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Horizontal Motion Control for Variable Vector Propeller of Submarine Based on Pitch Angle Varying

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Abstract

In order to decrease both the weight and the size of submarine, the variable vector propeller (i.e., VVP) has been applied on it. In this paper we will study the horizontal motion control for submarine with variable vector propeller. Firstly, we will build the reasonable simplification of the horizontal motion equations about submarine with variable vector propeller according to the need of control system. Secondly, we will build the rules of pitch matching for the variable vector propeller, so we can get the pitch state of VVP, and we will give the thrust of VVP in this pitch state. Thirdly, considering the characteristic of nonlinearity, time-variation, coupling, stochastic disturb of wave and ocean current, and dubious parameters in horizontal motion control system of submarine with variable vector propeller, the backstepping sliding mode controller based on model reference adaptive is designed for it. The adaptive rules are introduced in the backstepping sliding mode control during the design of controller. By applying the backstepping sliding mode controller based on adaptive rules for horizontal motion of submarine with VVP, we can overcome the influence caused by some factors such as uncertainty and complexity of control system, and hold the performance of control. The formulation consults proved that backstepping sliding mode controller based on model reference adaptive can have better adaptive ability for uncertainty and complexity than the fuzzy sliding controller.

Key Words: Variable Vector Propeller, Pitch Angle, Horizontal Motion Control, Backstepping Sliding Mode Controller and Model Reference Adaptive.

1 Introduction

Now, submarine must have the ability of executing complex order in the ocean exploitation. Because the submarine is running to larger and heavier weight with the increasing of diving depth, it will receive high restriction at applying. To solve the problem of light and the miniaturization of submarine variable vector propeller is used in submarine. Submarine with two VVP can produce variable vector thrust including pre-and-post, right-and-left and up-and-down so that the submarine move alone six degree freedom. Then the total member of propeller is reduced, and the weight and cubic of submarine can be diminished without affection on requirement of configure and intensity by using VVP.

Since the VVP is useful for submarine to reduce the weight and the cubic, the study about submarine with VVP has been widely developed. For example Stenovec,1987[1]; Nanbozhiai,1988[2]; Dubian,1989[3],[4]; Yongsong,1990[5]; Huang, 2001[6]; Liu & Zheng,2005[10]; Liu & Zheng,2007[9]; Zheng,2011[11],[12]; . However, the existing results are mainly developed for VVP mechanism or ordinary movement control. In practice, most moved processes are complex and coupling so that the performance depends on design of controller. The well-known intelligent control method for submarine with VVP could be fuzzy, NN, etc.

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Liu Sheng et al have studied the principle and realized mechanism of pitch regulation control system, gave the mathematical model and simulation, and studied the Forward Kinematics of VVP based on HGANN[13]. Zheng Xiuli et al have studied the pitch movement control of submarine with VVP by variable universe fuzzy sliding mode[14]. But the study about horizontal motion control of submarine with VVP is not mature.

In this paper the bacdstepping sliding mode control based on model reference adaptive about horizontal motion of submarine with VVP is studied. Because the controlled target (submarine with VVP) of horizontal motion control system is nonlinear, uncertain and complex, regulatory controller with bad parameter tuning and performance can hardly achieve control requirement. Besides disturbance effect, fuzzy sliding mode control can not solve the complex of system. Aim at above problem, bacdstepping sliding mode controller based on model reference adaptive is choose in this paper which combined model reference adaptive controller with sliding mode control, introducing backstepping method, it can stably track the giving trajectory, so the system can achieve optimum control effect. The simulation proved that the control method can produce better effect.

2 Model of Horizontal Motion for Submarine with VVP

2.1 Simplification Model of Horizontal Motion for Submarine

Horizontal motion is moving of submarine of submarine in horizontal-face that is $E\xi\eta$ plane in the Ground coordinate[14], at the same time the oxy plane in the unfix coordinate is moving plane. It includes the translational motion of buoyant center and pitch of circling the oz axis. Submarine don't produce pitching force F_z , pitching moment M_y and roll moment M_x under horizontal motion condition, so all the parameter of pitching movement and roll is zero. Now we can consider the vertical movement of submarine which nearly couldn't affect the horizontal movement, so we will neglect the effectiveness of the vertical movement. But the horizontal motion can be affected by the disturbance of ocean current, we will think the effectiveness of ocean current when we study the horizontal motion.

The horizontal motion includes yaw movement and lateral movement, so we must simplify the six freedom model of submarine with VVP[14], so the horizontal three freedom model can be shown in follows.

State variable is given by:

$$\eta = [x \quad y \quad \psi]^T \qquad \mathrm{v} = [u \quad v \quad r]^T \tag{1}$$

The thrust and moment of three degree of freedom are given by:

$$\boldsymbol{\tau}_{\mathrm{T}} = [F_x \quad F_y \quad Q_z]^T \tag{2}$$

$$R_E^B = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \mathrm{C(v)}=\begin{bmatrix} 0 & 0 & -m_2 v \\ 0 & 0 & m_1 u \\ m_2 v & -m_1 u & 0 \end{bmatrix}$$

$$\mathrm{M} = \begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & 0 \\ 0 & 0 & m_{66} \end{bmatrix} = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \tag{3}$$

Where $\mathrm{R}_{\mathrm{E}}^{\mathrm{B}}$, M and $\mathrm{C(v)}$ denote space coordinate translation matrix, inertial matrix and coriolis matrix, respectively.

Damping matrix of three degree of freedom is given by:

$$\mathrm{D(v)} = -\begin{bmatrix} \lambda_{11}\dot{u} & 0 & 0 \\ 0 & \lambda_{22}\dot{v} & 0 \\ 0 & 0 & \lambda_{66}\dot{r} \end{bmatrix} \tag{4}$$

The thrust and moment of three degree of freedom can be rewritten as the following state equation:

$$\tau_{\mathrm{T}} = B\boldsymbol{\tau} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & L \end{bmatrix} \begin{bmatrix} F_x \\ F_y \end{bmatrix} = \begin{bmatrix} F_x \\ F_y \\ Q_z \end{bmatrix} \tag{5}$$

Where $\boldsymbol{\tau}^T = [F_x \quad F_y] = \{\tau_x \quad \tau_y\}$, $Q_z = F_y \times L$, L is distance from thrust action point to gravity center.

We suppose the gravity center and buoyant center are superposition and the gravity equal to the buoyancy, $\boldsymbol{\tau}_G$ is zero, and the effect of ocean current couldn't be ignored, so the $\boldsymbol{\tau}_E$ isn't zero, and the horizontal three freedom model of submarine with VVP can be shown in follows:

$$\begin{cases} \dot{\eta} = R_E^B \mathrm{v} \\ \dot{\mathrm{v}} = -\mathrm{M}^{-1}(\mathrm{D(v)} + \mathrm{C(v)})\mathrm{v} + \mathrm{M}^{-1}\mathrm{B}\,\tau \end{cases} \tag{6}$$

The state vector is shown in follows:

$$\dot{x} = Ax + b\boldsymbol{\tau} \tag{7}$$

Where $A=\begin{bmatrix} R_E^B \\ -M^{-1}(D(v)+C(v))v \end{bmatrix}$, $b=\begin{bmatrix} 0 \\ M^{-1}B \end{bmatrix}$.

Using two variable vector propellers which work in cosine cycling pitch state, the submarine can achieve the yaw movement by the electric compass sensor.

2.2 Hydrodynamic Modeling of VVP in Cycling Pitch State

In order to achieve the horizontal motion the VVP of submarine must work in cosine cycling pitch state. We need build the hydrodynamic model of VVP in cosine cycling pitch state. We will build the model by regression fitting method, and make the significance test to validate the model's rationality and realization.

m-Order double independent variable regression model about thrust coefficient that is $K_T(J,\theta_p)$ of VVP is shown as:

$$\begin{aligned} K_T(J,\theta_p) &= b_0 + b_1 J + b_2\theta_p + b_3 J^2 + b_4\theta_p{}^2 + b_5 J\theta_p + \cdots \\ &+ b_{n-m}J^m + b_{n-m+1}\theta_p{}^m + b_{n-m+2}J^{m-1}\theta_p + \cdots + b_n J\theta_p{}^{m-1} + \varepsilon \end{aligned} \tag{8}$$

Where m is the order of regression model, $b_i(i=0,1,\cdots,n)$ is the interaction influence coefficient, and $n=\sum_{k=1}^{m}(k+1)$ If J = 0, and△q = 0, then $\theta_p=\Delta\theta\cos(\varphi-\varphi_0)=0$, the thrust of VVP is zero, and the working state is neutral, the thrust coefficient is zero that is $K_T=0$, and $b_0=0$; if $\cos(\varphi-\varphi_0)=1$, then the pitch angle of regression model is cycling pitch angle which is not correlation with phase angle. We can get the regression model about thrust coefficient by diagram between thrust coefficient and cycling pitch angle and advance ratio.

We can get the regression coefficient using the least square method. The hydrodynamic performance of VVP is mainly decided by its leaf inclination that is δ, based on related data, horizontal force of VVP could get best results when δ is $45°$, and axial force of VVP is best when δ is $0°$. We can get the thrust coefficient of cycling pitch angle state[6],[7],[8], the sampled data is shown as Tab.1.

Tab.1 Sampling Data of Main/Flap Rudder Lifting Coefficient

J	$\Delta\theta$	K_T	J	$\Delta\theta$	K_T	J	$\Delta\theta$	K_T
0	5	0.21	0	8	0.225	0	10	0.23
0.05	5	0.195	0.05	8	0.215	0.05	10	0.22
0.1	5	0.18	0.1	8	0.195	0.1	10	0.205

0.15	5	0.17	0.15	8	0.178	0.15	10	0.185
0.2	5	0.155	0.2	8	0.168	0.2	10	0.174
0.25	5	0.14	0.25	8	0.155	0.25	10	0.16
0.3	5	0.125	0.3	8	0.13	0.3	10	0.138
0.35	5	0.11	0.35	8	0.112	0.35	10	0.12
0.4	5	0.09	0.4	8	0.092	0.4	10	0.1
0.45	5	0.07	0.45	8	0.071	0.45	10	0.08
0.5	5	0.05	0.5	8	0.05	0.5	10	0.05
0.55	5	0.03	0.55	8	0.03	0.55	10	0.03

Using fitting we can get regression model of thrust efficient:

$$\begin{aligned} K_T(J,\theta_p) = &-0.1214J - 0.0807\theta_p - 0.2515J^2 - 0.0099\,\theta_p^{\,2} - 0.0240\,J\theta_p \\ &+ 0.1655J^3 + 0.0004\theta_p^{\,3} - 0.0077J^2\theta_p + 0.0013J\theta_p^{\,2} \end{aligned} \tag{9}$$

Similarly, we can get regression model of moment coefficient:

$$\begin{aligned} K_Q(J,\theta_p) = &\ 0.0125J + 0.0055\theta_p - 0.0350J^2 - 0.0007\theta_p^{\,2} - 0.0029J\theta_p \\ &- 0.0061J^3 + 0.0012J^2\theta_p + 0.0002J\theta_p^{\,2} \end{aligned} \tag{10}$$

We need make significance test after modeling for the rationality and realization. Using correlation-coefficient method and F-test method we make the significance test, and the results are shown as Tab.2. From Tab.2 we can get $R > R_\alpha$, $F > F_\alpha$, so the regression effects of least squares are significance, and the model is realizable and rational.

Tab.2 Remarkable Tests for Regressive Models of Hydrodynamic Coefficients of VVP ($\alpha = 0.05$)

Hydrodynamic Coefficient Regression Model	K_T	K_Q
Variance Statistics	L_{yy}	0.1723
	U	0.1720
	Q	2.8687e-004
Model Test	F $(F_\alpha = 2.35)$	9593.9 $(F > F_\alpha)$

	R $(R_\alpha = 0.3667)$	$0.9992\,(R > R_\alpha)$

The thrust and resistance are shown in follows:

$$F_T = 3\rho n^2 D^4(-0.1214J - 0.0807\Delta\theta\sin(\varphi) - 0.2515J^2 - 0.0099\,\Delta\theta^2\sin^2(\varphi) - 0.0240\,J\Delta\theta\sin(\varphi) + 0.1655J^3 + 0.0004\Delta\theta^3\sin^3(\varphi) - 0.0077J^2\Delta\theta\sin(\varphi) + 0.0013J\Delta\theta^2\sin^2(\varphi)) \quad (11)$$

$$F_Q = 0.35\rho n^2 D^6(0.0125J + 0.0055\Delta\theta\sin(\varphi) - 0.0350J^2 - 0.0007\Delta\theta^2\sin^2(\varphi) - 0.0029J\Delta\theta\sin(\varphi) - 0.0061J^3 + 0.0012J^2\Delta\theta\sin(\varphi) + 0.0002J\Delta\theta^2\sin^2(\varphi))/3 \quad (12)$$

Usually, ocean current is interference, without rotating, fixed coordinate system, the speed of ocean current is:

$$U_c = \begin{bmatrix} u_c & \omega_c \end{bmatrix}^T = \begin{bmatrix} u_x & u_y & u_z & 0 & 0 & 0 \end{bmatrix}^T \quad (13)$$

Without ocean current the relative speed of submarine is u, v, w, we supposed the current is parallel to the horizontal plane of Ground coordinate. If the relative speed of submarine is $U_r(u_r, v_r, w_r)$, then

$$u_r = u - U_x \qquad v_r = v - U_y \qquad w_r = w - U_z \quad (14)$$

So the disturbance force an moment of ocean current is:

$$\boldsymbol{\tau}_E = \begin{bmatrix} w_1 & w_2 & w_3 & 0 & 0 & 0 \end{bmatrix}^T \quad (15)$$

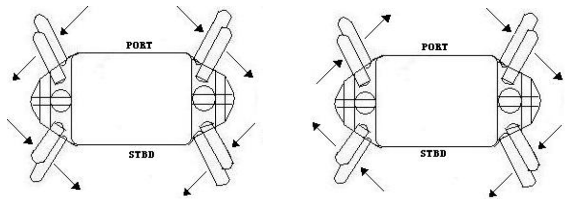
Where, w_1, w_2, w_3 is force of three axis direction causing by ocean current.

When submarine work in ocean it could go drift motion and diverge from the giving track and heading because the effectiveness of flow pressure caused by ocean current.

3 Horizontal Motion Control for VVP of Submarine

3.1 The Horizontal Motion Principle of Submarine with VVP

The horizontal motion of submarine with VVP includes lateral movement and yaw motion, lateral motion is shown as in “a” of Fig. 1. When the two VVP is in cosine cycling pitch state and rotary direction is reverse, or the phase contrast of cycling pitch angles of the two VVP is 180°, that is to say two rotary incline plane of VVP transverse the same angle circling Y axis, VVP produce left or right thrust, so the submarine can move alone left-right direction.



a. lateral movement b. yaw movement

Fig. 1: Thrust state of lateral and yaw movement

Because the characteristic of VVP is the pitch angle of blade is variable, the thrust magnitude of VVP is affected by the actual speed (i.e. u), relative water velocity (i.e. u_y), and rotational speed (i.e. n). If the thrust produced by the one VVP made the submarine move left or right at velocity (i.e. u_{1L}), the thrust produced by the other VVP made the submarine move left or right at velocity u_{2L} , the left or right velocity of submarine is shown in formula (16).

$$u = u_{1L} + u_{2L} \tag{16}$$

At he meantime the speed in Y axis produced by single VVP is shown as formula (17).

$$u_Y = [-V_Y + \sqrt{V_Y{}^2 + 4\{F_Y/(2\rho S)\}}\,]/2 \tag{17}$$

Axial included velocity of Y axis is:

$$u_{Y1} = Au_Y \tag{18}$$

Where, V_Y is infinite upstream influent speed in Y axis, A is coefficient of correction of induced velocity, $S = D_P \cdot R(1-b)$, which is the area by liquid fluid medium washed, D_P is the diameter of VVP, F_y is thrust in Z axis direction, ρ is fluid density.

So the pitch angle of VVP can be shown as formula (19).

$$\beta(\varphi) = \beta_0 + \Delta\beta\cos(\varphi - \varphi_0) \tag{19}$$

The pitch angle is realized by cosine cycling pitch state of two VVP, and the yawing motion can be keep by differential cosine cycling pitch of two VVP, an the heading can be keep using a electronic compass sensor.

Where the cycling pitch angle and the axial vertical force are big, the coefficient of correction close to 1; when the cycling pitch angle is small and the effect of induced velocity can be ignored, the coefficient of correction close to 0.

Yawing motion is shown as in "b" of Fig. 1, when the two VVP is in different cycling pitch state and the phase contrast of cycling pitch angles of the two VVP is 180°, the thrust produced by one VVP is leftward and the thrust produced by the other VVP is rightward, so the submarine is yawing motion leftward. Contrarily the submarine is yawing motion rightward. The one VVP produced some yawing angle, so the other VVP produced opposite direction yawing angle. If the yawing angle produced by one VVP is γ_1, and the yawing angle produced by the other VVP is γ_2, the yawing angle of submarine is shown in follows.

$$\gamma = \gamma_1 + \gamma_2 \tag{20}$$

3.2 Design of Adaptive Backstepping Sliding Model Controller for Horizontal Motion of Submarine with VVP

Because the object of control system is to keep the horizontal rotary motion, we suppose the x axis and y axis components F_x and F_y is the control input (i.e. $\boldsymbol{\tau}_{\mathbf{T}}$), and $Q_z = F_y \times L$, the control rule can be designed for control input F_x and F_y.

We take yawing angle and angular velocity to control the horizontal motion of submarine with VVP, shown as in Fig.3. We can calculate input that is thrust and moment according to giving yawing angle.

We consider the controlled object given by:

$$\begin{cases} \dot{x} = Ax + y \\ \dot{y} = bu \end{cases} \tag{21}$$

Where $b \neq 0$.

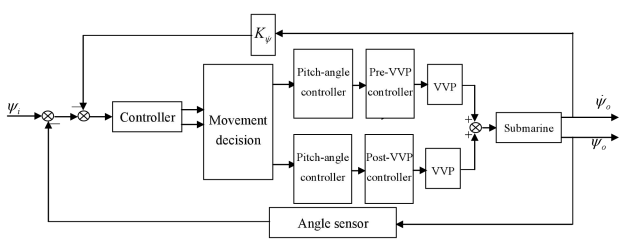


Fig. 3: Horizontal plane motion control block diagram

Design procedure:

Supposed z_d is position Command, the position error shown as:

$$z_1 = x - z_d \tag{22}$$

Then $\dot{z}_1 = \dot{x} - \dot{z}_d = Ax + y - \dot{z}_d$.

Definition1. Virtual controller can be expressed by:

$$\alpha_1 = -c_1 z_1 + \dot{z}_d \tag{23}$$

Where $c_1 > 0$.

We introduce the function z_2, which is defined as follow as:

$$z_2 = y - \alpha_1 \tag{24}$$

Let us choose the Lyapunov function candidates as:

$$V_1 = (1/2) z_1^2 \tag{25}$$

Then $\dot{V}_1 = z_1 \dot{z}_1 = z_1 (Ax + y - \dot{z}_d) = z_1 (Ax + z_2 + \alpha_1 - \dot{z}_d)$

So $\dot{V}_1 = -c_1 z_1^2 + (z_2 + Ax) z_1$.

If $z_2 = 0$, we couldn't prove $\dot{V}_2 \le 0$. Differentiating z_2:

$$\dot{z}_2 = \dot{y} - \dot{\alpha}_1 = bu + c_1 \dot{z}_1 - \ddot{z}_d \tag{26}$$

Let us choose Lyapunov function candidates as:

$$V_2 = V_1 + (1/2)\sigma^2 \tag{27}$$

Where σ is switching function, which is defined as:

$$\sigma = k_1 z_1 + z_2 \tag{28}$$

Where $k_1 > 0$, then

$$
\begin{aligned}
\dot V_2 &= \dot V_1 + \sigma\dot\sigma = -c_1 z_1^2 + (z_2 + Ax)z_1 + \sigma(k_1\dot z_1 + \dot z_2) = -c_1 z_1^2 + (z_2 + Ax)z_1 \\
&+\sigma(k_1(Ax + z_2 + \alpha_1 - \dot x_d) + bu + c_1(Ax + z_2 + \alpha_1 - \dot x_d) - \ddot x_d)
\end{aligned} \tag{29}
$$

In order to prove $\dot V_2 \le 0$, we design the control rules is shown as:

$$
\begin{aligned}
u &= -(1/b)(k_1(Ax + z_2 + \alpha_1 - \dot x_d) \\
&+c_1(Ax + z_2 + \alpha_1 - \dot x_d) - \ddot x_d - h(\sigma + c\,\mathrm{sgn}(\sigma)))
\end{aligned} \tag{30}
$$

Where h and c are positive constant.

Let u Substitute into $\dot V_2$ and rearranging, we get:

$$
\begin{aligned}
\dot V_2 &= -c_1 z_1^2 + z_1 z_2 - h\sigma^2 - hc\,|\sigma| + F\sigma - \bar F\,|\sigma| \\
&\le -c_1 z_1^2 + z_1 z_2 - h\sigma^2 - hc\,|\sigma|
\end{aligned} \tag{31}
$$

Lyapunov function is:

$$
V_3 = V_2 + (1/(2k_F))\tilde F^2 \tag{32}
$$

Where $\tilde F = F^* - \hat F$, $\hat F$ is estimated value of F, F is uncertain and disturbance of system, k_F is positive constant.

$$
\begin{aligned}
\dot V_3 &= \dot V_2 - (1/t)\tilde F\dot{\hat F} = -c_1 z_1^2 + (z_2 + Ax)z_1 + \sigma(k_1\dot z_1 + \dot z_2) \\
&= -c_1 z_1^2 + (z_2 + Ax)z_1 + \sigma(k_1(Ax + z_2 + \alpha_1 - \dot x_d) + bu \\
&+c_1(Ax + z_2 + \alpha_1 - \dot x_d) - \hat F - \ddot x_d) - (1/k_F)\tilde F(\dot{\hat F} - t\sigma)
\end{aligned} \tag{33}
$$

we design adaptive controller rules which is shown as:

$$
\begin{aligned}
u &= -(1/b)(k_1(Ax + z_2 + \alpha_1 - \dot x_d) + c_1(Ax + z_2 + \alpha_1 - \dot x_d) \\
&-\ddot x_d - \hat F - h(\sigma + c\,\mathrm{sgn}(\sigma)))
\end{aligned} \tag{34}
$$

The adaptive control rule is:

$$
\dot F = t\sigma \tag{35}
$$

We can get:

$$
\dot V_3 = z_1 z_2 - c_1 z_1^2 - h\sigma^2 - hc\,|\sigma| \tag{36}
$$

It can be rewritten as:

$$
\dot V_3 = z^T Q z - hc\,|\sigma| \le 0 \tag{37}
$$

Where $Q=\begin{bmatrix} c_1+hk_1^2 & hk_1-\frac{1}{2} \\ hk_1-\frac{1}{2} & h \end{bmatrix}$, $z^T=[z_1 \quad z_2]$.

3.3 Design of Backstepping Sliding based on Model Reference Adaptive Controller for Horizontal Motion of Submarine with VVP

The desired trajectory needed track is defined as $\eta_d=[x_d \quad y_d]$, which denote position of submarine at any moment. Let us choose reference system as decoupling Second Order Oscillators, and one oscillator response along reference position of x axis, the other response along reference position of y axis. It can be expressed as:

$$\ddot{X}_i+2\varsigma_i\omega_i\dot{X}_i+\omega_0 X_i=\omega_0{}^2 r_i \qquad i=1,2 \tag{38}$$

Where X_i is position of number i oscillator, ς_i is damping coefficient, ω_0 is natural frequency, r_i is reference input of number i oscillator, the reference system can be written as:

$$\begin{bmatrix} \dot{\eta}_r \\ \ddot{\eta}_r \end{bmatrix}=\begin{bmatrix} 0 & I_2 \\ -\omega_0^2 & -2\varsigma\omega_0 \end{bmatrix}\begin{bmatrix} \eta_r \\ \dot{\eta}_r \end{bmatrix}+\begin{bmatrix} 0 \\ \omega_0^2 \end{bmatrix}r_s \tag{39}$$

Where $\eta_r=[x_{r1} \quad x_{r2}]^T$, which is position of reference system, I_2 is identity matrix, $\varsigma=\begin{bmatrix} \varsigma_1 & 0 \\ 0 & \varsigma_2 \end{bmatrix}$, which is damping matrix, $\omega_0=\begin{bmatrix} \omega_1 & 0 \\ 0 & \omega_2 \end{bmatrix}$, which is natural frequency matrix, $r_s=[r_1 \quad r_2]$, which is reference inputs.

In order to track the expected system trajectory that is η_d, we choose reference signal is:

$$r_s=(\omega_0^2)^{-1}(\ddot{\eta}_d+\omega_0^2\eta_d+2\varsigma\omega_0\dot{\eta}_d) \tag{40}$$

We can get the error that is e_{rd} between system and desired trajectory, $e_{rd}=\left[\eta_r^T \quad \dot{\eta}_r^T\right]-\left[\eta_d^T \quad \dot{\eta}_d^T\right]$, so e_{rd} can be written as $\dot{e}_{rd}=A_r e_{rd}$, based on matrix A_r e_{rd} can be transformed to a zero, but in real system it is impossible, error within the permissible range we consider that system can achieve expected results. e and $\tilde{e}$ denote position error of Ground coordinate system and unfix coordinate system respectively, e and $\tilde{e}$ are defined as follows:

$$e=\eta_r-\eta_s, \ \tilde{e}=J_s^{-1}(\psi)e.$$

Where $\eta_s = [x \quad y]^T$,which is real position of submarine, $J_s(\psi)$ is submatrix of coordinate transformation matrix that is $J(\psi)$. e_s denote distance between real position and expected position, e_s is defined as $e_s = \|\tilde{e}\| = \|e\|$, then $\dot{e}_d = (1/e_d)\tilde{e}^T S\dot{\tilde{e}}$, β denote the angle between x axis and expected position, $\beta \in [-\pi, \pi]$, which is shown as in Fig.4.

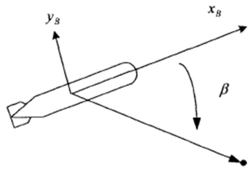


Fig.4 Angel between the x axis and the participate position

$(1/e_d)e^{-T} = [\cos\beta \quad \sin\beta]$, so position error and angle can be rewritten as:

$$\begin{cases} \dot{e}_d = [\cos\beta \quad \sin\beta]\dot{\tilde{e}} \\ \beta = \dfrac{1}{e_d}[-\sin\beta \quad \cos\beta]\dot{\tilde{e}} \end{cases} \tag{41}$$

In order to control e_d which is distance between real position and desired trajectory and β which is angle error, we introduce Lyapunov function as:

$$V_1(e_d, \beta) = e_d \sin^2(\beta/2) + (1/2)(e_d - a)^2 \tag{42}$$

Where a denote maximum error which system can tolerate and a >0, and $V_1(e_d, \beta) \geq 0$, with unique zero(a ,0). Using position error and angle we can get:

$$\dot{V}_1(e_d, \beta, \dot{\tilde{e}}) = [\sin^2(\beta/2) + e_d - a(1/2)\sin\beta]\mathrm{J}^{-1}(\beta)\dot{\tilde{e}} \tag{43}$$

Let $\dot{\tilde{e}}$ as real command, which should be expressed:

$$\dot{\tilde{e}}_{des} = -e_d \mathrm{J}(\beta) G_1 \begin{bmatrix} \sin^2(\frac{\beta}{2}) + e_d - a \\ \frac{1}{2}\sin\beta \end{bmatrix} \tag{44}$$

In order to make $\dot{\tilde{e}}$ to $\dot{\tilde{e}}_{des}$ we introduce another error variable r , which is shown as $r = \tilde{e} - \tilde{e}_{des}$, so we can get:

$$\dot{V}_1(e_d, \beta, r) = -e_d[\sin^2(\frac{\beta}{2}) + e_d - a\frac{1}{2}\sin\beta] \\ G_1 \begin{bmatrix} \sin^2(\frac{\beta}{2}) + e_d - a \\ \frac{1}{2}\sin\beta \end{bmatrix} + \lambda^T \mathrm{J}(\beta) \begin{bmatrix} \sin^2(\frac{\beta}{2}) + e_d - a \\ \frac{1}{2}\sin\beta \end{bmatrix} \tag{45}$$

But we can not guarantee $\dot{V}_2 \le 0$, we introduce another Lyapunov functuion as:

$$V_2(e_d,\beta,r) = e_d\sin^2(\beta/2) + (1/2)(e_d-a)^2 + (1/2)\sigma^2 \tag{46}$$

Differentiating $V_2(e_d,\beta,r)$:

$$\begin{aligned}&\dot{V}_2(e_d,\beta,r,\tau^*,t) = -e_d[\sin^2(\frac{\beta}{2}) + e_d - a\frac{1}{2}\sin\beta] \\ &G_1\begin{bmatrix}\sin^2(\frac{\beta}{2}) + e_d - a \\ \frac{1}{2}\sin\beta\end{bmatrix} + \sigma(\mathrm{J}(\beta))\begin{bmatrix}\sin^2(\frac{\beta}{2}) + e_d - a \\ \frac{1}{2}\sin\beta\end{bmatrix} + \dot{\sigma}(\tau^*,t))\end{aligned} \tag{47}$$

We choose $\tau^* = B_s^{-1}[(D_s(v_s)C_s(v))v_s + M_s(F_1(e_d,\beta,t) + \dot{r}S\tilde{e})]$, and define symbols as:

$\theta^* = B_sM_s$, $\theta_{21}^* = B_sD_{1s}, \theta_{22}^* = B_sD_{qs}, \theta_{23}^* = B_s\begin{bmatrix}m_1 & 0 \\ 0 & m_2\end{bmatrix}$,

$\theta_{31}^* = \dfrac{d_{13}}{m_3}B_sD_1, \theta_{32}^* = \dfrac{d_{q3}}{m_3}B_sM_s, \theta_{33}^* = \dfrac{m_2-m_1}{m_3}B_sM_s$,

$F_{21}(v_s) = v_s, F_{22}(v_s) = \|v_s\|v_s, F_{23}(v_s) = rS^Tv_s$,

$F_{31}(r,\tilde{e}) = rS^T\tilde{e}, F_{32}(r,\tilde{e}) = r\|r\|S^T\tilde{e}, F_{33}(r,\tilde{e}) = \mu\, vS^T\tilde{e}$,

So τ^* can be rewritten as:

$$\tau^* = \theta_1^*F_1(e_d,\beta,t) + \sum_{i=1}^{3}\theta_{2i}^*F_{2i}(v_s) + \sum_{j=1}^{3}\theta_{2j}^*F_{3j}(v,\tilde{e}) \tag{48}$$

The estimated value of θ^* is θ, estimated value of $\theta_1^*, \theta_{2i}^*, \theta_{3j}^*$ are $\theta_1, \theta_{2i}, \theta_{3j}$ which are substituted into τ^* and rearranging, we get:

$$\tau = \theta_1F_1(e_d,\beta,t) + \sum_{i=1}^{3}\theta_{2i}F_{2i}(v_s) + \sum_{j=1}^{3}\theta_{2j}F_{3j}(v,\tilde{e}) \tag{49}$$

$\tilde{\theta}$ denote error between estimated value of θ and real value of θ^* , $\tilde{\theta} = \theta - \theta^*$, in order that τ can be new control rule instead of τ^* we let $\tilde{\theta} \to 0$, and we choose a new Lyapunov function for getting update rule of θ:

$$\begin{aligned}&V_2(e_d,\beta,r,\tau,t) = e_d\sin^2(\frac{\beta}{2}) + \frac{1}{2}(e_d-a)^2 + \frac{1}{2}r^Tr + \frac{1}{2}tr(M_s^{-1}B_s^{-1}\tilde{\theta}_1\Gamma_1^{-1}\tilde{\theta}_1^T) \\ &+\sum_{i=1}^{3}\frac{1}{2}tr(M_s^{-1}B_s^{-1}\tilde{\theta}_{3i}\Gamma_{3i}^{-1}\tilde{\theta}_{3i}^T) + \sum_{j=1}^{3}\frac{1}{2}tr(M_s^{-1}B_s^{-1}\tilde{\theta}_{2j}\Gamma_{2j}^{-1}\tilde{\theta}_{2j}^T)\end{aligned} \tag{50}$$

Where $tr(\cdot)$ is trace-class operators, which can calculate the sum of the elements on the diagonal, $\Gamma_1, \Gamma_{2i}, \Gamma_{3i}$, i =1,2,3 are positive constant. We can get:

$$\dot{V}_2(e_d,\beta,r) = -e_d[\sin^2(\frac{\beta}{2}) + e_d - a\frac{1}{2}\sin\beta]G_1\begin{bmatrix}\sin^2(\frac{\beta}{2}) + e_d - a \\ \frac{1}{2}\sin\beta\end{bmatrix} + r^{\mathrm{T}}\begin{bmatrix}0 & \frac{L}{m_3}\end{bmatrix}\tau S(\tilde{e} \tag{51}$$
$$-\begin{bmatrix}a \\ 0\end{bmatrix}) - r^{\mathrm{T}}G_2 r - tr(M_s^{-1}B_s^{-1}\tilde{\theta}_1 F_1(e_d,\beta,t) - M_s^{-1}B_s^{-1}\tilde{\theta}_1\Gamma_1^{-1}\dot{\theta}_1^T - \sum_{i=1}^{3} tr(M_s^{-1}B_s^{-1}\tilde{\theta}_{2i}F_{2i}v(s)r^T$$
$$-M_s^{-1}B_s^{-1}\tilde{\theta}_{2i}\Gamma_{2i}^{-1}\dot{\theta}_{2i}^{\ T}) - \sum_{j=1}^{3} tr(M_s^{-1}B_s^{-1}\tilde{\theta}_{3j}F_{3j}(v,\tilde{e})r^T - M_s^{-1}B_s^{-1}\tilde{\theta}_{3j}\Gamma_{3j}^{-1}\dot{\theta}_{3j}^{\ T})$$

Finally we choose $\theta_1, \theta_{2i}, \theta_{3j}$ as the update rules:

$$\dot{\theta}_1 = \Gamma_1 r F_1^T(e_d,\beta,t),\ \dot{\theta}_{2i} = \Gamma_{2i} r F_{2i}^T(v_s) \quad i = 1,2,3 \tag{52}$$

$$\dot{\theta}_{3j} = \Gamma_{3j} r F_{3j}^T(v,\tilde{e}) \quad j = 1,2,3$$

So $\dot{V}_2$ can be expressed by:

$$\dot{V}_2(e_d,\beta,r,\tau,t) = -e_d[\sin^2(\frac{\beta}{2}) + e_d - a\frac{1}{2}\sin\beta] \tag{53}$$
$$G_1\begin{bmatrix}\sin^2(\frac{\beta}{2}) + e_d - a \\ \frac{1}{2}\sin\beta\end{bmatrix} - r^{\mathrm{T}}G_2 r + r^{\mathrm{T}}\begin{bmatrix}0 & \frac{L}{m_3}\end{bmatrix}\tau S(\tilde{e} - \begin{bmatrix}a \\ 0\end{bmatrix})$$

In conclusion Lyapunov stability of system can be keep based on $\dot{V}_2 \le 0$.

We can get above control rules without considering any exogenous disturbance, but when controlling a real process, it is usually affect by some exogenous disturbance such as surface wave, ocean current, and uncertainty of system parameters produce difficulty for controlling. In order to get better control results, some method will be employed to deal with the uncertainty in this study. We assumed submarine surface of dipped into water is enough depth, so attenuating the effect of disturbance caused by surface wave, it can be neglected. Bu the ocean current will take big effect for control results of submarine, we consider the ocean current disturbance for control rules and introduce adaptive control rule.

In this study we assume that speed and force direction of ocean current are unknown, the dynamic model which has added ocean current disturbance can be rewritten as:

$$M\dot{v} + D(v)v + C(v)v = B\tau + R_E^{B-1}w \tag{54}$$

Where, $w=[w_1,w_2,0]^T$, w is force and moment of ocean current to Ground coordinate system, introducing adaptive control law, then control inputs τ^* including ocean current effect can be rewritten as:

$$\tau^*=B_s^*(\tilde{e})M_s(F_1(e_d,\beta,t)-\frac{1}{m_3}(d_{13}r+d_{q3}|r|r+(m_2-m_1)\mu v)S\tilde{e})+B_s^*(\tilde{e})(D_s(v_s)$$
$$+C_s(v))v \quad +B_s^*(\tilde{e})J^{-1}(\psi)w=\theta_1^*F_1(e_d,\beta,t)+\sum_{i=1}^{3}\theta_{2i}^*F_{2i}(v_s)+\sum_{j=1}^{3}\theta_{3j}^*F_{3j}(v,e)+\theta_4^*F_4(\psi) \qquad (55)$$

Real control rule:

$$\tau=\theta_1 F_1(e_d,\beta,t)+\sum_{i=1}^{3}\theta_{2i}F_{2i}(v_s)+\sum_{j=1}^{3}\theta_{3j}F_{3j}(v,e)+\theta_4 F_4(\psi) \qquad (56)$$

Where θ_4 is estimate value of θ_4^* , in order to compensation estimation error $\tilde{\theta}_4=\theta_4-\theta_4^*$, define new Lyapunov function:

$$V_3(e_d,\beta,\gamma,\tau,t)=e_d\sin^2(\frac{\beta}{2})+\frac{1}{2}(e_d-\alpha)+\frac{1}{2}r^Tr+\frac{1}{2}tr(M_s^{-1}B_s^{-1}\tilde{\theta}_1\Gamma_1^{-1}\tilde{\theta}_1^T)$$
$$+\sum_{i=1}^{3}\frac{1}{2}tr(M_s^{-1}B_s^{-1}\tilde{\theta}_{2i}\Gamma_{2i}^{-1}\tilde{\theta}_{2i}^T)+\sum_{j=1}^{3}\frac{1}{2}tr(M_s^{-1}B_s^{-1}\tilde{\theta}_{3j}\Gamma_{3j}^{-1}\tilde{\theta}_{3j}^T)+\frac{1}{2}tr(M_s^{-1}B_s^{-1}\tilde{\theta}_4\Gamma_4^{-1}\tilde{\theta}_4^T) \qquad (57)$$

Finally, we choose new update rule is shown in follow:

$$\dot{\theta}_4=\Gamma_4\gamma F_4^T(\phi) \qquad (58)$$

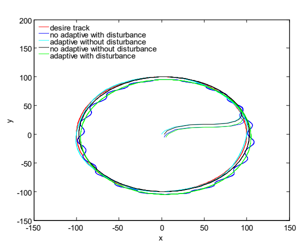
4 Computer Simulation

In this paper, the values of controller parameters and model parameters are given as follows:

$\omega_0=0.2\mathrm{I}_2$, $\varsigma=0.7\mathrm{I}_2$, $\Gamma_1=\Gamma_{2i}=\Gamma_{3j}=10\mathrm{I}_2$, $\theta_1=30\mathrm{I}_2$, $\theta_{21}=5\mathrm{I}_2$, $\theta_{22}=5\mathrm{I}_2$, $\theta_{23}=40\mathrm{I}_2$, $\theta_{31}=15\mathrm{I}_2$, $\theta_{32}=30\mathrm{I}_2$, $\theta_{33}=80\mathrm{I}_2$, $\alpha=0.6$

Subject the boundary radius from -100m to 100m, submarine speed is 3Knots, initial position (0,0,100).

In the simulation the pro- and post VVP are in different cosine cycle pitch state, so the pitch angle variable of VVP is same but the difference of phase is 180° , the speed of propeller is 200rpm, the speed of submarine is 3Knots. The curve of rotational motion, the curve of x axis error, the curve of y axis error and the curve of pitch angle are shown in Fig.5-Fig.8.



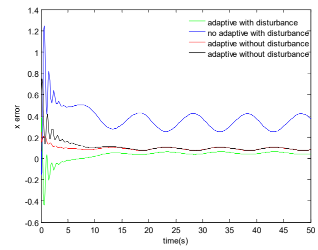
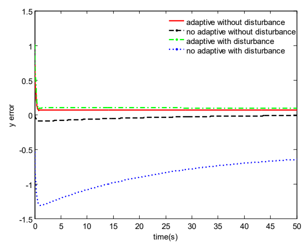


Fig. 5: Horizontal rotational motion curve **Fig. 6: x axis error curve**



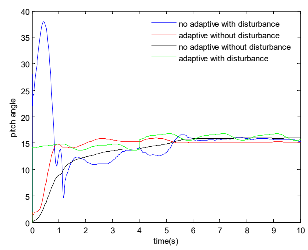


Fig. 7: y axis error curve **Fig. 8: Pitch curve of VVP**

Without ocean current disturbance the control effect is similarly compared backstepping sliding mode method with backstepping sliding mode based on model reference adaptive method. But with ocean current disturbance backstepping sliding mode based on model reference adaptive method can better track the desire trajectory, and the pitch angle is stable. We can conclude that this control method has high robust, the track effect is better than backstepping sliding mode method. By applying this control method, we can overcome the influence caused by some factors such as uncertainty and complexity of control system. So we can hold the performance of system and get the better control effect. The formulation consults proved that this controller can get better adaptive ability for uncertainty and complexity than the backstepping sliding mode controller.

5 Conclusion

In this paper, we have studied the problem of designing a backstepping sliding mode based on reference adaptive controller for horizontal motion of submarine with VVP. The outcome of the horizontal motion control is presented in terms of a set of nonlinear, complex, time varying and strong coupling problem. The adaptive rule is proposed to overcome the disturbance produced by ocean current. The problem approximately can be solved by using this control method. Finally, the proposed design method is applied to the horizontal motion of submarine with VVP, and the achieved simulation results show the effectiveness of the proposed controller.

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