SCIREA Journal of Computer
ISSN: 2995-6927
http://www.scirea.org/journal/Computer
October 30, 2024
Volume 9, Issue 4, August 2024
https://doi.org/10.54647/computer520426

Deductive Reasoning Based on the Aristotelian Modal Syllogism □AE◇O-2

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Abstract:

This paper first symbolizes the propositions involved in Aristotelian modal syllogisms from the perspective of mathematical structuralism, then proves the validity of the Aristotelian modal syllogism □AE◇O-2 by relevant definitions, and finally deduces the other 26 valid Aristotelian modal syllogisms from the syllogism □AE◇O-2 in line with some reasoning rules and definitions. This indicates that there are reducible relations between/among different syllogisms. This study contributes to the advancement of knowledge representation and reasoning in natural language.

Keywords:

Aristotelian modal syllogisms; syllogistic reasoning; reduction; validity

1. Introduction

As one of the most important ways of reasoning, syllogistic reasoning plays a significant role in social development and daily communication, which contains Aristotelian syllogisms (Łukasiewicz, 1957; Cao and Xu, 2024), generalized ones (Endrullis and Moss, 2015; Qiu and Ma, 2024), Aristotelian modal ones

(Brennan,1997; Johnson, 2004; Hao, 2024) and generalized modal ones (Xu and Zhang, 2023; Wang and Yuan, 2024). This paper mainly studies Aristotelian modal syllogisms, which have been investigated by Malink (2006), Wei and Zhang (2023), and so on.

Inspired by previous researches, this paper aims to study the reducibility of the Aristotelian modal syllogism □AE◇O-2 from the perspective of mathematical structuralism. To achieve this goal, the validity of the syllogism □AE◇O-2 needs to be proved in the first step. And the other 26 valid Aristotelian modal syllogisms can be derived from □AE◇O-2 by means of classical logic (Hamilton, 1978), modal logic (Chellas, 1980), and generalized quantifier theory (Peters and Westerståhl, 2006).

2. Symbolization of Aristotelian Modal Syllogisms

In the following, let *c*, *h*, and *y* be a lexical variable which is respectively an element from the set *C*, *H*, and *Y*. And *D* is a domain of these variables. Let κ, λ, μ, and ν be propositions, and '⊢κ' means that κ can be proved. 'if and only if' is shortened as 'iff'. And '*Q*' represents any one of Aristotelian quantifiers in Square{*no*}={*no*, *not all*, *all*, *some*}. The outer negation and inner negation of a quantifier *Q* are respectively denoted as ¬Q and Q¬.

Aristotelian syllogisms involve the following 4 propositions, that is, 'no *c*s are *h*s', 'not all *c*s are *h*s', 'all *c*s are *h*s', and 'some *c*s are *h*s' , which are respectively denoted as *no*(*c, h*), *not all*(*c, h*), *all*(*c, h*) and *some*(*c, h*) from the perspective of mathematical structuralism. An Aristotelian modal syllogism can be obtained by adding at least one and at most three non-overlapping modalities (i.e, necessary modality □ or possible modality ◇) to an Aristotelian syllogism (Weiand Zhang, 2023). Therefore, from the perspective of mathematical structuralism, Aristotelian modal syllogisms involve the following 12 propositions: *no*(*c, h*), *not all*(*c, h*), *all*(*c, h*), *some*(*c, h*), □*no*(*c, h*), □*not all*(*c, h*), □*all*(*c, h*), □*some*(*c, h*); ◇*no*(*c, h*), ◇*not all*(*c, h*), ◇*all*(*c, h*) and ◇*some*(*c, h*), and which are respectively referred to as Proposition E, O, A, I, □E, □O, □A, □I, ◇E, ◇O, ◇A, and ◇I. The definitions of figures of Aristotelian modal syllogisms are similar to those of Aristotelian syllogisms (He, 2018).

For example, the following Aristotelian modal syllogism □AE◇O-2 can be obtained by adding a non-overlapping necessary modality □ and a possible modality ◇ to the Aristotelian syllogism AEO-2. An instance of the syllogism in natural language is as follows:

Major premise: All dogs are necessarily animals.

Minor premise: No apples are animals.

Conclusion: Not all apples are possibly dogs.

Let *c*, *h*, *and y* be a variable representing a dog, an animal, and an apple, respectively. The above syllogism can be expressed as □*all*(*c, h*)→(*no*(*y*, *h*)→◇*not all*(*y*, *c*)), which can be abbreviated as □AE◇O-2. The other Aristotelian modal syllogisms can be similarly expressed.

3. Deductive Basis for Aristotelian Modal Syllogisms

On the basis of the syllogism $\Box$AE$\Diamond$O-2, this paper firstly proves its validity according to the following definitions, and then derives the other valid Aristotelian modal syllogisms from it in line with the following definitions, facts and rules which come from modal logic and generalized quantifier theory (Wei and Zhang, 2023).

Definition 1 (truth value):

(1.1) $\Box all(c, h)$ is true iff $C\subseteq H$ is true in all possible worlds;

(1.2) $no(c, h)$ is true iff $C\cap H=\varnothing$ is true in all real worlds;

(1.3) $\Diamond not\ all(c, h)$ is true iff $C\not\subseteq H$ is true in some possible world;

Definition 2 (inner negation): $Q\neg(c, h) =_{def} Q(c, D-h)$.

Definition 3 (outer negation): $\neg Q(c, h) =_{def}$ It is not that $Q(c, h)$.

Fact 1 (outer and inner negation):

(1.1) $\vdash\neg no(c, h)\leftrightarrow some(c, h)$;

(1.2) $\vdash no\neg(c, h)\leftrightarrow all(c, h)$;

(1.3) $\vdash\neg not\ all(c, h)\leftrightarrow all(c, h)$;

(1.4) $\vdash not\ all\neg(c, h)\leftrightarrow some(c, h)$;

(1.5) $\vdash\neg all(c, h)\leftrightarrow not\ all(c, h)$;

(1.6) $\vdash all\neg(c, h)\leftrightarrow no(c, h)$;

(1.7) $\vdash\neg some(c, h)\leftrightarrow no(c, h)$;

(1.8) $\vdash some\neg(c, h)\leftrightarrow not\ all(c, h)$.

Fact 2 (symmetry and dual):

(2.1) $\vdash some(c, h)\leftrightarrow some(h, c)$;

(2.2) $\vdash no(c, h)\leftrightarrow no(h, c)$;

(2.3) $\vdash\neg\Diamond Q(c, h)\leftrightarrow\Box\neg Q(c, h)$;

(2.4) $\vdash\neg\Box \mathrm{Q}(c, h)\leftrightarrow\Diamond\neg Q(c, h)$.

Fact 3 (subalternation):

(3.1) $\vdash all(c, h)\rightarrow some(c, h)$;

(3.2) $\vdash no(c, h)\rightarrow not\ all(c, h)$;

(3.3) $\vdash\Box Q(c, h)\rightarrow Q(c, h)$;

(3.4) $\vdash\Box Q(c, h)\rightarrow\Diamond Q(c, h)$;

(3.5) $\vdash Q(c, h)\rightarrow\Diamond Q(c, h)$.

Rule 1 (antecedent strengthening): If $\vdash(\kappa\rightarrow(\lambda\rightarrow\mu))$ and $\vdash(\nu\rightarrow\kappa)$, then $\vdash(\nu\rightarrow(\lambda\rightarrow\mu))$

or If $\vdash(\kappa\rightarrow(\lambda\rightarrow\mu))$ and $\vdash(\nu\rightarrow\lambda)$, then $\vdash(\kappa\rightarrow(\nu\rightarrow\mu))$.

Rule 2 (subsequent weakening): If $\vdash(\kappa\rightarrow(\lambda\rightarrow\mu))$ and $\vdash(\mu\rightarrow v)$, then $\vdash(\kappa\rightarrow(\lambda\rightarrow v))$.

Rule 3 (anti-syllogism): If $\vdash(\kappa\rightarrow(\lambda\rightarrow\mu))$, then $\vdash(\neg u\rightarrow(\lambda\rightarrow\neg\kappa))$ or $\vdash(\kappa\rightarrow(\neg u\rightarrow\neg\lambda))$.

3. Reducible Relations between/among Valid Modal Syllogisms

The following Theorem 1 proves the validity of the modal syllogism □AE◇O-2. Then the other 26 valid Aristotelian modal syllogisms are deduced from □AE◇O-2 in Theorem 2. That is to say that there are reducible relations between/among these 27 syllogisms.

Theorem 1 (□AE◇O-2): The Aristotelian modal syllogism □*all*(*c, h*)→(*no*(*y, h*)→◇*not all*(*y, c*)) is valid.

Proof: The expression '□AE◇O-2' is an abbreviation for the syllogism □*all*(*c, h*)→(*no*(*y, h*)→◇*not all*(*y, c*)). Suppose that □*all*(*c, h*) and *no*(*y, h*) are true, then $C\subseteq H$ is true in all possible worlds by Definition (1.1), and $Y\cap H=\varnothing$ is true in all real worlds by Definition (1.2). It is the fact that a real world is also a possible world, therefore, $Y\cap C=\varnothing$ is true in some possible world, i.e. ◇*no*(*y, c*) is true by Definition (1.3). It can be concluded that ◇*not all*(*y, c*) is true by Fact (3.2), just as desired.

Theorem 2: The following 26 valid Aristotelian modal syllogisms can be derived from □AE◇O-2:

(2.1) ⊢□AE◇O-2→□AE◇O-4

(2.2) ⊢□AE◇O-2→□AE◇O-4→□A□E◇O-4

(2.3) ⊢□AE◇O-2→□AE◇O-4→□A□E◇O-4→□E□A◇O-4

(2.4) ⊢□AE◇O-2→□AE◇O-4→□A□E◇O-4→□A□A◇I-4

(2.5) ⊢□AE◇O-2→□AE◇O-4→E□A◇O-4

(2.6) ⊢□AE◇O-2→□AE◇O-4→□A□AI-4

(2.7) ⊢□AE◇O-2→E□A◇O-3

(2.8) ⊢□AE◇O-2→□A□AI-1

(2.9) ⊢□AE◇O-2→□A□E◇O-2

(2.10) ⊢□AE◇O-2→□A□E◇O-2→□E□A◇O-2

(2.11) ⊢□AE◇O-2→□A□E◇O-2→□E□A◇O-2→□E□A◇O-1

(2.12) ⊢□AE◇O-2→□A□E◇O-2→□E□A◇O-3

(2.13) ⊢□AE◇O-2→□A□E◇O-2→□A□A◇I-1

(2.14) ⊢□AE◇O-2→□EA◇O-2

(2.15) ⊢□AE◇O-2→□EA◇O-2→□E□AO-1

(2.16) ⊢□AE◇O-2→□EA◇O-2→A□A◇I-3

(2.17) ⊢□AE◇O-2→□EA◇O-2→□A□A◇I-3

(2.18) ⊢□AE◇O-2→□EA◇O-2→□EA◇O-1

(2.19) ⊢□AE◇O-2→□EA◇O-2→□EA◇O-1→□E□AO-2

(2.20) ⊢□AE◇O-2→□EA◇O-2→A□A◇I-3→□AA◇I-3

(2.21) ⊢□AE◇O-2→□EA◇O-2→A□A◇I-3→□AA◇I-3→□EA◇O-3

(2.22) ⊢□AE◇O-2→□EA◇O-2→A□A◇I-3→□AA◇I-3→□EA◇O-3→□EA◇O-4

(2.23) ⊢□AE◇O-2→□EA◇O-2→A□A◇I-3→□AA◇I-3→□EA◇O-3→□AA◇I-1

(2.24) ⊢□AE◇O-2→□EA◇O-2→A□A◇I-3→□AA◇I-3→□EA◇O-3→□AA◇I-1

→□A□EO-2

(2.25) ⊢□AE◇O-2→□EA◇O-2→A□A◇I-3→□AA◇I-3→□EA◇O-3→□AA◇I-1

→□A□EO-2→□A□EO-4

(2.26) ⊢□AE◇O-2→□EA◇O-2→A□A◇I-3→□AA◇I-3→□EA◇O-3→□AA◇I-1

→□A□EO-2→□A□EO-4→A□A◇I-4

Proof:

[1] ⊢□*all*(*c*, *h*)→(*no*(*y*, *h*)→◇*not all*(*y*, *c*)) (i.e. □AE◇O-2, Theorem 1)

[2] ⊢□*all*(*c*, *h*)→(*no*(*h*, *y*)→◇*not all*(*y*, *c*)) (i.e. □AE◇O-4, by [1] and Fact (2.2))

[3] ⊢□*all*(*c*, *h*)→(□*no*(*h*, *y*)→◇*not all*(*y*, *c*)) (i.e. □A□E◇O-4, by [2] and Rule 1)

[4] ⊢¬◇*not all*(*y*, *c*)→(□*no*(*h*, *y*)→¬□*all*(*c*, *h*)) (by [3] and Rule 3)

[5] ⊢□¬*not all*(*y*, *c*)→(□*no*(*h*, *y*)→◇¬*all*(*c*, *h*)) (by [4], Fact (2.3) and Fact (2.4))

[6] ⊢□*all*(*y*, *c*)→(□*no*(*h*, *y*)→◇*not all*(*c*, *h*))

(i.e. □E□A◇O-4, by [5], Fact (1.3) and Fact (1.5))

[7] ⊢□*all*(*c*, *h*)→(¬◇*not all*(*y*, *c*)→¬□(*no*(*h*, *y*)) (by [3] and Rule 3)

[8] ⊢□*all*(*c*, *h*)→(□¬*not all*(*y*, *c*)→◇¬*no*(*h*, *y*)) (by [7], Fact (2.3) and Fact (2.4))

[9] ⊢□*all*(*c*, *h*)→(□all(*y*, *c*)→◇*some*(*h*, *y*))

(i.e. □A□A◇I-4, by [8], Fact (1.3) and Fact (1.1))

[10] ⊢¬◇*not all*(*y*, *c*)→(*no*(*h*, *y*)→¬□*all*(*c*, *h*)) (by [2] and Rule 3)

[11] ⊢□¬*not all*(*y*, *c*)→(*no*(*h*, *y*)→◇¬*all*¬*c*, *h*)) (by [10], Fact (2.3) and Fact (2.4))

[12] $\vdash\Box all(y, c)\rightarrow(no(h, y)\rightarrow\Diamond not\ all(c, h))$

(i.e. E□A◇O-4, by [11], Fact (1.3) and Fact (1.5))

[13] $\vdash\Box all(c, h)\rightarrow(\neg\Diamond not\ all(y, c)\rightarrow\neg no(h, y))$ (by [2] and Rule 3)

[14] $\vdash\Box all(c, h)\rightarrow(\Box\neg not\ all(y, c)\rightarrow\neg no(h, y))$ (by [10], Fact (2.3))

[15] $\vdash\Box all(c, h)\rightarrow(\Box all(y, c)\rightarrow some(h, y))$

(i.e. □A□AI-4, by [14], Fact (1.3) and Fact (1.1))

[16] $\vdash\neg\Diamond not\ all(y, c)\rightarrow(no(y, h)\rightarrow\neg\Box all(c, h))$ (by [1] and Rule 3)

[17] $\vdash\Box\neg not\ all(y, c)\rightarrow(no(y, h)\rightarrow\Diamond\neg all\neg c, h))$ (by [16], Fact (2.3) and Fact (2.4))

[18] $\vdash\Box all(y, c)\rightarrow(no(y, h)\rightarrow\Diamond not\ all(c, h))$

(i.e. E□A◇O-3, by [17], Fact (1.3) and Fact (1.5))

[19] $\vdash\Box all(c, h)\rightarrow(\neg\Diamond not\ all(y, c)\rightarrow\neg no(y, h))$ (by [1] and Rule 3)

[20] $\vdash\Box all(c, h)\rightarrow(\Box\neg not\ all(y, c)\rightarrow\neg no(y, h))$ (by [19] and Fact (2.3))

[21] $\vdash\Box all(c, h)\rightarrow(\Box all(y, c)\rightarrow some(y, h))$

(i.e. □A□AI-1, by [20], Fact (1.3) and Fact (1.1))

[22] $\vdash\Box all(c, h)\rightarrow(\Box no(y, h)\rightarrow\Diamond not\ all(y, c))$

(i.e. □A□E◇O-2, by [1], Fact (3.3) and Rule 1)

[23] $\vdash\Box no\neg(c, h)\rightarrow(\Box all\neg(y, h)\rightarrow\Diamond not\ all(y, c))$ (by [22], Fact (1.2) and Fact (1.6))

[24] $\vdash\Box no(c, D-h)\rightarrow(\Box all(y, D-h)\rightarrow\Diamond not\ all(y, c))$

(i.e. □E□A◇O-2, by [23] and Definition 2)

[25] $\vdash\Box no(D-h, c)\rightarrow(\Box all(y, D-h)\rightarrow\Diamond not\ all(y, c))$

(i.e. □E□A◇O-1, by [24] and Fact (2.2))

[26] $\vdash\neg\Diamond not\ all(y, c)\rightarrow(\Box no(y, h)\rightarrow\neg\Box all(c, h))$ (by [22] and Rule 3)

[27] $\vdash\Box\neg not\ all(y, c)\rightarrow(\Box no(y, h)\rightarrow\Diamond\neg all(c, h))$ (by [26], Fact (2.3) and Fact (2.4))

[28] $\vdash\Box all(y, c)\rightarrow(\Box no(y, h)\rightarrow\Diamond not\ all(c, h))$

(i.e. □E□A◇O-3, by [27], Fact (1.3) and Fact (1.5))

[29] $\vdash\Box all(c, h)\rightarrow(\neg\Diamond not\ all(y, c)\rightarrow\neg\Box no(y, h))$ (by [22] and Rule 3)

[30] $\vdash\Box \mathit{all}(c, h)\rightarrow(\Box\neg \mathit{not\ all}(y, c)\rightarrow\Diamond\neg \mathit{no}(y, h))$ (by [29], Fact (2.3) and Fact (2.4))

[31] $\vdash\Box \mathit{all}(c, h)\rightarrow(\Box \mathit{all}(y, c)\rightarrow\Diamond \mathit{some}(y, h))$

(i.e. □A□A◇I-1, by [30], Fact (1.3) and Fact (1.1))

[32] $\vdash\Box \mathit{no}\neg(c, h)\rightarrow(\mathit{all}\neg(y, h)\rightarrow\Diamond \mathit{not\ all}(y, c))$ (by [1], Fact (1.2) and Fact (1.6))

[33] $\vdash\Box \mathit{no}(c, D-h)\rightarrow(\mathit{all}(y, D-h)\rightarrow\Diamond \mathit{not\ all}(y, c))$

(i.e. □EA◇O-2, by [23] and Definition 2)

[34] $\vdash\Box \mathit{no}(c, D-h)\rightarrow(\neg\Diamond \mathit{not\ all}(y, c)\rightarrow\neg \mathit{all}(y, D-h))$ (by [33] and Rule 3)

[35] $\vdash\Box \mathit{no}(c, D-h)\rightarrow(\Box\neg \mathit{not\ all}(y, c)\rightarrow\neg \mathit{all}(y, D-h))$ (by [34] and Fact (2.3))

[36] $\vdash\Box \mathit{no}(c, D-h)\rightarrow(\Box \mathit{all}(y, c)\rightarrow \mathit{not\ all}(y, D-h))$

(i.e. □E□AO-1, by [35], Fact (1.3) and Fact (1.5))

[37] $\vdash\neg\Diamond \mathit{not\ all}(y, c)\rightarrow(\mathit{all}(y, D-h)\rightarrow\neg\Box \mathit{no}(c, D-h))$ (by [33] and Rule 3)

[38] $\vdash\Box\neg \mathit{not\ all}(y, c)\rightarrow(\mathit{all}(y, D-h)\rightarrow\Diamond\neg \mathit{no}(c, D-h))$

(by [37], Fact (2.3) and Fact (2.4))

[39] $\vdash\Box \mathit{all}(y, c)\rightarrow(\mathit{all}(y, D-h)\rightarrow\Diamond \mathit{some}(c, D-h))$

(i.e. A□A◇I-3, by [38], Fact (1.3) and Fact (1.1))

[40] $\vdash\Box \mathit{all}(y, c)\rightarrow(\Box \mathit{all}(y, D-h)\rightarrow\Diamond \mathit{some}(c, D-h))$ (i.e. □A□A◇I-3, by [39] and Rule 1)

[41] $\vdash\Box \mathit{no}(D-h, c)\rightarrow(\mathit{all}(y, D-h)\rightarrow\Diamond \mathit{not\ all}(y, c))$ (i.e. □EA◇O-1, by [33] and Fact (2.2))

[42] $\vdash\Box \mathit{no}(D-h, c)\rightarrow(\neg\Diamond \mathit{not\ all}(y, c)\rightarrow\neg \mathit{all}(y, D-h))$ (by [41] and Rule 3)

[43] $\vdash\Box \mathit{no}(D-h, c)\rightarrow(\Box\neg \mathit{not\ all}(y, c)\rightarrow\neg \mathit{all}(y, D-h))$ (by [42] and Fact (2.3))

[44] $\vdash\Box \mathit{no}(D-h, c)\rightarrow(\Box \mathit{all}(y, c)\rightarrow \mathit{not\ all}(y, D-h))$

(i.e. □E□AO-2, by [43], Fact (1.3) and Fact (1.5))

[45] $\vdash\Box \mathit{all}(y, c)\rightarrow(\mathit{all}(y, D-h)\rightarrow\Diamond \mathit{some}(D-h, c))$

(i.e. □AA◇I-3, by [39] and Fact (2.1))

[46] $\vdash\Box \mathit{no}\neg(y, c)\rightarrow(\mathit{all}(y, D-h)\rightarrow\Diamond \mathit{not\ all}\neg(D-h, c))$ (by [45] and Fact (1.2) and Fact (1.4))

[47] $\vdash\Box \mathit{no}(y, D-c)\rightarrow(\mathit{all}(y, D-h)\rightarrow\Diamond \mathit{not\ all}(D-h, D-c))$

(i.e. □EA◇O-3, by [46] and Definition 2)

[48] $\vdash\Box \mathit{no}(D-c, y)\rightarrow(\mathit{all}(y, D-h)\rightarrow\Diamond \mathit{not\ all}(D-h, D-c))$

(i.e. □EA◇O-4, by [47] and Fact (2.2))

[49]⊢¬◇*not all*(*D*−*h*, *D*−*c*)→(*all*(*y*, *D*−*h*)→¬□*no*(*y*, *D*−*c*)) (by [47] and Rule 3)

[50]⊢□¬*not all*(*D*−*h*, *D*−*c*)→(*all*(*y*, *D*−*h*)→◇¬*no*(*y*, *D*−*c*))

(by [49], Fact (2.3) and Fact (2.4))

[51]⊢□*all*(*D*−*h*, *D*−*c*)→(*all*(*y*, *D*−*h*)→◇*some*(*y*, *D*−*c*))

(i.e. □AA◇I-1, by [50] and Fact (1.3) and Fact (1.1))

[52] ⊢□*all*(*D*−*h*, *D*−*c*)→(¬◇*some*(*y*, *D*−*c*)→¬*all*(*y*, *D*−*h*) (by [51] and Rule 3)

[53] ⊢□*all*(*D*−*h*, *D*−*c*)→(□¬*some*(*y*, *D*−*c*)→¬*all*(*y*, *D*−*h*)) (by [52] and Fact (2.3))

[54] ⊢□*all*(*D*−*h*, *D*−*c*)→(□*no*(*y*, *D*−*c*)→*not all*(*y*, *D*−*h*))

(i.e. □A□EO-2, by [53] and Fact (1.5) and Fact (1.7))

[55] ⊢□*all*(*D*−*h*, *D*−*c*)→(□*no*(*D*−*c*, *y*)→*not all*(*y*, *D*−*h*))

(i.e. □A□EO-4, by [54] and Fact (2.2))

[56] ⊢□*all*(*D*−*h*, *D*−*c*)→(¬*not all*(*y*, *D*−*h*)→¬□*no*(*D*−*c*, *y*)) (by [55] and Rule 3)

[57] ⊢□*all*(*D*−*h*, *D*−*c*)→(¬*not all*(*y*, *D*−*h*)→◇¬*no*(*D*−*c*, *y*)) (by [56] and Fact (2.4))

[58] ⊢□*all*(*D*−*h*, *D*−*c*)→(*all*(*y*, *D*−*h*)→◇*some*(*D*−*c*, *y*))

(i.e. A□A◇I-4, by [57], Fact (1.1) and Fact (1.3))

Theorem 2 proves that the other 26 valid Aristotelian modal syllogisms can be deduced from the syllogism □AE◇O-2 by means of the above reductive operations. That is to say that valid modal syllogisms can be mutually deduced using related definitions and reasoning rules.

4. Conclusion and Future Work

This paper first symbolizes the propositions involved in Aristotelian modal syllogisms from the perspective of mathematical structuralism, then proves the validity of the Aristotelian modal syllogism □AE◇O-2 by relevant definitions, and finally deduces the other 26 valid Aristotelian modal syllogisms from the syllogism □AE◇O-2 in line with some reasoning rules and definitions. This indicates that there are reducible relations between/among different syllogisms.

If both premises of a syllogism are universal propositions and its conclusion is a particular proposition, then this syllogism is called a weak syllogism. For example, the following 9 Aristotelian modal syllogisms are weak syllogisms: A□A◇I-1, □A□AI-3, □AA◇I-4, A□E◇O-2, A□E◇O-4, E□A◇O-1, E□A◇

O-2, □E□AO-3, and □E□AO-4. After our preliminary study, the 9 syllogisms are valid, and they cannot be derived from the syllogism□AE◇O-2. Is there any other methods that can solve this problem? Can more valid Aristotelian modal syllogisms be inferred from the syllogism □AE◇O-2? These questions require further research.

Acknowledgement

This work was supported by Humanities and Social Sciences Research Project of the Chinese Ministry of Education under Grant No.19YJC740123.

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