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Knowledge Mining Based on the Classical Modal Syllogism □AI◇I-3

Xiaojun Zhang [1]**,Yuzhen Wang** [1,*]

[1] School of Philosophy, Anhui University

Email address: 591551032@qq.com (Xiaojun Zhang), 787828605@qq.com (Yuzhen Wang)

***Corresponding author**: Yuzhen Wang

Abstract

This paper first formalizes classical modal syllogisms from the perspective of knowledge representation. Subsequently, it employs modal logic and generalized quantifier theory to prove the validity of the classical modal syllogism □AI◇I-3. Finally, making best of some rules and facts in first-order logic and the definitions of inner and outer negation for classical quantifiers in generalized quantifier theory, at least the other 37 valid classical modal syllogisms can be derived from the validity of the syllogism □AI◇I-3. The method is not only concise and elegant, but also universally applicable to the study of various types of syllogisms. Undoubtedly, this research is beneficial for the further development of knowledge mining in artificial intelligence.

Keywords: classical modal syllogisms; validity; knowledge mining; knowledge reasoning

1. Introduction

There are various types of syllogisms in natural language, such as classical syllogisms (Patzig, 1969; Long, 2023; Hui, 2023), classical modal syllogisms (Łukasiewicz, 1957; Cheng, 2023), and generalized syllogisms (Xiaojun and Baoxiang, 2021), and so on. This paper mainly discusses classical modal syllogisms.

Classical modal syllogisms have been studied by many scholars. For example, Xiaojun (2020a, 2020b) and Cheng (2023) provide a formal study of classical modal syllogisms from the perspective of modern logic. Protin (2022) proposes a new deductive system to explain the validity of classical modal syllogisms. However, a consensus among scholars indicates that the current body of research fails to provide a coherent explanation of classical modal syllogisms.

This paper endeavors to offer a coherent explanation of classical modal syllogisms. To this end, on the basis of relevant definitions, facts, and reasoning rules, this paper first proves the validity of the modal syllogism □AI◇I-3, and then deduces other 37 valid syllogisms from the modal syllogism □AI◇I-3.

2. Knowledge Representation for Classical Modal Syllogisms

In the following, let Q be any of the four classical quantifiers (namely, *all*, *some*, *no*, *not all*), $\neg Q$ be its outer negation quantifier and $Q\neg$ be its inner one. Let z, k and b be lexical variables, and D be their domain. The sets composed of z, k and b are respectively Z, K, and B. '$=_{def}$' means that the left can be defined by the right. Let λ, θ, μ, and π be well-formed formulas (abbreviated as wff). '$\vdash \phi$' indicates that the formula ϕ is provable. The other cases are similar. The operators discussed in the paper represent fundamental symbols within the realms of set theory and modal logic, for instance, $\neg$, $\rightarrow$, $\wedge$, $\leftrightarrow$, □ and ◇ are operators of negation, conditionality, conjunction, bicondition, necessity and possibility, respectively.

Classical syllogisms involve 4 kinds of propositions as follows: 'all *z*s are *b*s', 'some *z*s are *b*s', 'no *z*s are *b*s' and 'not all *z*s are *b*s', which can be respectively formalized as *all(z, b)*, *some(z, b)*, *no(z, b)*, and *not all(z, b)*. These four propositions are respectively called Proposition *A*, *I*, *E*, *O*. Classical syllogisms comprise four distinct figures, which are defined as usual.

A classical modal syllogism is obtained from a classical syllogism by adding necessary modalities ($\Box$) and/or possible ones ($\Diamond$). More specifically, in addition to the four propositions previously mentioned, non-trivial modal syllogisms also encompass at least one of the following eight categories of propositions: $\Box$*all(z, b)*, $\Box$*some(z, b)*, $\Box$*no(z, b)*, $\Box$*not all(z, b)*, $\Diamond$*all(z, b)*, $\Diamond$*some(z, b)*, $\Diamond$*no(z, b)*, and $\Diamond$*not all(z, b)*. And they are respectively called Proposition $\Box$A, $\Box$I, $\Box$E, $\Box$O, $\Diamond$A, $\Diamond$I, $\Diamond$E, and $\Diamond$O. Then, for example, the expansion of the syllogism $\Box$AI$\Diamond$I-3 is that $\Box$*all(k, b)*$\wedge$*some(k, z)*$\rightarrow$$\Diamond$*some(z, b)*. An instance of the syllogism is as follows:

Major premise: All drugs passing strict scrutiny are necessarily safe.

Minor premise: Some drugs passing strict scrutiny are new anti-cancer drugs.

Conclusion: Some new anti-cancer drugs are possibly safe.

Let *k* be the variable of a drug that passes strict scrutiny, *b* be that of a safe drug, and *z* be that of a new anti-cancer drug. Then, this instance can be formalized as $\Box$*all(k, b)*$\wedge$*some(k, z)*$\rightarrow$$\Diamond$*some(z, b)*, which is abbreviated as $\Box$AI$\Diamond$I-3. Other representations are similar to this.

3. Formal System of Classical Modal Syllogistic

This formal system is composed of the following: initial symbols, formation rules, related definitions, basic axioms and deductive rules.

3.1 Initial Symbols

[1] lexical variables: *z*, *k*, *b*

[2] quantifier: *all*, *some*

[3] modality: $\Box$

[4] operators: $\neg$, $\rightarrow$

[5] brackets: (,)

3.2 Formation Rules

[1] If *Q* is a quantifier, *z* and *b* are lexical variables, then *Q(z, b)* is a wff.

[2] If π is a wff, then so are $\neg\pi$ and $\Box\pi$.

[3] If μ and π are wffs, then so is $\mu\rightarrow\pi$.

[4] Only the formulas constructed based on the above three rules are wffs.

3.3 Basic Axioms

[1] A1: If π is a valid formula in first-order logic, then $\vdash \pi$.

[2] A2: $\vdash \Box all(k, b) \wedge some(k, z) \rightarrow \Diamond some(z, b)$ (that is, the syllogism □AI◇I-3).

3.4 Rules of Deduction

Rule 1 (antecedent strengthening): From $\vdash (\lambda \wedge \theta \rightarrow \mu)$ and $\vdash (\pi \rightarrow \lambda)$ infer $\vdash (\pi \wedge \theta \rightarrow \mu)$.

Rule 2 (subsequent weakening): From $\vdash (\lambda \wedge \theta \rightarrow \mu)$ and $\vdash (\mu \rightarrow \pi)$ infer $\vdash (\lambda \wedge \theta \rightarrow \pi)$.

Rule 3 (anti-syllogism): From $\vdash (\lambda \wedge \theta \rightarrow \mu)$ infer $\vdash (\neg \mu \wedge \lambda \rightarrow \neg \theta)$.

3.5 Relevant Definitions

D1 (conjunction): $(\lambda \wedge \theta) =_{def} \neg(\lambda \rightarrow \neg \theta)$;

D2 (bicondition): $(\lambda \leftrightarrow \theta) =_{def} (\lambda \rightarrow \theta) \wedge (\theta \rightarrow \lambda)$;

D3 (inner negation): $(Q\neg)(z, b) =_{def} Q(z, D-b)$;

D4 (outer negation): $(\neg Q)(z, b) =_{def}$ It is not that $Q(z, b)$;

D5 (truth value): *some(z, b)* is true iff $Z \cap B \neq \varnothing$ is true in any real world;

D6 (truth value): $\Box all(z, b)$ is true iff $Z \subseteq B$ is true in any possible world;

D7 (truth value): $\Diamond some(z, b)$ is true iff $Z \cap B \neq \varnothing$ is true in at least one possible world.

3.6 Relevant Facts

Fact 1 (inner negation):

[1.1] $\vdash all(z, b) \leftrightarrow no\neg(z, b)$;

[1.2] $\vdash no(z, b) \leftrightarrow all\neg(z, b)$;

[1.3] $\vdash some(z, b) \leftrightarrow not\ all\neg(z, b)$;

[1.4] $\vdash not\ all(z, b) \leftrightarrow some\neg(z, b)$.

Fact 2 (outer negation):

[2.1] $\vdash \neg all(z, b) \leftrightarrow not\ all(z, b)$;

[2.2] $\vdash \neg not\ all(z, b) \leftrightarrow all(z, b)$;

[2.3] $\vdash \neg no(z, b) \leftrightarrow some(z, b)$;

[2.4] $\vdash$ $\neg some(z, b)\leftrightarrow no(z, b)$.

Fact 3 (symmetry):

[3.1] $\vdash$ $some(z, b)\leftrightarrow some(b, z)$;

[3.2] $\vdash$ $no(z, b)\leftrightarrow no(b, z)$.

Fact 4 (subordination) :

[4.1] $\vdash$ $all(z, b)\rightarrow some(z, b)$;

[4.2] $\vdash$ $no(z, b)\rightarrow not\ all(z, b)$;

[4.3] $\vdash$ $\Box Q(z, b)\rightarrow Q(z, b)$;

[4.4] $\vdash$ $\Box Q(z, b)\rightarrow \Diamond Q(z, b)$;

[4.5] $\vdash$ $Q(z, b)\rightarrow \Diamond Q(z, b)$.

Fact 5 (dual):

[5.1] $\vdash$ $\neg\Diamond\neg Q(z, b)\leftrightarrow\Box Q(z, b)$;

[5.2] $\vdash$ $\neg\Box\neg Q(z, b)\leftrightarrow\Diamond Qz, b)$;

[5.3] $\vdash$ $\neg\Box Q(z, b)\leftrightarrow\Diamond\neg Q(z, b)$;

[5.4] $\vdash$ $\neg\Diamond Q(z, b)\leftrightarrow\Box\neg Q(z, b)$.

These facts are well-known within the domains of first-order logic and generalized quantifier theory. So we omit their proofs.

4. Knowledge Reasoning Based on the Classical Modal Syllogism □AI◇I-3

In the following, Theorem 1 proves that the syllogism □AI◇I-3 is valid. [2.1] '$\vdash$ □AI◇I-3→□AI◇I-1' in Theorem 2 suggests that the validity of the latter can be proved based on that of the former. In other words, there is a reducible relationship between them.

Theorem 1 (□AI◇I-3): The classical modal syllogism $\Box all(k, b)\wedge some(k, z)\rightarrow\Diamond some(z, b)$ is valid.

Proof: Suppose that $\Box all(k, b)$ and $some(k, z)$ are true, then $K\subseteq B$ is true at any possible world and $K\cap Z\neq\varnothing$ is true at any real world in line with Definition D6 and D5, respectively. Because all real worlds are possible worlds. It follows that $Z\cap B\neq\varnothing$ is true in at least one possible world. Hence ◇*some(z, b)* is true in the light of Definition D7. This proves that the syllogism □*all(k, b)*∧*some(k, z)*→◇*some(z, b)* is valid, just as expected.

Theorem 2: There are at least the following 37 valid classical modal syllogisms inferred from the syllogism □AI◇I-3:

[2.1] ⊢ □AI◇I-3→□AI◇I-1

[2.2] ⊢ □AI◇I-3→I□A◇I-3

[2.3] ⊢ □AI◇I-3→□AA◇I-3

[2.4] ⊢ □AI◇I-3→□A□I◇I-3

[2.5] ⊢ □AI◇I-3→□EI◇O-3

[2.6] ⊢ □AI◇I-3→□EI◇O-1

[2.7] ⊢ □AI◇I-3→□E□AE-2

[2.8] ⊢ □AI◇I-3→□AI◇I-1→I□A◇I-4

[2.9] ⊢ □AI◇I-3→□AI◇I-1→□AA◇I-1

[2.10] ⊢ □AI◇I-3→□AI◇I-1→□A□I◇I-1

[2.11] ⊢ □AI◇I-3→□AI◇I-1→□A□EE-2

[2.12] ⊢ □AI◇I-3→I□A◇I-3→A□A◇I-3

[2.13] ⊢ □AI◇I-3→I□A◇I-3→O□A◇O-3

[2.14] ⊢ □AI◇I-3→I□A◇I-3→□EI◇O-2

[2.15] ⊢ □AI◇I-3→I□A◇I-3→□E□AE-1

[2.16] ⊢ □AI◇I-3→□AA◇I-3→□A□A◇I-3

[2.17] ⊢ □AI◇I-3→□AA◇I-3→□EA◇O-3

[2.18] ⊢ □AI◇I-3→□AA◇I-3→□EA◇O-1

[2.19] ⊢ □AI◇I-3→□AA◇I-3→□E□AO-2

[2.20] ⊢ □AI◇I-3→□A□I◇I-3→□I□A◇I-3

[2.21] ⊢ □AI◇I-3→□A□I◇I-3→□E□I◇O-3

[2.22] ⊢ □AI◇I-3→□A□I◇I-3→□E□I◇O-1

[2.23] ⊢ □AI◇I-3→□A□I◇I-3→□E□A◇E-2

[2.24] ⊢ □AI◇I-3→□EI◇O-3→□EI◇O-4

[2.25] ⊢ □AI◇I-3→□AI◇I-1→I□A◇I-4→A□A◇I-4

[2.26] ⊢ □AI◇I-3→□AI◇I-1→I□A◇I-4→□I□A◇I-4

[2.27] ⊢ □AI◇I-3→□AI◇I-1→I□A◇I-4→□A□EE-4

[2.28] ⊢ □AI◇I-3→□AI◇I-1→□AA◇I-1→□A□A◇I-1

[2.29] ⊢ □AI◇I-3→□AI◇I-1→□AA◇I-1→□A□EO-2

[2.30] ⊢ □AI◇I-3→□AI◇I-1→□A□I◇I-1→□A□E◇E-2

[2.31] ⊢ □AI◇I-3→I□A◇I-3→A□A◇I-3→E□A◇O-3

[2.32] ⊢ □AI◇I-3→I□A◇I-3→A□A◇I-3→□EA◇O-2

[2.33] ⊢ □AI◇I-3→I□A◇I-3→A□A◇I-3→□E□AO-1

[2.34] ⊢ □AI◇I-3→I□A◇I-3→O□A◇O-3→□O□A◇O-3

[2.35] ⊢ □AI◇I-3→I□A◇I-3→O□A◇O-3→□AO◇O-2

[2.36] ⊢ □AI◇I-3→I□A◇I-3→O□A◇O-3→□A□AA-1

[2.37] ⊢ □AI◇I-3→I□A◇I-3→□EI◇O-2→□E□I◇O-2

Proof:

[1] ⊢ □*all(k, b)*∧*some(k, z)*→◇*some(z, b)* (i.e. □AI◇I-3, Basic Axiom A1)

[2] ⊢ □*all(k, b)*∧*some(z, k)*→◇*some(z, b)* (i.e. □AI◇I-1, by [1] and Fact [3.1])

[3] ⊢ □*all(k, b)*∧*some(k, z)*→◇*some(b, z)* (i.e. I□A◇I-3, by [1] and Fact [3.1])

[4] ⊢ □*all(k, b)*∧*all(k, z)*→◇*some(z, b)* (i.e. □AA◇I-3, by [1], Fact [4.1] and Rule 1)

[5] ⊢ □*all(k, b)*∧□*some(k, z)*→◇*some(z, b)* (i.e. □A□I◇I-3, by [1], Fact [4.3] and Rule 1)

[6] ⊢ □*no¬(k, b)*∧*some(k, z)*→◇*not all¬(z, b)* (by [1], Fact [1.1] and [1.3])

[7] ⊢ □*no(k, D−b)*∧*some(k, z)*→◇*not all(z, D−b)* (i.e. □EI◇O-3, by [6] and Definition D3)

[8] ⊢ ¬◇*some(z, b)*∧*some(k, z)*→¬□*all(k, b)* (by [1] and Rule 3)

[9] ⊢ □¬*some(z, b)*∧*some(k, z)*→◇¬*all(k, b)* (by [8], Fact [5.4] and [5.3])

[10] ⊢ □*no(z, b)*∧*some(k, z)*→◇*not all(k, b)* (i.e. □EI◇O-1, by [9], Fact [2.4] and [2.1])

[11] ⊢ ¬◇*some(z, b)*∧□*all(k, b)*→¬*some(k, z)* (by [1] and Rule 3)

[12] ⊢ □¬*some(z, b)*∧□*all(k, b)*→¬*some(k, z)* (by [11] and Fact [5.4])

[13] ⊢ □*no(z, b)*∧□*all(k, b)*→*no(k, z)* (i.e. □E□AE-2, by [12] and Fact [2.4])

[14] ⊢ □*all(k, b)*∧*some(z, k)*→◇*some(b, z)* (i.e. I□A◇I-4, by [2] and Fact [3.1])

[15] ⊢ $\Box all(k, b)\wedge all(z, k)\rightarrow\Diamond some(z, b)$ (i.e. □AA◇I-1, by [2], Fact [4.1] and Rule 1)

[16] ⊢ $\Box all(k, b)\wedge\Box some(z, k)\rightarrow\Diamond some(z, b)$ (i.e. □A□I◇I-1, by [2], Fact [4.3] and Rule 1)

[17] ⊢ $\neg\Diamond some(z, b)\wedge\Box all(k, b)\rightarrow\neg some(z, k)$ (by [2] and Rule 3)

[18] ⊢ $\Box\neg some(z, b)\wedge\Box all(k, b)\rightarrow\neg some(z, k)$ (by [17] and Fact [5.4])

[19] ⊢ $\Box no(z, b)\wedge\Box all(k, b)\rightarrow no(z, k)$ (i.e. □A□EE-2, by [18] and Fact [2.4])

[20] ⊢ $\Box all(k, b)\wedge all(k, z)\rightarrow\Diamond some(b, z)$ (i.e. A□A◇I-3, by [3], Fact [4.1] and Rule 1)

[21] ⊢ $\Box all(k, b)\wedge \text{not all}\neg(k, z)\rightarrow\Diamond \text{not all}\neg(b, z)$ (by [3] and Fact [1.3])

[22] ⊢ $\Box all(k, b)\wedge \text{not all}(k, D-z)\rightarrow\Diamond \text{not all}(b, D-z)$ (i.e. O□A◇O-3, by [21] and Definition D3)

[23] ⊢ $\neg\Diamond some(b, z)\wedge some(k, z)\rightarrow\neg\Box all(k, b)$ (by [3] and Rule 3)

[24] ⊢ $\Box\neg some(b, z)\wedge some(k, z)\rightarrow\Diamond\neg all(k, b)$ (by [23], Fact [5.4] and ([5.3])

[25] ⊢ $\Box no(b, z)\wedge some(k, z)\rightarrow\Diamond \text{not all}(k, b)$ (i.e. □EI◇O-2, by [24], Fact [2.4] and [2.1])

[26] ⊢ $\neg\Diamond some(b, z)\wedge\Box all(k, b)\rightarrow\neg some(k, z)$ (by [3] and Rule 3)

[27] ⊢ $\Box\neg some(b, z)\wedge\Box all(k, b)\rightarrow\neg some(k, z)$ (by [26] and Fact [5.4])

[28] ⊢ $\Box no(b, z)\wedge\Box all(k, b)\rightarrow no(k, z)$ (i.e. □E□AE-1, by [27] and Fact [2.4])

[29] ⊢ $\Box all(k, b)\wedge\Box all(k, z)\rightarrow\Diamond some(z, b)$ (i.e. □A□A◇I-3, by [4], Fact [4.3] and Rule 1)

[30] ⊢ $\Box no\neg(k, b)\wedge \mathrm{all}(k, z)\rightarrow\Diamond \text{not all}\neg(z, b)$ (by [4], Fact [1.1] and [1.3])

[31] ⊢ $\Box no(k, D-b)\wedge all(k, z)\rightarrow\Diamond \text{not all}(z, D-b)$ (i.e. □EA◇O-3, by [30] and Definition D3)

[32] ⊢ $\neg\Diamond some(z, b)\wedge all(k, z)\rightarrow\neg\Box all(k, b)$ (by [4] and Rule 3)

[33] ⊢ $\Box\neg some(z, b)\wedge all(k, z)\rightarrow\Diamond\neg all(k, b)$ (by [32], Fact [5.4] and [5.3])

[34] ⊢ $\Box no(z, b)\wedge all(k, z)\rightarrow\Diamond \text{not all}(k, b)$ (i.e. □EA◇O-1, by [33], Fact [2.4] and [2.1])

[35] ⊢ $\neg\Diamond some(z, b)\wedge\Box all(k, b)\rightarrow\neg all(k, z)$ (by [4] and Rule 3)

[36] ⊢ $\Box\neg some(z, b)\wedge\Box all(k, b)\rightarrow\neg all(k, z)$ (by [35] and Fact [5.4])

[37] ⊢ $\Box no(z, b)\wedge\Box all(k, b)\rightarrow \text{not all}(k, z)$ (i.e. □E□AO-2, by [36], Fact [2.4] and [2.1])

[38] ⊢ $\Box all(k, b)\wedge\Box some(k, z)\rightarrow\Diamond some(b, z)$ (i.e. □I□A◇I-3, by [5] and Fact [3.1])

[39] ⊢ $\Box no\neg(k, b)\wedge\Box some(k, z)\rightarrow\Diamond \text{not all}\neg(z, b)$ (by [5], Fact [1.1] and [1.3])

[40] ⊢ $\Box no(k, D-b)\wedge\Box some(k, z)\rightarrow\Diamond \text{not all}(z, D-b)$

(i.e. □E□I◇O-3, by [39] and Definition D3)

[41] ⊢ $\neg\Diamond some(z, b)\wedge\Box some(k, z)\rightarrow\neg\Box all(k, b)$ (by [5] and Rule 3)

[42] ⊢ $\Box\neg some(z, b)\wedge\Box some(k, z)\rightarrow\Diamond\neg all(k, b)$ (by [41], Fact [5.4] and [5.3])

[43] ⊢ $\Box no(z, b)\wedge\Box some(k, z)\rightarrow\Diamond not\ all(k, b)$ (i.e. □E□I◇O-1, by [42], Fact [2.4] and [2.1])

[44] ⊢ $\neg\Diamond some(z, b)\wedge\Box all(k, b)\rightarrow\neg\Box some(k, z)$ (by [5] and Rule 3)

[45] ⊢ $\Box\neg some(z, b)\wedge\Box all(k, b)\rightarrow\Diamond\neg some(k, z)$ (by [44], Fact [5.4]and [5.3])

[46] ⊢ $\Box no(z, b)\wedge\Box all(k, b)\rightarrow\Diamond no(k, z)$ (i.e. □E□A◇E-2, by [45] and Fact [2.4])

[47] ⊢ $\Box no(D{-}b, k)\wedge some(k, z)\rightarrow\Diamond not\ all(z, D{-}b)$ (i.e. □EI◇O-4, by [7] and Fact [3.2])

[48] ⊢ $\Box all(k, b)\wedge all(z, k)\rightarrow\Diamond some(b, z)$ (i.e. A□A◇I-4, by [14], Fact [4.1] and Rule 1)

[49] ⊢ $\Box all(k, b)\wedge\Box some(z, k)\rightarrow\Diamond some(b, z)$ (i.e. □I□A◇I-4, by [14], Fact [4.3] and Rule 1)

[50] ⊢ $\neg\Diamond some(b, z)\wedge\Box all(k, b)\rightarrow\neg some(z, k)$ (by [14] and Rule 3)

[51] ⊢ $\Box\neg some(b, z)\wedge\Box all(k, b)\rightarrow\neg some(z, k)$ (by [50] and Fact [5.4])

[52] ⊢ $\Box no(b, z)\wedge\Box all(k, b)\rightarrow no(z, k)$ (i.e. □A□EE-4, by [51] and Fact [2.4])

[53] ⊢ $\Box all(k, b)\wedge\Box all(z, k)\rightarrow\Diamond some(z, b)$ (i.e. □A□A◇I-1, by [15], Fact [4.3] and Rule 1)

[54] ⊢ $\neg\Diamond some(z, b)\wedge\Box all(k, b)\rightarrow\neg all(z, k)$ (by [15] and Rule 3)

[55] ⊢ $\Box\neg some(z, b)\wedge\Box all(k, b)\rightarrow\neg all(z, k)$ (by [54] and Fact [5.4])

[56] ⊢ $\Box no(z, b)\wedge\Box all(k, b)\rightarrow not\ all(z, k)$ (i.e. □A□EO-2, by [55], Fact [2.4] and [2.1])

[57] ⊢ $\neg\Diamond some(z, b)\wedge\Box all(k, b)\rightarrow\neg\Box some(z, k)$ (by [16] and Rule 3)

[58] ⊢ $\Box\neg some(z, b)\wedge\Box all(k, b)\rightarrow\Diamond\neg some(z, k)$ (by [57], Fact [5.4] and [5.3])

[59] ⊢ $\Box no(z, b)\wedge\Box all(k, b)\rightarrow\Diamond no(z, k)$ (i.e. □A□E◇E-2, by [58] and Fact [2.4])

[60] ⊢ $\Box all(k, b)\wedge no\neg(k, z)\rightarrow\Diamond not\ all\neg(b, z)$ (by [20], Fact [1.1] and [1.3])

[61] ⊢ $\Box all(k, b)\wedge no(k, D{-}z)\rightarrow\Diamond not\ all(b, D{-}z)$ (i.e. E□A◇O-3, by [60] and Definition D3)

[62] ⊢ $\neg\Diamond some(b, z)\wedge all(k, z)\rightarrow\neg\Box all(k, b)$ (by [20] and Rule 3)

[63] ⊢ $\Box\neg some(b, z)\wedge all(k, z)\rightarrow\Diamond\neg all(k, b)$ (by [62], Fact [5.4] and [5.3])

[64] ⊢ $\Box no(b, z)\wedge all(k, z)\rightarrow\Diamond not\ all(k, b)$ (i.e.□EA◇O-2, by [63], Fact [2.4] and [2.1])

[65] ⊢ $\neg\Diamond some(b, z)\wedge\Box all(k, b)\rightarrow\neg all(k, z)$ (by [20] and Rule 3)

[66] ⊢ $\Box\neg some(b, z)\wedge\Box all(k, b)\rightarrow\neg all(k, z)$ (by [65] and Fact [5.4])

[67] ⊢ $\Box no(b, z)\wedge\Box all(k, b)\rightarrow not\ all(k, z)$ (i.e. □E□AO-1, by [66], Fact [2.4] and [2.1])

[68] ⊢ $\Box all(k, b)\wedge\Box not\ all(k, D{-}z)\rightarrow\Diamond not\ all(b, D{-}z)$

(i.e. □O□A◇O-3, by [22], Fact [4.3] and Rule 1)

[69] ⊢ $\neg\Diamond$*not all(b, D−z)*$\wedge$*not all(k, D−z)*$\rightarrow\neg\Box$*all(k, b)* (by [22] and Rule 3)

[70] ⊢ $\Box\neg$*not all(b, D−z)*$\wedge$*not all(k, D−z)*$\rightarrow\Diamond\neg$*all(k, b)* (by [69], Fact [5.4] and [5.3])

[71] ⊢ $\Box$*all(b, D−z)*$\wedge$*not all(k, D−z)*$\rightarrow\Diamond$*not all(k, b)*

(i.e. □AO◇O-2, by [70], Fact [2.2] and [2.1])

[72] ⊢ $\neg\Diamond$*not all(b, D−z)*$\wedge\Box$*all(k, b)*$\rightarrow\neg$*not all(k, D−z)* (by [22] and Rule 3)

[73] ⊢ $\Box\neg$*not all(b, D−z)*$\wedge\Box$*all(k, b)*$\rightarrow\neg$*not all(k, D−z)* (by [72] and Fact [5.4])

[74] ⊢ $\Box$*all(b, D−z)*$\wedge\Box$*all(k, b)*$\rightarrow$*all(k, D−z)* (i.e. □A□AA-1, by [73] and Fact [2.2])

[75] ⊢ $\Box$*no(b, z)*$\wedge\Box$*some(k, z)*$\rightarrow\Diamond$*not all(k, b)* (i.e. □E□I◇O-2, by [25], Fact [4.3] and Rule 1)

At this point, the other 37 valid classical modal syllogisms have been derived from the validity of the syllogism □AI◇I-3. By continuing to apply similar reasoning methods, one can deduce other valid syllogisms. Similar to Theorem 1, the validity of these newly derived syllogisms can also be proved through relevant definitions.

5. Conclusion

Making best of modal logic, set theory and generalized quantifier theory, this paper initially proves the validity of the classical modal syllogism □AI◇I-3. Subsequently, with the aid of the relevant definitions, facts, and reasoning rules, it derives the other 37 valid classical modal syllogisms from the validity of the syllogism □AI◇I-3. The results obtained by these deductive methods are logically consistent. This approach is not only concise and elegant, but also universally applicable for studying other types of syllogisms.

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References

[1] G. Patzig, Aristotle's Theory of the Syllogism, D. Reidel, Dordrecht, 1969.

[2] L. Wei, Formal system of categorical syllogistic logic based on the syllogism AEE-4. Open Journal of Philosophy, Vol. 13, No.1, 2023, pp.97-103.

[3] H. Li, Reduction between categorical syllogisms based on the syllogism EIO-2. Applied Science and Innovative Research, Vol. 7, No.1, 2023, pp.30-37.

[4] J. Łukasiewicz, Aristotle's Syllogistic: From the Standpoint of Modern Formal Logic (Second edition), Clerndon Press, Oxford, 1957.

[5] C. Zhang, Formal Research on Aristotelian Modal Syllogism from the Perspective of Mathematical Structuralism, Doctoral Dissertation, Anhui University, 2023. (in Chinese)

[6] X. J. Zhang, and B. X. Wu, Study on Chinese Discourse Inference, People's Publishing House, Beijing, 2021. (in Chinese)

[7] X. J. Zhang, Reducible relations between/among Aristotle's modal syllogisms. SCIREA Journal of Computer, Vol. 5, No. 1, 2020a, pp.1-33.

[8] X. J. Zhang, Screening out all valid Aristotelian modal syllogisms. Applied and Computational Mathematics, Vol. 8, No. 6, 2020b, pp.95-104.

[9] C. L. Protin, A Logic for Aristotle's Modal Syllogistic. History and Philosophy of Logic, Vol. 44, No. 3, 2022, pp.225-246.