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Knowledge Mining about the Generalized Modal Syllogism E□M◇F-2 with the Quantifiers in Square{*fewer than half of the*} and Square{*no*}

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Abstract:

On the basis of set theory, generalized quantifier theory, and modal logic, this paper mainly focuses on the knowledge mining about generalized modal syllogism with the quantifiers in Square{*fewer than half of the*} and Square{*no*}. To this end, this paper firstly proves the validity of the non-trivial syllogism E□M◇F-2, and deduces other 22 valid non-trivial generalized modal syllogisms based on relative reduction operations. The reason why syllogisms with different figures and forms can be mutually reduced is that any quantifier in Square{*fewer than half of the*} and Square{*no*} can define the other three quantifiers, and the necessary and possible modality are mutually dual. Since all the proofs in this article are deductive reasoning, their conclusions are consistent.

Keywords: knowledge mining; Square{*fewer than half of the*}; Square{*no*}; generalized modal syllogisms

1. Introduction

Syllogistic reasoning is a widespread and significant form of reasoning in human daily thinking and scientific reasoning (Łukasiewicz, 1957; Westerståhl, 1989). There are various types of syllogisms, such as Aristotelian syllogisms (Hao, 2023), generalized syllogisms (Murinová and Novák, 2012), Aristotelian modal syllogisms (Johnson, 2004; Zhang, 2018), generalized modal syllogisms (Hao, 2024a), and so on. So far, there are few works of generalized modal syllogisms, and this paper focuses on them.

Generalized modal syllogism includes both generalized quantifiers and modalities. There are two kinds of quantifiers in natural language, that is, Aristotelian quantifiers (i.e. *all, some, no, and not all*) and generalized quantifiers (such as, *most*, *both*, *fewer than half of the).* The former is a special case of the latter, while the latter is an extension of the former. In other words, Aristotelian quantifiers are trivial generalized quantifiers. One can obtain a generalized modal syllogism by adding at least one and at most three non-overlapping modalities (that is, necessary modality □ or possible modality ◇) to a generalized syllogism (Hao, 2024b).

2. Knowledge Representation of Generalized Modal Syllogisms

In this paper, *b, t*, and *z* denote lexical variables, and *D* represents the domain. The sets composed of *b*, *t*, and *z* are denoted as *B*, *T,* and *Z*, respectively. Let θ、γ、δ and ϕ be well-formed formulas (shorted as wff). '$|B\cap Z|$'represents the cardinality for the intersection of the set *B* and *Z.* '$\vdash\theta$' says that the formula is provable, and '$\gamma =_{def} \phi$' states that γ can be defined by ϕ. Others are similar. The operators in the paper such as, ¬, →, ∧, ↔ are symbols in modal logic (Chagrov and Zakharyaschev, 1997) and set theory (Halmos).

The generalized modal syllogisms studied in this paper involves the following 24 propositions: (1) *all(b, z), some(b, z), no(b, z), not all(b, z), fewer than half of the(b, z), at least half of the(b, z)*, *most(b, z)*, *at most half of the most(b, z),* which are respectively abbreviated as Proposition *A, I, E, O, F, S*, *M,* and *H.* (2) □*all(b, z),* □*some(b, z),* □*no(b, z),* □*not all(b, z),* □*fewer than half of the(b, z),* □*at least half of the(b, z)*, □*most(b, z)*, □*at most half of the most(b, z),* which are respectively abbreviated as Proposition □*A,* □*I,* □*b,* □*O,* □*F,* □*S*, □*M,* and □*H*; (3)◇*all(b, z),* ◇*some(b, z),* ◇*no(b, z),* ◇*not all(b, z),* ◇*fewer than half of the(b, z),* ◇*at least half of the(b, z)*, ◇*most(b, z)*, ◇*at most half of the most(b, z),* which are respectively abbreviated as Proposition ◇*A,* ◇*I,* ◇*E,* ◇*O,* ◇*F,* ◇*S*, ◇*M,* and ◇*H.*

A non-trivial generalized modal syllogism contains at least one non-trivial generalized quantifier and one modality. This paper mainly studies knowledge mining about the generalized modal syllogism E□M◇F-2 with the quantifiers in Square{*fewer than half of the*} and Square {*no*}. An instance of the syllogism E□M◇F-2 in natural language is as follows:

Major premise: No cat is a dog.

Minor premise: Most pet animals are necessarily dogs.

Conclusion: Fewer than half of pet animals are possibly cats.

Let *z* be a lexical variable that stands for cats in the domain, *t* be a lexical variable that denotes dogs in the domain, and *b* be a lexical variable that represents pet animals in the domain. Then this syllogism can be formalized as '*no(z, t)*∧□*most(b, t)*→◇*fewer than half of the(b, z)*', which is abbreviated as E□M◇F-2. Others are similar.

3. Generalized Modal Syllogism System

For any quantifier *Q*, there are three kinds of negative quantifiers, that is, outer negation ¬*Q*, inner negation *Q*¬, and dual negation ¬*Q*¬. Any quantifier *Q* and its three negative ones can form a Square{*Q*}={*Q*, ¬*Q*, *Q*¬, ¬*Q*¬}. In other words, any quantifier in Square{*Q*} can define the other three quantifiers. For example, Square{*fewer than half of the*}={*fewer than half of the, at least half of the*, *most*, *at most half of the*}, and Square{*no*}={*no*, *some*, *all*, *not all*}. The generalized modal syllogisms studied in this paper only involves 8 quantifiers in Square{*fewer than half of the*} and Square{*no*}. The definable relationship between these quantifiers can be found in the following Fact 1 and Fact 2.

3.1 Primitive Symbols

(1) lexical variables: *b*, *t*, z

(2) quantifier: *no*

(3) quantifier:*fewer than half of the*

(4) modality: □

(5) unary negative operator: ¬

(6) binary implication operator: →

(7) brackets: (,)

3.2 Formation Rules

(1) If Q is a quantifier, b and z are lexical variables, then $Q(b, z)$ is a wff.

(2) If ϕ is a wff, then so are $\neg\phi$ and $\Box\phi$.

(3) If θ and δ are wffs, then so is $\theta\rightarrow\delta$.

(4) Only the formulas obtained by the above rules are wffs.

3.3 Basic Axioms

A1: If θ is a valid formula in first-order logic, then $\vdash\theta$.

A2: $\vdash no(z, t)\wedge\Box most(b, t)\rightarrow\Diamond fewer\ than\ half\ of\ the(b, z)$ (that is, the syllogism E□M◇F-2).

3.4 Rules of Deduction

Rule 1 (subsequent weakening): $\vdash(\theta\wedge\delta\rightarrow\psi)$ can be inferred from $\vdash(\theta\wedge\delta\rightarrow\phi)$ and $\vdash(\phi\rightarrow\psi)$.

Rule 2 (anti-syllogism): $\vdash(\neg\phi\wedge\theta\rightarrow\neg\delta)$ can be inferred from $\vdash(\theta\wedge\delta\rightarrow\phi)$.

Rule 3 (anti-syllogism): $\vdash(\neg\phi\wedge\delta\rightarrow\neg\theta)$ can be inferred from $\vdash(\theta\wedge\delta\rightarrow\phi)$.

3.5 Relevant Definitions

D1: $(\theta\wedge\delta)=_{def}\neg(\theta\rightarrow\neg\delta)$;

D2: $(\theta\leftrightarrow\delta) =_{def} (\theta\rightarrow\delta)\wedge(\delta\rightarrow\theta)$;

D3: $(Q\neg)(b, z)=_{def} Q(b, D-z)$;

D4: $(\neg Q)(b, z)=_{def}$ It is not that $Q(b, z)$;

D5: $\Diamond Q(b, z)=_{def} \neg\Box\neg Q(b, z)$;

D6: $no(b, z)$ is true when and only when $B\cap Z=\varnothing$ is true in any real world;

D7: $\Box most(b, z)$ is true when and only when $|B\cap Z|>0.5|B|$ is true in any possible world;

D8: $\Diamond fewer\ than\ half\ of\ the(b, z)$ is true when and only when $|B\cap Z|<0.5|B|$ is true in at least one possible world.

3.6 Relevant Facts

Fact 1 (Inner Negation):

(1.1) $all(b, z) = no\neg(b, z)$; (1.2) $no(b, z) = all\neg(b, z)$;

(1.3) $some(b, z) = not\ all\neg(b, z)$; (1.4) $not\ all(b, z) = some\neg(b, z)$;

(1.5) $most(b, z) = fewer\ than\ half\ of\ the\neg(b, z)$; (1.6) $fewer\ than\ half\ of\ the(b, z) = most\neg(b, z)$;

(1.7) $at\ least\ half\ of\ the(b, z) = at\ most\ half\ of\ the\neg(b, z)$;

(1.8) $at\ most\ half\ of\ the(b, z) = at\ least\ half\ of\ the\neg(b, z)$.

Fact 2 (Outer Negation):

(2.1) $\neg all(b, z) = not\ all(b, z)$; (2.2) $\neg not\ all(b, z) = all(b, z)$;

(2.3) $\neg no(b, z) = some(b, z)$; (2.4) $\neg some(b, z) = no(b, z)$;

(2.5) $\neg most(b, z) = at\ most\ half\ of\ the(b, z)$; (2.6) $\neg at\ most\ half\ of\ the(b, z) = most(b, z)$;

(2.7) $\neg fewer\ than\ half\ of\ the(b, z) = at\ least\ half\ of\ the(b, z)$;

(2.8) $\neg at\ least\ half\ of\ the(b, z) = fewer\ than\ half\ of\ the(b, z)$;

Fact 3 (Symmetry):

(3.1) $some(b, z) \leftrightarrow some(z, b)$; (3.2) $no(b, z) \leftrightarrow no(z, b)$.

Fact 4 (Dual):

(4.1) $\neg\Box Q(b, z) = \Diamond\neg Q(b, z)$; (4.2) $\neg\Diamond Q(b, z) = \Box\neg Q(b, z)$.

Fact 5: $\vdash\Box Q(b, z) \rightarrow Q(b, z)$.

Fact 6: $\vdash\Box Q(b, z) \rightarrow \Diamond Q(b, z)$.

Fact 7: $\vdash Q(b, z) \rightarrow \Diamond Q(b, z)$.

Fact 8 (Subordination):

(8.1) $\vdash all(b, z) \rightarrow some(b, z)$; (8.2) $\vdash no(b, z) \rightarrow not\ all(b, z)$;

(8.3) $\vdash all(b, z) \rightarrow most(b, z)$; (8.4) $\vdash most(b, z) \rightarrow some(b, z)$;

(8.5) $\vdash at\ least\ half\ of\ the(b, z) \rightarrow some(b, z)$; (8.6) $\vdash all(b, z) \rightarrow at\ least\ half\ of\ the(b, z)$;

(8.7) $\vdash at\ most\ half\ of\ the(b, z) \rightarrow not\ all(b, z)$; (8.8) $\vdash fewer\ than\ half\ of\ the(b, z) \rightarrow not\ all(b, z)$.

The above facts are the fundamental knowledge of generalized quantifier theory (Peters and Westerståhl, 2006) and modal logic, and their proofs are omitted.

4. Knowledge Reasoning about the Generalized Modal Syllogism E□M◇F-2

The following Theorem 1 states that the syllogism E□M◇F-2 is valid. In the following Theorem 2, E□M◇F-2→E□M◇F-1 means that the validity of the syllogism E□M◇F-1 can be inferred from that of E□M◇F-2. One can say that there are reducible relations between the two syllogisms. The others are similar.

Theorem 1 (E□M◇F-2): The generalized modal syllogism *no(z, t)*∧□*most(b, t)*→◇*fewer than half of the(b, z)* is valid.

Proof: According to Example 1, E□M◇F-2 is the abbreviation of the second figure syllogism *no(z, t)*∧□*most(b, t)*→◇*fewer than half of the(b, z)*. Suppose that *no(z, t)* and □*most(b, t)* are true, then $Z \cap T = \varnothing$ is true in any real world and $|B \cap T| > 0.5|B|$ is true in any possible world in line with Definition D6 and D7, respectively. Because all real worlds are possible worlds. Now it follows that $|B \cap Z| < 0.5|B|$ is true in at least one possible world. Thus, ◇*fewer than half of the(b, z)* is true according to Definition D8. This proves that the syllogism *no(z, t)*∧□*most(b, t)*→◇*fewer than half of the(b, z)* is valid.

Theorem 2: There are at least the following 22 valid syllogisms can be deduced from the syllogism E□M◇F-2:

(2.1) ⊢E□M◇F-2→E□M◇F-1

(2.2) ⊢E□M◇F-2→E□M◇O-2

(2.3) ⊢E□M◇F-2→E□M◇F-1→E□M◇O-1

(2.4) ⊢E□M◇F-2→E□M□O-2→E□A◇H-1

(2.5) ⊢E□M◇F-2→E□M□O-2→E◇A◇H-1→E□A◇H-2

(2.6) ⊢E□M◇F-2→E□M◇O-2→□M□AI-3

(2.7) ⊢E□M◇F-2→E□M◇O-2→□M□AI-3→ □A□MI-3

(2.8) ⊢E□M◇F-2→E□M◇O-2→A□F◇O-2

(2.9) ⊢E□M◇F-2→E□M□O-2→E□A◇H-1→A□A◇S-1

(2.10) ⊢E□M◇F-2→E□M□O-2→E□A◇H-1→A□A◇S-1→□F□AO-3

(2.11) ⊢E□M◇F-2→A□F◇F-2

(2.12) ⊢E□M◇F-2→E□M□O-2→E◇A◇H-1→E□A◇H-2→A□E◇H-2

(2.13) ⊢E□M◇F-2→E□M□O-2→E◇A◇H-1→E□A◇H-2→A□E◇H-2→A□E◇H-4

(2.14) ⊢E□M◇F-2→E□M◇O-2→□M□AI-3→□M□A◇I-3

(2.15) $\vdash$E□M◇F-2→E□M◇O-2→□M□AI-3→□M□A◇I-3→□A□M◇I-3

(2.16) $\vdash$E□M◇F-2→E□M□O-2→E□A◇H-1→E□A◇O-1

(2.17) $\vdash$E□M◇F-2→E□M□O-2→E□A◇H-1→E□A◇O-1→E□A◇O-2

(2.18) $\vdash$E□M◇F-2→E□M□O-2→E□A◇H-1→A□A◇S-1→A□A◇I-1

(2.19) $\vdash$E□M◇F-2→E□M□O-2→E◇A◇H-1→E□A◇H-2→A□E◇H-2→A□E◇O-2

(2.20) $\vdash$E□M◇F-2→E□M□O-2→E◇A◇H-1→E□A◇H-2→A□E◇H-2→A□E◇O-2 →A□E◇O-4

(2.21) $\vdash$E□M◇F-2→A□F◇F-2→A□F◇O-2

(2.22) $\vdash$E□M◇F-2→E□M□O-2→E□A◇H-1→A□A◇S-1→□F□AO-3→□F□A◇O-3

Proof:

[1] $\vdash$*no(z, t)*∧□*most(b, t)*→◇*fewer than half of the(b, z)* (i.e. E□M◇F-2, basic axiom)

[2] $\vdash$*no(t, z)*∧□*most(b, t)*→◇*fewer than half of the(b, z)* (i.e. E□M◇F-1, by [1] and Fact 3)

[3] $\vdash$*no(z, t)*∧□*most(b, t)*→◇*not all(b, z)* (i.e. E□M◇O-2, by [1] and Fact 8)

[4] $\vdash$*no(t, z)*∧□*most(b, t)*→◇*not all(b, z)* (i.e. E□M◇O-1, by [2] and Fact 8)

[5] $\vdash$□¬*not all(b, z)*∧*no(z, t)*→◇¬*most(b, t)* (by [3], Rule 2 and Fact 4)

[6] $\vdash$□*all(b, z)*∧*no(z, t)*→◇*at most half of the(b, t)* (i.e. E□A◇H-1, by [5] and Fact 2)

[7] $\vdash$□*all(b, z)*∧*no(t, z)*→◇*at most half of the(b, t)* (i.e. E□A◇H-2, by [6] and Fact 3)

[8] $\vdash$□¬*not all(b, z)*∧□*most(b, t)*→¬*no(z, t)* (by [3], Rule 3 and Fact 4)

[9] $\vdash$□*all(b, z)*∧□*most(b, t)*→some*(z, t)* (i.e. □M□AI-3, by [8] and Fact 2)

[10] $\vdash$□*all(b, z)*∧□*most(b, t)*→some*(t, z)* (i.e. □A□MI-3, by [9] and Fact 3)

[11] $\vdash$*all¬(z, t)*∧□*fewer than half of the¬(b, t)*→◇*not all(b, z)* (by [3] and Fact 1)

[12] $\vdash$*all(z, D−t)*∧□*fewer than half of the(b, D−t)*→◇*not all(b, z)*

(i.e. A□F◇O-2, by [11] and D3)

[13] $\vdash$□*all(b, z)*∧*all¬(z, t)*→◇*at least half of the¬(b, t)* (by [6] and Fact 1)

[14] $\vdash$□*all(b, z)*∧*all(z, D−t)*→◇*at least half of the(b, D−t)*

(i.e. A□A◇S-1, by [13] and D3)

[15] $\vdash$□¬*at least half of the(b, D−t)*∧□*all(b, z)*→¬*all(z, D−t)* (by [14], Rule 2 and Fact 4)

[16] $\vdash$□*fewer than half of the(b, D−t)*∧□*all(b, z)*→*not all(z, D−t)*

(i.e. □F□AO-3, by [15] and Fact 2)

[17] $\vdash$*all¬(z, t)*∧□*fewer than half of the¬(b, t)*→◇*fewer than half of the(b, z)* (by[1] and Fact 1)

[18] $\vdash$*all(z, D−t)*∧□*fewer than half of the(b, D−t)*→◇*fewer than half of the(b, z)*

(i.e. A□F◇F-2, by [17] and D3)

[19] $\vdash$□*no¬(b, z)*∧*all¬(t, z)*→◇*at most half of the(b, t)* (by [7] and Fact 1)

[20] $\vdash$□*no(b, D−z)*∧*all(t, D−z)*→◇*at most half of the(b, t)* (i.e. A□E◇H-2, by [19] and D3)

[21] $\vdash$□*no(D−z, b)*∧*all(t, D−z)*→◇*at most half of the(b, t)* (i.e. A□E◇H-4, by [20] and Fact 3)

[22] $\vdash$some$(z, t)\rightarrow\Diamond$some$(z, t)$ (by Fact 6)

[23] $\vdash\Box all(b, z)\wedge\Box most(b, t)\rightarrow\Diamond$some$(z, t)$ (i.e. □M□A◇I-3, by [9], [22]and Rule 1)

[24] $\vdash\Box all(b, z)\wedge\Box most(b, t)\rightarrow\Diamond$some$(t, z)$ (i.e. □A□M◇I-3, by [23] and Fact 3)

[25] $\vdash\Diamond$*at most half of the*$(b, t)\rightarrow\Diamond$*not all*$(b, t)$ (by Fact 8)

[26] $\vdash\Box all(b, z)\wedge no(z, t)\rightarrow\Diamond$*not all*$(b, t)$ (i.e. E□A◇O-1, by [6], [25] and Rule 1)

[27] $\vdash\Box all(b, z)\wedge no(t, z)\rightarrow\Diamond$*not all*$(b, t)$ (i.e. E□A◇O-2, by [26] and Fact 3)

[28] $\vdash\Diamond$*at least half of the*$(b, D-t)\rightarrow\Diamond some(b, D-t)$ (by Fact 8)

[29] $\vdash\Box all(b, z)\wedge all(z, D-t)\rightarrow\Diamond some(b, D-t)$ (i.e. A□A◇I-1, by [14], [28] and Rule 1)

[30] $\vdash\Box no(b, D-z)\wedge all(t, D-z)\rightarrow\Diamond$*not all*$(b, t)$ (i.e. A□E◇O-2, by [20], [25] and Rule 1)

[31] $\vdash\Box no(D-z, b)\wedge all(t, D-z)\rightarrow\Diamond$*not all*$(b, t)$ (i.e. A□E◇O-4, by [30] and Fact 3)

[32] $\vdash\Diamond$*fewer than half of the*$(b, z)\rightarrow\Diamond$*not all*$(b, z)$ (by Fact 8)

[33] $\vdash all(z, D-t)\wedge\Box$*fewer than half of the*$(b, D-t)\rightarrow\Diamond$*not all*$(b, z)$

(i.e. A□F◇O-2, by [18], [32] and Rule 1)

[34] $\vdash$*not all*$(z, D-t)\rightarrow\Diamond$*not all*$(z, D-t)$ (by Fact 6)

[35] $\vdash\Box$*fewer than half of the*$(b, D-t)\wedge\Box all(b, z)\rightarrow\Diamond$*not all*$(z, D-t)$

(i.e. □F□A◇O-3, by [16], [34] and Rule 1)

Now, the above 22 generalized modal syllogisms have been derived from the valid syllogism E□M◇F-2. It shows that there are reducible relations between/among valid generalized modal syllogisms of different figures and forms.

4. Conclusion

On the basis of set theory, generalized quantifier theory, and modal logic, this paper mainly focuses on the knowledge mining about generalized modal syllogisms with the quantifiers in Square{*fewer than half of the*} and Square{*no*}. For this purpose, this paper firstly proves the validity of the non-trivial syllogism E□M◇F-2 in Theorem 1, and then deduces other 22 valid non-trivial generalized modal syllogisms based on relative reduction operations (such as facts and definitions) in Theorem 2. The reason why syllogisms with different figures and forms can be mutually reduced is that any quantifier in Square{*fewer than half of the*} and Square{*no*} can define the other three quantifiers, and the necessary and possible modality are mutually dual. Since all the proofs in this article are deductive reasoning, their conclusions are consistent.

Can this research method provide a unified research paradigm for knowledge mining of other generalized modal syllogisms involving other quantifiers (such as *at most 1/3 of the*, *fewer than 3/5 of the*) ? Is the generalized syllogism fragment system studied in this paper sound and complete? These problems require further discussion.

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