SCIREA Journal of Computer
http://www.scirea.org/journal/Computer
November 13, 2022
Volume 7, Issue 4, August 2022
https://doi.org/10.54647/computer52321

Reduction between the Syllogism *OAO-3* and the Remaining 23 Valid Syllogisms

Xiaojun Zhang[1], Long Wei[1], Yijiang Hao[2]
[1]School of Philosophy, Anhui University, Hefei, China
[2]Institute of Philosophy, Chinese Academy of Social Sciences, Beijing, China
Email address: 591551032@qq.com (Xiaojun Zhang), 657703460@qq.com (Long Wei), 2127848827@qq.com (Yijiang Hao)

Abstract

With the help of the definitions of three negative quantifiers of Aristotelian quantifiers (i.e. *all*, *no*, *some* and *not all*), the symmetry of *no* and *some*, and some basic inference rules in propositional logic, one can deduce the remaining 23 valid syllogisms only from the syllogism *OAO-3*. In other words, there is reducible relations between/among different forms and different figures of valid traditional syllogisms. And these reducible relations actually reflect the transformation relations between the monotonicity of the four Aristotelian quantifiers. This paper provides a computational level of reasoning for syllogistic logic and an important theoretical basis for knowledge representation and knowledge reasoning in computers.

Keywords: Aristotelian quantifiers; symmetry; reducible relations; generalized quantifier theory

1. Introduction

Syllogistic arguments have played an important role in logic and human reasoning from Aristotle onwards [1]. In natural languages, there are various syllogisms, such as traditional syllogisms [2-4], syllogisms with verbs [5], generalized syllogisms [6-7] and modal syllogisms [8-10], and so on, in which traditional syllogisms are time-honored and the most intensively researched in logic due to its prominence in human reasoning. This paper only studies the traditional syllogisms, thus all of the syllogisms in the following refers to traditional syllogisms.

Different studies of traditional syllogistic logic have already been shown, for example by in Łukasiewicz ([2]), Corcoran ([11]), van Benthem ([12]), Westerståhl ([4], [13]), Moss([3], [5]) Beihai et al ([14]), and Xiaojun ([15-18]), and so on. This paper makes a formal study of traditional syllogistic logic from the perspective of generalized quantifier theory [19-20] and set theory, and provides a computational level of reasoning. The research method is simple and universal. And this paper is not only beneficial to the further study of other kinds of syllogisms, but also provides an important theoretical basis for knowledge representation and knowledge reasoning in computers.

2. Symbolization of Syllogisms

Traditional syllogisms involve the four kinds of categorical propositions, *A*, *E*, *I*, *O*. Let *x* and *y* be the subject and predicate variables of the categorical proposition, respectively. The categorical propositions *A* refers to that *all x*s are *y*, and can be symbolized as *all(x, y), in which x* and *y* are lexical variables. The categorical propositions *E* refers to that *no x*s are *y*, and can be denoted by *no(x, y)*. The categorical propositions *I* refers to that *some x*s are *y*, and can be symbolized as *some(x, y)*. The categorical propositions *O* refers to that *not all x*s are *y*, and can be denoted by *not all(x, y)*. From the perspective of generalized quantifier theory, *all*, *no*, *some* and *not all* are called Aristotelian quantifiers, and traditional syllogisms embody the semantic and inferential properties of these four Aristotelian quantifiers.

The figure of a syllogism is determined by the position of the middle term. A syllogism is called the first figure if the middle term is the subject of the major premise and the predicate of the minor premise, respectively. A syllogism is called the second figure if the middle term is the predicate of both the major and minor premise. A syllogism is called the third figure if the middle term is the subject of both the major and minor premise. A syllogism is called the fourth figure if the middle term is the predicate of the major premise and the subject of the minor premise, respectively. For example, the syllogism *EIO-3* stands for the third figure syllogism, and its major premise, minor premise and conclusion is the categorical proposition *E*, *I* and *O*, respectively. And the syllogism can be symbolized as *no(y, z)*∧*some(y, x)*→*not all(x, z)*, in which *x*, *y*, and *z* are lexical variables. The other notations and symbolization are similar.

In this paper, $=_{def}$ means that the content on the left can be defined by the content on the right.

3. Formation Rules and Definitions

In order to explore the reducible relations between/among different syllogisms, it is necessary to give primitive symbols, formation rules, related definitions as the following.

3.1 Primitive Symbols

(1) lexical variables: *x*, *y*, *z*

(2) unary negative operator: ¬

(3) binary implication operator: ∧, →

(4) quantifier: *not all*

(5) brackets: (,)

3.2 Formation Rules

(1) If *Q* is a quantifier, *x* and *y* are lexical variables, then *Q(x, y)* is a well-formed formula.

(2) If *p* and *q* are well-formed formulas, then ¬p, *p*∧*q* and *p*→*q* are well-formed formulas.

(3) Only the formulas obtained through (1) and (2) are well-formed formulas.

For example, *not all(x, y)*, and *not all(y, z)*∧*all(y, x)*→*not all(x, z)* are well-formed formulas, which read respectively as '*not all x*s are *y*', and 'if *not all y*s are *z*, and *all y*s are *x*, then *not all x*s are *z*. Others are similar.

3.3 Related Definitions

Let *D* be the domain of lexical variables, and *Q* be a quantifier, then ¬*Q, Q*¬ and Q^d stands for the outer quantifier of *Q*, the inner quantifier of *Q* and the dual quantifier of *Q,* respectively.

Definition 1 (outer negative quantifier): $\neg Q(x, y) =_{def}$ It is not that $Q(x, y)$;

Definition 2 (inner negative quantifier): $Q\neg(x, y) =_{def} Q(x, D-y)$;

Definition 3 (dual negative quantifier): $Q^d(x, y) =_{def} \neg Q\neg(x, y)$;

According to the above definitions and generalized quantifier theory, the negative relations of the four Aristotelian quantifiers (i.e, *all*, *no*, *some* and *not all*) in traditional syllogisms are as follows: (1) *all* and *not all*, *no* and *some* are mutually outer negative. More specifically, *all*=¬*not all*, *not all*=¬*all*; *no*=¬*some, some*=¬*no*; (2) *no* and *all*, *some* and *not all* are mutually inner negative. More specifically, *no*=*all*¬, all=no¬; *some*=*not all*¬; *not all*=*some*¬; (3) *all* and *some*, *no* and *not all* are mutually dual negative. More specifically, *all*=¬*some*¬, *some*=¬*all*¬; *no*=¬*not all*¬, *not all*=¬*no*¬.

4. System of Traditional Syllogistic Logic

In the following, ⊢ is to represent a proposition or syllogism that can be proved. For example, the syllogism OAO-3 can be proved, and symbolized as ⊢ not all(y, z)∧some(y, x)→not all(x, z). The other notations are similar.

4.1 Basic Axioms

(1) A0: if *p* is a valid formula in propositional logic, then ⊢ *p*.

(2) A1: ⊢ *all(x, x)*.

(3) A2: ⊢ *some(x, x)*.

(4) A3 (that is, the syllogism *OAO-3*): ⊢ *not all(y, z)∧some(y, z)→not all(x, z)*.

4.2 Inference Rules

In the following rules, *p, q, r* and *s* are well-formed formulas.

(1) Replacement rule: if *p* is obtained from *q* by means of "replacing one variable with another", then ⊢ *p* can be derived from ⊢ *q*.

(2) Modus Ponens: From ⊢ (p→q) and ⊢ p infer ⊢ *q*.

(3) Substitution of equivalents: From ⊢ (...*p*...) and *p*↔*q* infer ⊢ (...*q*...), and vice versa.

(4) Double negative: From ⊢ ¬¬*p* infer ⊢ *p* , and vice versa.

(5) Antecedent interchange: From ⊢ (*p*∧*q*→*r*) infer ⊢ (*q*∧*p*→*r*).

(6) Subsequent weakening: From ⊢ $(p\wedge q\rightarrow r)$ and ⊢ $(r\rightarrow s)$ infer ⊢ $(p\wedge q\rightarrow s)$.

(7) Reverse rule: From ⊢ $(p\rightarrow q)$ infer ⊢ $(\neg q\rightarrow\neg p)$.

(8) Reverse rule 1 of syllogism: From ⊢ $(p\wedge q\rightarrow r)$ infer ⊢ $(\neg r\wedge p\rightarrow\neg q)$.

(9) Reverse rule 2 of Syllogism: From ⊢ $(p\wedge q\rightarrow r)$ infer ⊢ $(\neg r\wedge q\rightarrow\neg p)$.

4.3 Relevant Facts

The following Fact 1 and 2 are the definitions in generalized quantifier theory. Fact 3 embodies the symmetry of *some* and *no*. Fact 4 is the basic fact of predicate logic. And the four facts can be easily proved by means of the above definitions, inference rules and axioms. Therefore, the detailed proofs of these facts will not be given here.

Fact 1 (inner negation):

(1) ⊢ *all(x, y)*↔*no*¬*(x, y)*; (2) ⊢ *no(x, y)*↔*all*¬*(x, y)*;

(3) ⊢ *some(x, y)*↔*not all*¬*(x, y)*; (4) ⊢ *not all(x, y)*↔*some*¬*(x, y)*.

Fact 2 (outer negation):

(1) ⊢ *all(x, y)*↔¬*not all(x, y)*; (2) ⊢ *not all(x, y)*↔¬*all(x, y)*;

(3) ⊢ *some(x, y)*↔¬*no(x, y)*; (4) ⊢ *no(x, y)*↔¬*some(x, y)*.

Fact 3 (symmetry of *some* and *no*):

(1) (symmetry of *some*): ⊢ *some(x, y)*↔*some(y, x)*;

(2) (symmetry of *no*): ⊢ *no(x, y)*↔*no(y, x)*.

Fact 4 (assertoric subalternations):

(1) ⊢ *no(x, y)*→*not all(x, y)*; (2) ⊢ *all(x, y)*→*some(x, y)*.

4.4 Reducible Relations between/among Valid Syllogisms from the Syllogism *OAO-3*

In this paper, *OAO-3*⇒*IAI-3* means that the validity of syllogism *IAI-3* can be deduced from the validity of syllogism *OAO-3*. And one can say that there is a reducible relation between the two syllogisms. Others are similar. On the basis of generalized quantifier theory, set theory and the above inference rules in propositional logic, one can prove the reductions between/among valid traditional syllogisms as the following Theorem 1.

Theorem 1: The other 23 valid syllogisms can be deduced only from the syllogism *OAO-3*. According to the order of proof, the reducible relations between/among syllogisms as the following:

[1] *OAO-3⇒IAI-3*

[2] *OAO-3⇒IAI-3⇒IAI-4*

[3] *OAO-3⇒IAI-3⇒AII-3*

[4] *OAO-3⇒IAI-3⇒AII-3⇒AII-1*

[5] *OAO-3⇒AOO-2*

[6] *OAO-3⇒AOO-2⇒EIO-2*

[7] *OAO-3⇒AOO-2⇒EIO-2⇒EIO-1*

[8] *OAO-3⇒AOO-2⇒EIO-2⇒EIO-1⇒EIO-3*

[9] *OAO-3⇒AOO-2⇒EIO-2⇒EIO-4*

[10] *OAO-3⇒AAA-1*

[11] *OAO-3⇒AAA-1⇒AAI-1*

[12] *OAO-3⇒AAA-1⇒AAI-1⇒AAI-4*

[13] *OAO-3⇒AAA-1⇒EAE-1*

[14] *OAO-3⇒AAA-1⇒EAE-1⇒EAE-2*

[15] *OAO-3⇒AAA-1⇒EAE-1⇒EAE-2⇒AEE-2*

[16] *OAO-3⇒AAA-1⇒EAE-1⇒AEE-4*

[17] *OAO-3⇒AAA-1⇒EAE-1⇒EAO-1*

[18] *OAO-3⇒AAA-1⇒EAE-1⇒EAE-2⇒EAO-2*

[19] *OAO-3⇒AAA-1⇒EAE-1⇒EAE-2⇒AEE-2⇒AEO-2*

[20] *OAO-3⇒AAA-1⇒EAE-1⇒AEE-4⇒AEO-4*

[21] *OAO-3⇒AAA-1⇒EAE-1⇒EAE-2⇒AEE-2⇒AEO-2⇒EAO-3*

[22] *OAO-3⇒AAA-1⇒EAE-1⇒EAE-2⇒AEE-2⇒AEO-2⇒EAO-3⇒EAO-4*

[23] *OAO-3⇒AAA-1⇒EAE-1⇒EAE-2⇒AEE-2⇒AEO-2⇒EAO-3⇒AAI-3*

Proof:

(1) *not all(y, z)∧all(y, x)→not all(x, z)* (i.e. Axiom 3)

(2) *some¬(y, z)∧all(y, x)→some¬(x, z)* (by (1) and *not all*=*some¬*)

(3) *some(y, D−z)∧all(y, x)→some(x, D−z)* (by (2) and Definition 2)

(4) *some(y, z)∧all(y, x)→some(x, z)* (i.e. *IAI-3*, by (3) and Replacement rule)

(5) $some(y, z) \leftrightarrow some(z, y)$ (by the symmetry of *some*)

(6) $some(z, y) \wedge all(y, x) \rightarrow some(x, z)$ (i.e. *IAI-4*, by (4), (5) and Substitution of equivalents)

(7) $some(x, z) \leftrightarrow some(z, x)$ (by the symmetry of *some*)

(8) $some(y, z) \wedge all(y, x) \rightarrow some(z, x)$ (by (4), (7) and Substitution of equivalents)

(9) $all(y, x) \wedge some(y, z) \rightarrow some(z, x)$ (i.e. *AII-3*, by (8) and Antecedent interchange)

(10) $all(y, x) \wedge some(z, y) \rightarrow some(z, x)$ (i.e. *AII-1*, by(5), (9) and Substitution of equivalents)

(11) $\neg not\ all(x, z) \wedge not\ all(y, z) \rightarrow \neg all(y, x)$ (by (1) and Reverse rule 1 of syllogism)

(12) $all(x, z) \wedge not\ all(y, z) \rightarrow not\ all(y, x)$

(i.e. *AOO-2*, by (11), $\neg not\ all = all$, $\neg all = not\ all$, and Substitution of equivalents)

(13) $no\neg(x, z) \wedge some\neg(y, z) \rightarrow not\ all(y, x)$

(by (12), $all = no\neg$, $not\ all = some\neg$, and Substitution of equivalents)

(14) $no(x, D-z) \wedge some(y, D-z) \rightarrow not\ all(y, x)$ (by (13) and Definition 2)

(15) $no(x, z) \wedge some(y, z) \rightarrow not\ all(y, x)$ (i.e. *EIO-2*, by (14) and Replacement rule)

(16) $no(x, z) \leftrightarrow no(z, x)$ (by the symmetry of *no*)

(17) $no(z, x) \wedge some(y, z) \rightarrow not\ all(y, x)$ (i.e. *EIO-1*, by (15), (16) and Substitution of equivalents)

(18) $no(z, x) \wedge some(z, y) \rightarrow not\ all(y, x)$ (i.e. *EIO-3*, by (5), (17) and Substitution of equivalents)

(19) $no(x, z) \wedge some(z, y) \rightarrow not\ all(y, x)$ (i.e. *EIO-4*, by (5), (15) and Substitution of equivalents)

(20) $\neg not\ all(x, z) \wedge all(y, x) \rightarrow \neg not\ all(y, z)$ (by (1) and Reverse rule 2 of syllogism)

(21) $all(x, z) \wedge all(y, x) \rightarrow all(y, z)$

(i.e. *AAA-1*, by (20), $\neg not\ all = all$ and Substitution of equivalents)

(22) $all(y, z) \rightarrow some(y, z)$ (Fact 4 and Replacement rule)

(23) $all(x, z) \wedge all(y, x) \rightarrow some(y, z)$ (i.e. *AAI-1*, by (21) and Subsequent weakening)

(24) $all(x, z) \wedge all(y, x) \rightarrow some(z, y)$ (by (5), (23) and Substitution of equivalents)

(25) $all(y, x) \wedge all(x, z) \rightarrow some(z, y)$ (i.e. AAI-4, by (24) and Antecedent interchange)

(26) $no\neg(x, z) \wedge all(y, x) \rightarrow no\neg(y, z)$ (by (21), $all = no\neg$ and Substitution of equivalents)

(27) $no(x, D-z) \wedge all(y, x) \rightarrow no(y, D-z)$ (by (26) and Definition 2)

(28) $no(x, z) \wedge all(y, x) \rightarrow no(y, z)$ (i.e. *EAE-1*, by (27) and Replacement rule)

(29) $no(z, x) \wedge all(y, x) \rightarrow no(y, z)$ (i.e. *EAE-2*, by (16), (28) and Substitution of equivalents)

(30) $no(y, z) \leftrightarrow no(z, y)$ (by the symmetry of *no*)

(31) $no(z, x) \wedge all(y, x) \rightarrow no(z, y)$ (by (29), (30) and Substitution of equivalents)

(32) $all(y, x) \wedge no(z, x) \rightarrow no(z, y)$ (i.e. *AEE-2*, by (31) Antecedent interchange)

(33) $no(x, z) \wedge all(y, x) \rightarrow no(z, y)$ (by (28), (30) and Substitution of equivalents)

(34) $all(y, x) \wedge no(x, z) \rightarrow no(z, y)$ (i.e. *AEE-4*, by (33) and Antecedent interchange)

(35) $no(y, z) \rightarrow not\ all(y, z)$ (Fact 4 and Replacement rule)

(36) $no(x, z) \wedge all(y, x) \rightarrow not\ all(y, z)$ (i.e. *EAO-1*, by (28), (35) and Subsequent weakening)

(37) *no(z, x)∧all(y, x)→not all(y, z)* (i.e. *EAO-2*, by (29), (35) and Subsequent weakening)

(38) *no(z, y)→not all(z, y)* (Fact 4 and Replacement rule)

(39) *all(y, x)∧no(z, x)→not all(z, y)* (i.e. *AEO-2*, by (32), (38) and Subsequent weakening)

(40) *all(y, x)∧no(x, z)→not all(z, y)* (i.e. *AEO-4*, by (34), (38) and Subsequent weakening)

(41) *¬not all(z, y)∧no(z, x)→¬all(y, x)* (by (39) and and Reverse rule 2 of syllogism)

(42) *all(z, y)∧no(z, x)→not all(y, x)*

(by (41), *¬not all=all*,*¬all=not all*, and Substitution of equivalents)

(43) *no(z, x)∧all(z, y)→not all(y, x)* (i.e. *EAO-3*, by (42) and Antecedent interchange)

(44) *no(x, z)∧all(z, y)→not all(y, x)* (i.e. *EAO-4*, by (16), (43) and Replacement rule)

(45) *all¬(z, x)∧all(z, y)→some¬(y, x)*

(by (43), *no=all¬*, *not all=some¬*, and Substitution of equivalents)

(46) *all(z, D−x)∧all(z, y)→some(y, D−x)* (by (45) and Definition 2)

(47) *all(z, x)∧all(z, y)→some(y, x)* (i.e. *AAI-3*, by (46) and Replacement rule)

5. Conclusion and Future Work

The main work and conclusions of this paper are as follows: (1) with the help of the definitions of three negative quantifiers of Aristotelian quantifiers (i.e. *all*, *no*, *some* and *not all*), the symmetry of *no* and *some*, and some basic inference rules in propositional logic, one can deduce the remaining 23 valid syllogisms only from the syllogism *OAO-3.* (2) there is reducible relations between/among different forms and different figures of valid traditional syllogisms. (3) these reducible relations actually reflect the transformation relations between the monotonicity of the four Aristotelian quantifiers. These relations once again exemplifies and highlights the dialectical materialist viewpoint that 'things are universally connected'.

This paper provides a simple and clear research paradigm for other kinds of syllogisms, such as generalized syllogisms and modal syllogisms. How to deal with them needs further study.

Acknowledgements

This work was supported by the National Social Science Foundation of China under Grant No.18ZDA032.

Reference

[1] G. Patzig, 1969, Aristotle's Theory of the Syllogism, J. Barnes (trans.), Dordrecht: D. Reidel.

[2] J. Łukasiewicz, 1957. Aristotle's Syllogistic from the Standpoint of Modern Formal Logic, Oxford: Clarendon Press.

[3] L. S. Moss, 2008, "Completeness theorems for syllogistic fragments", in F. Hamm and S. Kepser (eds.), Logics for Linguistic Structures, Berlin: Mouton de Gruyter, 143-173.

[4] D. Westerståhl, 1989, "Aristotelian syllogisms and generalized quantifiers", Studia Logica, Vol. XLVII, 4: 577-585.

[5] L. S. Moss, 2010, "Syllogistic logics with verbs", Journal of Logic and Computation, 20(4): 947 -967.

[6] P. Murinová P, and V. Novák, 2012, "A formal theory of generalized intermediate syllogisms", Fuzzy Sets and Systems, 186: 47-80.

[7] J. Endrullis, and L. S. Moss, 2015, "Syllogistic logic with 'Most'", in V. de Paiva et al. (eds.), Logic, Language, Information, and Computation, 124-139.

[8] F. Johnson, 2004, "Aristotle's modal syllogisms", Handbook of the History of Logic, 1: 247- 308.

[9] Xiaojun Zhang, 2020, "Reducible relations between/among Aristotle's Modal Syllogisms", SCIREA Journal of Computer, 5(1): 1-33.

[10] Xiaojun Zhang, 2020, "Screening out all valid Aristotelian modal syllogisms", Applied and Computational Mathematics, 8(6): 95-104.

[11] J. Corcoran, 1972, "Completeness of an ancient logic", Journal of Symbolic Logic, 37(2): 696- 702.

[12] J. van Benthem, 1984, "Questions about quantifiers", Journal of Symbol Logic, 49(2): 443-466.

[13] D. Westerståhl, 2007, "Quantifiers in formal and natural languages", in D. M. Gabbay and F. Guenthner (eds.), Handbook of Philosophical Logic, 14: 227-242.

[14] Beihai Zhou, Qiang Wang, Zhi Zheng., 2018, "Aristotle's division lattice and Aristotelian logic", Logic research, 2: 2-20. (in Chinese)

[15] Xiaojun Zhang, 2018, “Axiomatization of Aristotelian syllogistic logic based on generalized quantifier theory”, Applied and Computational Mathematics, 7(3): 167-172.

[16] Xiaojun Zhang, Sheng Li, 2016, “Research on the formalization and axiomatization of traditional syllogisms”, Journal of Hubei University (Philosophy and social sciences), 6: 32-37. (in Chinese)

[17] Mengyao Huang, Xiaojun Zhang., 2020, “Assertion or rejection of Łukasiewicz's assertoric syllogism system ŁA”, Journal of Chongqing University of Science and Technology (Social Sciences Edition), 2: 10-18. (in Chinese)

[18] Xiaojun Zhang, 2020, Research on Extended Syllogism for Natural Language Information Processing, Beijing: Science Press. (in Chinese)

[19] Xiaojun Zhang, 2014, A Study of Generalized Quantifier Theory, Xiamen: Xiamen University Press.

[20] S. Peters, and D. Westerståhl, 2006, Quantifiers in Language and Logic, Oxford: Claredon Press.