q(B, G)$.

Fact 7: (1) $\vdash$ *all*$(B, G) \rightarrow$ *some*(B, G); (2) $\vdash$ *no*$(B, G) \rightarrow$ *not all*(B, G).

Fact 8: (1) *some*$(B, G) \leftrightarrow$ *some*(G, B); (2) *no*$(B, G) \leftrightarrow$ *no*(G, B).

The following rules are basic rules in classical propositional logic which are suitable for generalized modal syllogistic. Let x, y, z and w be proposition variables, then

Rule 1: If $\vdash (x \wedge y \rightarrow z)$, then $\vdash (\neg z \wedge x \rightarrow \neg y)$ or $\vdash (\neg z \wedge y \rightarrow \neg x)$.

Rule 2: If $\vdash (x \wedge y \rightarrow z)$ and $\vdash (z \rightarrow w)$, then $\vdash (x \wedge y \rightarrow w)$.

3. The Other 22 Generalized Modal Syllogisms Deduced from □EF◇O-1

The following theorem 1 means that the syllogism □EF◇O-1 is valid. Theorem 2 formally presents the other 22 valid generalized modal syllogisms deduced from □EF◇O-1. For example, '□EF◇O-1⇒□AF◇I-1' in the Theorem 2(1) represents that the validity of syllogism □AF◇I-1 can be deduced from the validity of syllogism □EF◇O-1. The other cases in Theorem 2 are similar.

Theorem 1 (□EF◇O-1): The generalized modal syllogism $\Box$*no*$(D, G) \wedge$ *fewer than half of the*$(B, D) \rightarrow \Diamond$*not all*$(B, G)$ is valid.

Proof: Example 1 shows that the expansion of □EF◇O-1 is $\Box$*no*$(D, G) \wedge$ *fewer than half of the*$(B, D) \rightarrow \Diamond$*not all*$(B, G)$. Assume that $\Box$*no*(D, G) and *fewer than half of the*(B, D) are true. In terms of the clause (3) in Definition 1, $D \cap G = \varnothing$ is true in any possible world. Similarly, $|B \cap D| < 0.5|B|$ is true in the light of the clause (5) in Definition 1. It is obvious that $D \cap G = \varnothing$ and $|B \cap D| < 0.5|B|$ are true. Therefore, $B \not\subseteq G$. This can be proved by *reductio ad absurdum*. Assume that $B \not\subseteq G$ is not true. That is, $B \subseteq G$ is true, and it has been proved that $D \cap G = \varnothing$. Thus, it follows that $B \cap G = \varnothing$, which contradicts $|B \cap D| < 0.5|B|$. So $B \subseteq G$ is not true. This means that $B \not\subseteq G$ is true. Then according to the clause (4) in Definition 1, *not all*(B, G) is true. Therefore, $\Diamond$*not all*(B, G) comes out true in virtue of Fact 6, as required.

Theorem 2: The validity of the following 22 generalized modal syllogisms can be inferred from □EF◇O-1:

(1) □EF◇O-1⇒□AF◇I-1

(2) □EF◇O-1⇒□EF◇O-2

(3) □EF◇O-1⇒□AF◇I-3

(4) □EF◇O-1⇒□E□AL-2

(5) □EF◇O-1⇒□AF◇I-1⇒F□A◇I-4

(6) □EF◇O-1⇒□AF◇I-1⇒□EF◇O-3

(7) □EF◇O-1⇒□AF◇I-1⇒□A□EL-2

(8) □EF◇O-1⇒□EF◇O-2⇒□AM◇O-2

(9) □EF◇O-1⇒□EF◇O-2⇒F□A◇I-3

(10) □EF◇O-1⇒□EF◇O-2⇒□E□AL-1

(11) □EF◇O-1⇒□AF◇I-1⇒F□A◇I-4⇒□A□EL-4

(12) □EF◇O-1⇒□AF◇I-1⇒F□A◇I-4⇒□EF◇O-4

(13) □EF◇O-1⇒□AF◇I-1⇒□A□EL-2⇒□A□E◇L-2

(14) □EF◇O-1⇒□EF◇O-2⇒□AM◇O-2⇒□A□AH-1

(15) □EF◇O-1⇒□EF◇O-2⇒□AM◇O-2⇒M□A◇O-3

(16) □EF◇O-1⇒□AF◇I-1⇒F□A◇I-4⇒□A□EL-4⇒□A□E◇L-4

(17) □EF◇O-1⇒□AF◇I-1⇒□A□EL-2⇒□A□E◇L-2⇒□E□A◇L-2

(18) □EF◇O-1⇒□AF◇I-1⇒□A□EL-2⇒□A□E◇L-2⇒□A□F◇I-1

(19) □EF◇O-1⇒□AF◇I-1⇒□A□EL-2⇒□A□E◇L-2⇒□E□F◇O-3

(20) □EF◇O-1⇒□EF◇O-2⇒□AM◇O-2⇒□A□AH-1⇒□A□A◇H-1

(21) □EF◇O-1⇒□AF◇I-1⇒□A□EL-2⇒□A□E◇L-2⇒□A□F◇I-1⇒□F□A◇I-4

(22) □EF◇O-1⇒□AF◇I-1⇒□A□EL-2⇒□A□E◇L-2⇒□E□F◇O-3⇒□F□E◇O-1

Proof:

[1] ⊢ □*no*(*D*, *G*)∧*fewer than half of the*(*B*, *D*)→◇*not all*(*B*, *G*) (i. e. □EF◇O-1)

[2] ⊢ *no*(*D*, *G*)=*all*¬(*D*, *G*) (by Fact 1(2))

[3] ⊢ *not all*(*B*, *G*)=*some*¬(*B*, *G*) (by Fact 1(4))

[4] ⊢ □*all*¬(*D*, *G*)∧*fewer than half of the*(*B*, *D*)→◇*some*¬(*B*, *G*) (by [1], [2] and [3])

[5] ⊢ □*all*(*D*, *U*−*G*)∧*fewer than half of the*(*B*, *D*)→◇*some*(*B*, *U*−*G*)

(i. e. □AF◇I-1, by Definition 2 and [4])

[6] ⊢ $no(D, G)\leftrightarrow no(G, D)$ (by symmetry of *no*)

[7] ⊢ $\Box no(G, D)\wedge fewer\ than\ half\ of\ the(B, D)\rightarrow\Diamond not\ all(B, G)$ (i. e. □EF◇O-2, by [1] and [6])

[8] ⊢ $\neg\Diamond not\ all(B, G)\wedge fewer\ than\ half\ of\ the(B, D)\rightarrow\neg\Box no(D, G)$ (by Rule 1 and [1])

[9] ⊢ $\Box\neg not\ all(B, G)\wedge fewer\ than\ half\ of\ the(B, D)\rightarrow\Diamond\neg no(D, G)$ (by [8] and Fact 3)

[10] ⊢ $\neg not\ all(B, G)=all(B, G)$ (by Fact 2(1))

[11] ⊢ $\neg no(D, G)=some(D, G)$ (by Fact 2(3))

[12] ⊢ $\Box all(B, G)\wedge fewer\ than\ half\ of\ the(B, D)\rightarrow\Diamond some(D, G)$

(i. e. □AF◇I-3, by [9], [10] and [11])

[13] ⊢ $\neg\Diamond not\ all(B, G)\wedge\Box no(D, G)\rightarrow\neg fewer\ than\ half\ of\ the(B, D)$ (by Rule 1 and [1])

[14] ⊢ $\Box\neg not\ all(B, G)\wedge\Box no(D, G)\rightarrow\neg fewer\ than\ half\ of\ the(B, D)$ (by [13] and Fact 3(2))

[15] ⊢ $\neg fewer\ than\ half\ of\ the(B, D)=at\ least\ half\ of\ the(B, D)$ (by Fact 2(5))

[16] ⊢ $\Box all(B, G)\wedge\Box no(D, G)\rightarrow at\ least\ half\ of\ the(B, D)$ (i. e. □E□AL-2, by [10], [14] and [15])

[17] ⊢ $some(B, U-G)\leftrightarrow some(U-G, B)$ (by symmetry of *some*)

[18] ⊢ $\Box all(D, U-G)\wedge fewer\ than\ half\ of\ the(B, D)\rightarrow\Diamond some(U-G, B)$

(i. e. F□A◇I-4, by [5] and [17])

[19] ⊢ $\neg\Diamond some(B, U-G)\wedge fewer\ than\ half\ of\ the(B, D)\rightarrow\neg\Box all(D, U-G)$ (by Rule 1 and [5])

[20] ⊢ $\Box\neg some(B, U-G)\wedge fewer\ than\ half\ of\ the(B, D)\rightarrow\Diamond\neg all(D, U-G)$ (by [19] and Fact 3)

[21] ⊢ $\neg some(B, U-G)=no(B, U-G)$ (by Fact 2(4))

[22] ⊢ $\neg all(D, U-G)=not\ all(D, U-G)$ (by Fact 2(2))

[23] ⊢ $\Box no(B, U-G)\wedge fewer\ than\ half\ of\ the(B, D)\rightarrow\Diamond not\ all(D, U-G)$

(i. e. □EF◇O-3, by [20], [21] and [22])

[24] ⊢ $\neg\Diamond some(B, U-G)\wedge\Box all(D, U-G)\rightarrow\neg fewer\ than\ half\ of\ the(B, D)$ (by Rule 1 and [5])

[25] ⊢ $\Box\neg some(B, U-G)\wedge\Box all(D, U-G)\rightarrow\neg fewer\ than\ half\ of\ the(B, D)$ (by [24] and Fact 3(2))

[26] ⊢ $\Box no(B, U-G)\wedge\Box all(D, U-G)\rightarrow at\ least\ half\ of\ the(B, D)$

(i. e. □A□EL-2, by [15], [21] and [25])

[27] ⊢ $no(G, D)=all\neg(G, D)$ (by Fact 1(2))

[28] ⊢ $fewer\ than\ half\ of\ the(B, D)=most\neg(B, D)$ (by Fact 1(5))

[29] $\vdash$ $\Box all\neg(G, D) \wedge most\neg(B, D) \rightarrow \Diamond not\ all(B, G)$ (by [7], [27] and [28])

[30] $\vdash$ $\Box all(G, U\text{–}D) \wedge most(B, U\text{–}D) \rightarrow \Diamond not\ all(B, G)$

(i. e. □AM◇O-2, by Definition 2 and [29])

[31] $\vdash$ $\neg\Diamond not\ all(B, G) \wedge fewer\ than\ half\ of\ the(B, D) \rightarrow \neg\Box no(G, D)$ (by [7] and Rule 1)

[32] $\vdash$ $\Box\neg not\ all(B, G) \wedge fewer\ than\ half\ of\ the(B, D) \rightarrow \Diamond\neg no(G, D)$ (by [31] and Fact 3)

[33] $\vdash$ $\Box all(B, G) \wedge fewer\ than\ half\ of\ the(B, D) \rightarrow \Diamond some(G, D)$

(i. e. F□A◇I-3, by Fact 2(1), Fact 2(3) and [32])

[34] $\vdash$ $\neg\Diamond not\ all(B, G) \wedge \Box no(G, D) \rightarrow \neg fewer\ than\ half\ of\ the(B, D)$ (by [7] and Rule 1)

[35] $\vdash$ $\Box\neg not\ all(B, G) \wedge \Box no(G, D) \rightarrow \neg fewer\ than\ half\ of\ the(B, D)$ (by [34] and Fact 3(2))

[36] $\vdash$ $\Box all(B, G) \wedge \Box no(G, D) \rightarrow at\ least\ half\ of\ the(B, D)$

(i. e. □E□AL-1, by [35], Fact 2(1) and Fact 2(5))

[37] $\vdash$ $\neg\Diamond some(U\text{–}G, B) \wedge \Box all(D, U\text{–}G) \rightarrow \neg fewer\ than\ half\ of\ the(B, D)$ (by [18] and Rule 1)

[38] $\vdash$ $\Box\neg some(U\text{–}G, B) \wedge \Box all(D, U\text{–}G) \rightarrow \neg fewer\ than\ half\ of\ the(B, D)$ (by Fact 3(2) and [37])

[39] $\vdash$ $\Box no(U\text{–}G, B) \wedge \Box all(D, U\text{–}G) \rightarrow at\ least\ half\ of\ the(B, D)$

(i. e. □A□EL-4, by [38], Fact 2(4) and Fact 2(5))

[40] $\vdash$ $\neg\Diamond some(U\text{–}G, B) \wedge fewer\ than\ half\ of\ the(B, D) \rightarrow \neg\Box all(D, U\text{–}G)$ (by [18] and Rule 1)

[41] $\vdash$ $\Box\neg some(U\text{–}G, B) \wedge fewer\ than\ half\ of\ the(B, D) \rightarrow \Diamond\neg all(D, U\text{–}G)$ (by Fact 3 and [40])

[42] $\vdash$ $\Box no(U\text{–}G, B) \wedge fewer\ than\ half\ of\ the(B, D) \rightarrow \Diamond not\ all(D, U\text{–}G)$

(i. e. □EF◇O-4, by Fact 2(4), Fact 2(2) and [41])

[43] $\vdash$ $at\ least\ half\ of\ the(B, D) \rightarrow \Diamond at\ least\ half\ of\ the(B, D)$ (by Fact 6)

[44] $\vdash$ $\Box no(B, U\text{–}G) \wedge \Box all(D, U\text{–}G) \rightarrow \Diamond at\ least\ half\ of\ the(B, D)$

(i. e. □A□E◇L-2, by [26], [43] and Rule 1)

[45] $\vdash$ $\neg\Diamond not\ all(B, G) \wedge \Box all(G, U\text{–}D) \rightarrow \neg most(B, U\text{–}D)$ (by [30] and Rule 1)

[46] $\vdash$ $\Box\neg not\ all(B, G) \wedge \Box all(G, U\text{–}D) \rightarrow \neg most(B, U\text{–}D)$ (by [45] and Fact 3(2))

[47] $\vdash$ $\Box all(B, G) \wedge \Box all(G, U\text{–}D) \rightarrow at\ most\ half\ of\ the(B, U\text{–}D)$

(i. e. □A□AH-1, by Fact 2(1), Fact 2(7) and [46])

[48] $\vdash$ $\neg\Diamond not\ all(B, G) \wedge most(B, U\text{–}D) \rightarrow \neg\Box all(G, U\text{–}D)$ (by [30] and Rule 1)

[49] $\vdash$ $\Box\neg not\ all(B, G) \wedge most(B, U\text{–}D) \rightarrow \Diamond\neg all(G, U\text{–}D)$ (by Fact 3 and [48])

[50] $\vdash$ $\Box all(B, G) \wedge most(B, U-D) \rightarrow \Diamond not\ all(G, U-D)$

(i. e. M□A◇O-3, by Fact 2(1), Fact 2(2) and [49])

[51] $\vdash$ $\Box no(U-G, B) \wedge \Box all(D, U-G) \rightarrow \Diamond at\ least\ half\ of\ the(B, D)$

(i. e. □A□E◇L-4, by [39], [43] and Rule 2)

[52] $\vdash$ $\Box all\neg(B, U-G) \wedge \Box no\neg(D, U-G) \rightarrow \Diamond at\ least\ half\ of\ the(B, D)$

(by [44], Fact 1(2) and Fact 1(1))

[53] $\vdash$ $\Box all(B, U-(U-G)) \wedge \Box no(D, U-(U-G)) \rightarrow \Diamond at\ least\ half\ of\ the(B, D)$

(by Definition 2 and [52])

[54] $\vdash$ $\Box all(B, G) \wedge \Box no(D, G) \rightarrow \Diamond at\ least\ half\ of\ the(B, D)$ (i. e. □E□A◇L-2, by [53])

[55] $\vdash$ $\neg\Diamond at\ least\ half\ of\ the(B, D) \wedge \Box all(D, U-G) \rightarrow \neg\Box no(B, U-G)$ (by [44] and Rule 1)

[56] $\vdash$ $\Box\neg at\ least\ half\ of\ the(B, D) \wedge \Box all(D, U-G) \rightarrow \Diamond\neg no(B, U-G)$ (by [55] and Fact 3)

[57] $\vdash$ $\Box fewer\ than\ half\ of\ the(B, D) \wedge \Box all(D, U-G) \rightarrow \Diamond some(B, U-G)$

(i. e. □A□F◇I-1, by Fact 2(6), Fact 2(3) and [56])

[58] $\vdash$ $\neg\Diamond at\ least\ half\ of\ the(B, D) \wedge \Box no(B, U-G) \rightarrow \neg\Box all(D, U-G)$ (by [44] and Rule 1)

[59] $\vdash$ $\Box\neg at\ least\ half\ of\ the(B, D) \wedge \Box no(B, U-G) \rightarrow \Diamond\neg all(D, U-G)$ (by [58] and Fact 3)

[60] $\vdash$ $\Box fewer\ than\ half\ of\ the(B, D) \wedge \Box no(B, U-G) \rightarrow \Diamond not\ all(D, U-G)$

(i. e. □E□F◇O-3, by Fact 2(6), Fact 2(2) and [59])

[61] $\vdash$ $at\ most\ half\ of\ the(B, U-D) \rightarrow \Diamond at\ most\ half\ of\ the(B, U-D)$ (by Fact 6)

[62] $\vdash$ $\Box all(B, G) \wedge \Box all(G, U-D) \rightarrow \Diamond at\ most\ half\ of\ the(B, U-D)$

(i. e. □A□A◇H-1, by [47], [61] and Rule 2)

[63] $\vdash$ $some(B, U-G) \leftrightarrow some(U-G, B)$ (by symmetry of *some*)

[64] $\vdash$ $\Box fewer\ than\ half\ of\ the(B, D) \wedge \Box all(D, U-G) \rightarrow \Diamond some(U-G, B)$

(i. e. □F□A◇I-4, by [57] and [63])

[65] $\vdash$ $no(B, U-G) \leftrightarrow no(U-G, B)$ (by symmetry of *no*)

[66] $\vdash$ $\Box fewer\ than\ half\ of\ the(B, D) \wedge \Box no(U-G, B) \rightarrow \Diamond not\ all(D, U-G)$

(i. e. □F□E◇O-1, by [60] and [65])

The above reasoning process indicates that by making use of the above facts, definitions and inference rules, the other 22 valid generalized modal syllogisms can be deduced merely from the valid generalized modal syllogism □EF◇O-1. This embodies that there are reducible relations between these 23 syllogisms.

4. Conclusion

For the sake of obtaining valid generalized modal syllogisms, the article first proves the validity of the generalized modal syllogism □EF◇O-1 by means of set theory and modal logic, and then deduces the other 22 valid generalized modal syllogisms from the syllogism □EF◇O-1 in accordance with modern modal logic, generalized quantifier theory, and so on. The reason why there are reducibilities between different generalized modal syllogisms is that: (1) any of the Aristotelian quantifiers (i.e., *all*, *no*, *some*, *not all*) is definable by the other three Aristotelian quantifiers; (2) any of the four generalized quantifiers mentioned in this article (i.e. *fewer than half of the*, *at least half of the*, *most*, *at most half of the*) is definable by the other three generalized quantifiers; (3) the transformation relationship between necessity (□) and possibility (◇); (4) the symmetry of *some* and *no.*

The article presents a formal research method for generalized modal syllogistic, which not only provides a unified mathematical research paradigm for other generalized modal syllogisms and even other kinds of syllogisms such as generalized syllogisms (Endrullis and Moss, 2015), but also meet with the demands for formalization transformation of modern logic in the era of artificial intelligence. Therefore, this study has considerable theoretical and practical values.

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