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Knowledge Reasoning Based on the Generalized Syllogism *AHH-2*

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Abstract

This paper uses set theory to provide knowledge representation methods for common generalized syllogisms in natural language. Then, the validity of the generalized syllogism *AHH-2* with the non-trivial generalized quantifier *at most half of the* is proved by the truth definitions of categorical propositions, and then the other 25 valid generalized syllogisms are derived from this syllogism. The reason why these results are consistent is that deductive reasoning is used throughout the proof process. In fact, more valid generalized syllogisms can be inferred from the syllogism *AHH-2* if this similar method is used to continue reasoning. The formal transformation of generalized syllogisms in this paper is in line with the demand for knowledge reasoning in the era of artificial intelligence.

Keywords: generalized syllogisms; validity; knowledge representation; knowledge reasoning

1. Introduction

The basic steps of computer processing natural language are as follows: Firstly, express a sentence of natural language in a formal way; secondly, formalize the expression as an algorithm; finally, program based on the algorithm[1]. In order to adapt to the development of artificial intelligence, modern logic is formalizing various types of information. Formalization is a fundamental feature of modern logic. The mission of studying thinking patterns can be accomplished through the process of formalization and the exploration of formal systems[2].

Generalized syllogisms are common forms of reasoning in natural language([3-4]). Just as generalized quantifiers are extensions of Aristotelian ones (that is, *no*, *all*, *not all*, *some*) ([5]), generalized syllogisms are extensions of Aristotelian ones ([6-8]). This paper only studies generalized syllogisms.

There are a huge number of generalized quantifiers in natural language, and then non-trivial generalized syllogisms studied in the paper only involve the Aristotelian quantifier *all* and the non-trivial generalized quantifier *at most half of the* and their inner, outer, and dual negative quantifiers, that is, *no*, *not all*, *some, at least half of the, fewer than half of the, at most half of the*, and *most,* respectively.

2. Preliminaries

In this following, let *d*, *r*, and *t* be lexical variables, and *U* be their domain. The sets composed of *d*, *r*, and *t* are respectively *D, R*, and *T*. Let α, δ, θ and ψ be well-formed formulas (shortened as wff). Let Q be a generalized quantifier, $\neg Q$ and $Q\neg$ be its outer and inner quantifier, respectively . '$\vdash\delta$' says that the wff δ is provable, and '$=_{def}$' that the left defined by the right. The operators '$\square$' and '$\Diamond$' are respectively necessary modality, and possible one. The other operators (such as $\neg$, $\rightarrow$, $\wedge$, $\leftrightarrow$) are symbols in proposition logic ([9]). If not otherwise specified, the following syllogisms refer to generalized ones.

The generalized syllogisms studied in this paper only include the following 8 types of propositions: *all(d, t)*, *some(d, t), not all(d, t), no(d, t)*, *at most half of the(d, t), most(d, t), at least half of the(d, t), fewer than half of the(d, t),* and they are respectively shortened to: Proposition *A*, *I*, *O*, *E*, *H*, *M*, *S*, and *F.* And non-trivial generalized syllogisms at least contain one of the last four propositions. For instance, the second figure generalized syllogism *all*(*t, r*)$\wedge$*at most half of the*(*d, r*)$\rightarrow$*at most half of the*(*d, t*) is abbreviated as *AHH-2.* An example of this syllogism is as follows:

Major premise: All cats prefer to eat fishes.

Minor premise: At most half of the pets in this farm are cats.

Conclusion: At most half of the pets in this farm prefer to eat fishes.

3. Generalized Syllogism System with the Quantifier '*at most half of the*'

This system includes the following: primitive symbols, formation and deductive rules, and basic axioms, etc.

3.1 Primitive Symbols

(1) lexical variables: *d*, *r*, *t*

(2) quantifiers: *all, at most half of the*

(3) operators: $\neg$, $\rightarrow$

(4) brackets: (,)

3.2 Formation Rules

(1) If Q is a quantifier, d and t are lexical variables, then $Q(d, t)$ is a wff.

(2) If δ is a wff, then so is $\neg\delta$.

(3) If α and δ are wffs, then so is $\alpha\rightarrow\delta$.

(4) Only the formulas formed based on the above rules are wffs.

3.3 Basic Axioms

A1: If δ is a valid formula in propositional logic, then $\vdash\delta$.

A2: $\vdash$*all(t, r)*$\wedge$*at most half of the(d, r)*$\rightarrow$*at most half of the(d, t)* (i.e. the syllogism *AHH-2*).

3.4 Rules of Deduction

Rule 1 (subsequent weakening): From $\vdash(\alpha\wedge\delta\rightarrow\theta)$ and $\vdash(\theta\rightarrow\psi)$ infer $\vdash(\alpha\wedge\delta\rightarrow\psi)$;

Rule 2 (anti-syllogism): From $\vdash(\alpha\wedge\delta\rightarrow\theta)$ infer $\vdash(\neg\theta\wedge\alpha\rightarrow\neg\delta)$;

Rule 3 (anti-syllogism): From $\vdash(\alpha\wedge\delta\rightarrow\theta)$ infer $\vdash(\neg\theta\wedge\delta\rightarrow\neg\alpha)$.

3.5 Relevant Definitions

D1 (conjunction): $(\alpha\wedge\delta)=_{def}\neg(\alpha\rightarrow\neg\delta)$;

D2 (bicondition): $(\alpha\leftrightarrow\delta)=_{def}(\alpha\rightarrow\delta)\wedge(\delta\rightarrow\alpha)$;

D3 (inner negation): $(Q\neg)(d, t)=_{def}Q(d, U-t)$;

D4 (outer negation): $(\neg Q)(d, t)=_{def}$ It is not that Q(d, t);

D5 (truth value): $all(d, t)=_{def}D\subseteq T$;

D6 (truth value): $some(d, t)=_{def}D\cap T\neq\varnothing$;

D8 (truth value): $no(d, t)=_{def}D\cap T=\varnothing$;

D9 (truth value): $not\ all(d, t)=_{def}D\nsubseteq T$;

D10 (truth value): *most*(d, t) is true iff $|D\cap T|\geq 0.6|D|$ is true;

D11 (truth value): *at most half of the*(d, t) is true iff $|D\cap T|<0.4|D|$;

D12 (truth value): *fewer than half of the*(d, t) is true iff $|D\cap T|<0.5|D|$ is true;

D13 (truth value): *at least half of the*(d, t) is true iff $|D\cap T|\geq 0.5|D|$ is true.

3.5 Relevant Facts

Fact 1(inner negation):

(1.1) $all(d, t)=no\neg(d, t)$;

(1.2) *no(d, t)=all¬(d, t)*;

(1.3) *some(d, t)=not all¬(d, t)*;

(1.4) *not all(d, t)=some¬(d, t)*;

(1.5) *most(d, t)=fewer than half of the¬(d, t)*;

(1.6) *fewer than half of the(d, t)=most¬(d, t)*;

(1.7) *at least half of the(d, t)=at most half of the¬(d, t)*;

(1.8) *at most half of the(d, t)=at least half of the¬(d, t)*.

Fact 2 (outer negation):

(2.1) *¬all(d, t)=not all(d, t)*;

(2.2) *¬not all(d, t)=all(d, t)*;

(2.3) *¬no(d, t)=some(d, t)*;

(2.4) *¬some(d, t)=no(d, t)*;

(2.5) *¬most(d, t)=at most half of the(d, t)*;

(2.6) *¬at most half of the(d, t)=most(d, t)*.

(2.7) *¬fewer than half of the(d, t)=at least half of the(d, t)*;

(2.8) *¬at least half of the(d, t)=fewer than half of the(d, t)* ;

Fact 3 (symmetry):

(3.1) *some(d, t)↔some(t*, *d*);

(3.2) *no(d, t)↔no(t*, *d*).

Fact 4 (Subordination) :

(4.1) ⊢*all(d, t)→some(d, t)*;

(4.2) ⊢*no(d, t)→not all(d, t)*;

(4.3) ⊢*all(d, t)→most(d, t)*;

(4.4) ⊢*most(d, t)→some(d, t)*;

(4.5) ⊢*at least half of the(d, t)→some(d, t)*;

(4.6) ⊢*all(d, t)→at least half of the(d, t)*;

(4.7) ⊢*at most half of the(d, t)→not all(d, t)*;

(4.8) ⊢*fewer than half of the(d, t)→not all(d, t)*.

The above facts are elementary knowledge in propositional logic ([8-9]) and generalized quantifier theory ([10]), then their proofs are omitted.

4. Knowledge Reasoning for Generalized Syllogisms

The following Theorem 1 proves the validity of the generalized syllogism *AHH-2*. Theorem 2 shows that other valid generalized syllogisms can be deduced from the syllogism *AHH-2*. In other words, there are reducible relationships between/among valid generalized syllogisms.

Theorem 1 (*AHH-2*)： The generalized syllogism *all*(*t, r*)∧*at most half of the*(*d, r*)→*at most half of the*(*d, t*) is valid.

Proof: Suppose that *all*(*t, r*) and *at most half of the*(*d, r*) are true, then $T\subseteq R$ and $|D\cap R|<0.4|D|$ are true in line with Definition D5 and D11, respectively. Thus, it is easy to see that $|D\cap T|<0.4|D|$ is true. Therefore, *at most half of the(d, t)* is true in the light of Definition D11, as required.

Theorem 2: The following 34 valid generalized syllogisms can be deduced from the syllogism *AHH-2*:

(1) ⊢*AHH-2*→*AHO-2*
(2) ⊢*AHH-2*→*AMM-1*
(3) ⊢*AHH-2*→*AMM-1*→*AMI-1*
(4) ⊢*AHH-2*→*AMM-1*→*AMI-1*→*MAI-4*
(5) ⊢*AHH-2*→*HMO-3*
(6) ⊢*AHH-2*→*HMO-3*→*ESH-2*
(7) ⊢*AHH-2*→*HMO-3*→*ESH-2*→*ESO-2*
(8) ⊢*AHH-2*→*HMO-3*→*ESH-2*→*ESH-1*
(9) ⊢*AHH-2*→*HMO-3*→*ESH-2*→*ESH-1*→*ESO-1*
(10) ⊢*AHH-2*→*AHO-2*→*AAM-1*
(11) ⊢*AHH-2*→*AHO-2*→*MAO-3*
(12) ⊢*AHH-2*→*AMM-1*→*EMF-1*
(13) ⊢*AHH-2*→*AMM-1*→*EMF-1*→*EMF-2*
(14) ⊢*AHH-2*→*AMM-1*→*EMF-1*→*EMO-1*
(15) ⊢*AHH-2*→*AMM-1*→*EMF-1*→*EMI-1*→*EMO-2*
(16) ⊢*AHH-2*→*HMO-3*→*SMI-3*
(17) ⊢*AHH-2*→*HMO-3*→*SMI-3*→*MSI-3*
(18) ⊢*AHH-2*→*HMO-3*→*ESH-2*→*ESO-2*→*EAF-1*
(19) ⊢*AHH-2*→*HMO-3*→*ESH-2*→*ESO-2*→*EAF-1*→*EAF-2*
(20) ⊢*AHH-2*→*HMO-3*→*ESH-2*→*ESO-2*→*SAI-3*
(21) ⊢*AHH-2*→*HMO-3*→*ESH-2*→*ESO-2*→*SAI-3*→*ASI-3*
(22) ⊢*AHH-2*→*AMM-1*→*EMF-1*→*EMO-1*→*AMI-3*

(23) ⊢*AHH-2→AMM-1→EMF-1→EMO-1→AMI-3→MAI-3*

(24) ⊢*AHH-2→AMM-1→EMF-1→EMO-1→EAH-2*

(25) ⊢*AHH-2→AMM-1→EMF-1→EMO-1→EAH-2→EAH-1*

Proof:

[1] ⊢*all(t, r)∧at most half of the(d, r)→at most half of the(d, t)* (i.e. *AHH-2*, Axiom A2)

[2] ⊢*all(t, r)∧at most half of the(d, r)→not all(d, t)* (i.e. *AHO-2*, by [1] and Fact (4.7))

[3] ⊢*¬at most half of the(d, t)∧all(t, r)→¬at most half of the(d, r)* (by [1] and Rule 2)

[4] ⊢*most(d, t)∧all(t, r)→most(d, r)* (i.e. *AMM-1*, by [3] and Fact (2.6))

[5] ⊢*most(d, t)∧all(t, r)→some(d, r)* (i.e. *AMI-1*, by [4] and Fact (4.4))

[6] ⊢*most(d, t)∧all(t, r)→some(r, d)* (i.e. *MAI-4*, by [5] and Fact (3.1))

[7] ⊢*¬at most half of the(d, t)∧at most half of the(d, r)→¬all(t, r)* (by [1] and Rule 3)

[8] ⊢*most(d, t)∧at most half of the(d, r)→not all(t, r)* (i.e. *HMO-3*, by [7], Fact (2.6) and (2.1))

[9] ⊢*no¬(t, r)∧at least half of the¬(d, r)→at most half of the(d, t)* (by [1], Fact (1.1) and (1.8))

[10] ⊢*no(t, U−r)∧at least half of the(d, U−r)→at most half of the(d, t)*
(i.e. *ESH-2*, by [9] and Definition D3)

[11] ⊢*no(t, U−r)∧at least half of the(d, U−r)→not all(d, t)* (i.e. *ESO-2*, by [10] and Fact (4.7))

[12] ⊢*no(U−r, t)∧at least half of the(d, U−r)→at most half of the(d, t)*
(i.e. *ESH-1*, by [10] and Fact (3.2))

[13] ⊢*no(U−r, t)∧at least half of the(d, U−r)→not all(d, t)* (i.e. *ESO-1*, by [12] and Fact (4.7))

[14] ⊢*¬not all(d, t)∧all(t, r)→¬at most half of the(d, r)* (by [2] and Rule 2)

[15] ⊢*all(d, t)∧all(t, r)→most(d, r)* (i.e. *AAM-1*, by [14], Fact (2.2) and (2.6))

[16] ⊢*¬not all(d, t)∧at most half of the(d, r)→¬all(t, r)* (by [2] and Rule 3)

[17] ⊢*all(d, t)∧at most half of the(d, r)→not all(t, r)* (i.e. *MAO-3*, by [16], Fact (2.2) and (2.1))

[18] ⊢*most(d, t)∧no¬(t, r)→fewer than half of the¬(d, r)* (by [4], Fact (1.1) and (1.5))

[19] ⊢*most(d, t)∧no(t, U−r)→fewer than half of the(d, U−r)*
(i.e. *EMF-1*, by [18] and Definition D3)

[20] ⊢*most(d, t)∧no(U−r, t)→fewer than half of the(d, U−r)* (i.e. *EMF-2*, by [19] and Fact (3.2))

[21] ⊢*most(d, t)∧no(t, U−r)→not all(d, U−r)* (i.e. *EMO-1*, by [19] and Fact (4.5))

[22] ⊢*most(d, t)∧no(U−r, t)→not all(d, U−r)* (i.e. *EMO-2*, by [21] and Fact (3.2))

[23] ⊢*most(d, t)∧at least half of the¬(d, r)→some¬(t, r)* (by [8], Fact (1.8) and (1.4))

[24] ⊢*most(d, t)∧at least half of the(d, U−r)→some(t, U−r)*
(i.e. *SMI-3*, by [23], Fact (2.6) and (2.1))

[25] ⊢*most(d, t)∧at least half of the(d, U−r)→some(U−r, t)* (i.e. *MSI-3*, by [24] and Fact (3.1))

[26] ⊢*¬not all(d, t)∧no(t, U−r)→¬at least half of the(d, U−r)* (by [11] and Rule 2)

[27] ⊢*all(d, t)∧no(t, U−r)→fewer than half of the(d, U−r)*
(i.e. *EAF-1*, by [26], Fact (2.2) and (2.8))

[28] ⊢*all(d, t)∧no(U−r, t)→fewer than half of the(d, U−r)* (i.e. *EAF-2*, by [27] and Fact (3.2))

[29] ⊢*¬not all(d, t)∧at least half of the(d, U−r)→¬no(t, U−r)* (by [11] and Rule 3)

[30] ⊢*all(d, t)∧at least half of the(d, U¬r)→some(t, U¬r)* (i.e. *SAI-3*, by [29], Fact (2.2) and (2.3))

[31] ⊢*all(d, t)∧at least half of the(d, U¬r)→some(U¬r, t)* (i.e. *ASI-3*, by [30] and Fact (3.1))

[32] ⊢*¬not all(d, U¬r)∧most(d, t)→¬no(t, U¬r)* (by [21] and Rule 2)

[33] ⊢*all(d, U¬r)∧most(d, t)→some(t, U¬r)* (i.e. *AMI-3*, by [32], Fact (2.2) and (2.3))

[34] ⊢*all(d, U¬r)∧most(d, t)→some(U¬r, t)* (i.e. *MAI-3*, by [32] and Fact 3.2))

[35] ⊢*¬not all(d, U¬r)∧no(t, U¬r)→¬most(d, t)* (by [21] and Rule 3)

[36] ⊢*all(d, U¬r)∧no(t, U¬r)→at most half of the(d, t)* (i.e. *EAH-2*, by [35], Fact (2.2) and (2.5))

[37] ⊢*all(d, U¬r)∧no(U¬r, t)→at most half of the(d, t)* (i.e. *EAH-1*, by [36] and Fact (3.2))

Theorem 2 shows that the above 25 valid generalized syllogisms can be inferred from the valid generalized syllogism *AHH-2*.

5. Conclusion and Future Work

Making most of relevant definitions, reasoning rules, and facts in generalized quantifier theory and propositional logic, this paper firstly proves the validity of the generalized syllogism *AHH-2* in Theorem 1, and then derives the other 25 valid generalized ones in Theorem 2. The reason why these results are consistent is that deductive reasoning is used throughout the proof process. In fact, more valid generalized syllogisms can be inferred from the syllogism *AHH-2* if one continues to derive by means of the above methods. Similar to Theorem 1, the validity of these syllogisms can be proven by means of the above definitions and facts.

This formal method not only provides a theoretical basis for knowledge representation and reasoning in artificial intelligence, but also a unified mathematical paradigm for other types of syllogisms, such as generalized modal syllogisms, syllogisms with verbs. We will provide specific details in other papers.

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References

[1] Y. Fan, Artificial Intelligence and ChatGPT, Tsinghua University Press, Beijing, 2023.

[2] Z. X. Cai, and G. Y. Xu Guangyou. Artificial Intelligence and Its Applications (4th Edition) Beijing: Tsinghua University Press, 2010.

[3] L. S. Moss, Syllogistic logics with verbs. Journal of Logic and Computation, Vol 20. No. 4, 2010, pp. 947-967.

[4] J. Endrullis, and L. S. Moss, Syllogistic logic with 'most'. In: V. de Paiva et al. (eds.), Logic,

Language, Information, and Computation, 2015, pp.124-139.

[5] E. L. Keenan, and D. Westerståhl, Generalized quantifiers in linguistics and logic. In: J. van Benthem and A. ter Meulen (eds.), Handbook of Logic and Language, Vol. 2, 2011, pp.859-910.

[6] J. Łukasiewicz, Aristotle's Syllogistic: From the Standpoint of Modern Formal Logic (2nd edition), Clerndon Press, Oxford, 1957.

[7] D. Westerståhl, Quantifiers in formal and natural languages. In: D. M. Gabbay and F. Guenthner (eds.), Handbook of Philosophical Logic, Vol.14, 2007, pp.227-242.

[8] C. Zhang,The remaining 23 valid Aristotelian syllogisms can be deduced only from the syllogism IAI-3. SCIREA Journal of Computer, Vol 7. No. 5, 2022, pp.85-95.

[9] A. G. Hamilton, Logic for Mathematicians, Cambridge University Press, Cambridge, 1978.

[10] S. Peters, and D. Westerståhl, Quantifiers in Language and Logic, Claredon Press, Oxford, 2006.