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The Remaining 23 Valid Aristotelian Syllogisms can be Deduced only from the Syllogism *IAI-3*

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Abstract

Syllogism reasoning is a common and important form of reasoning in natural language and logic. This paper shows that the remaining 23 valid syllogisms can be deduced merely from the syllogism *IAI-3* by making the best of propositional logic and generalized quantifier theory, so as to achieve the goal of deeply discussing the reducible relations between the syllogism *IAI-3* and the other syllogisms. More specifically, on the basis of formalizing syllogisms, this paper makes full of rules of deduction in classical propositional logic, the definitions of outer and inner negative quantifiers of Aristotelian quantifiers, and the symmetry of Aristotelian quantifiers *no* and *some* in generalized quantifier theory, and then establishes a concise formalized axiom system for Aristotelian syllogistic logic. This innovative research not only shows that formalized logic has the characteristics of structuralism, but also provides a concise and general mathematical paradigm for studying other syllogistic logics, and also provides theoretical support for knowledge and information processing in artificial intelligence and computer science.

Keywords: Aristotelian syllogisms; Aristotelian quantifiers; reducible relations; negative quantifiers; symmetry of quantifiers

1. Introduction

Syllogistic reasoning is a common and important form of reasoning in natural language and logic [1-3], which has been widely studied and plays an important role in logic from Aristotle on onwards[4-10]. There are many kinds of syllogisms, such as Aristotelian syllogisms[6], generalized syllogisms[3-5], modal syllogisms [12-13], rational syllogisms [14], and so on. This paper focuses on the study of Aristotelian syllogisms. And in the following, unless otherwise specified, syllogism refers to Aristotelian syllogisms.

Aristotelian syllogisms involve sentences of the following four forms: *all x*s are *y, no x*s are *y, some x*s are *y,* and *not all x*s are *y,* and these four forms of sentences are referred to as proposition *A*, *E*, *I* and *O* respectively, in which *all*, *no*, *some* and *not all* are called Aristotelian quantifiers [12, 15]. It can be seen that Aristotelian syllogistic logic mainly studies the semantic properties and reasoning properties of the four Aristotelian quantifiers[16-17]. It is widely known that there are 256 kinds of Aristotelian syllogisms, and only just 24 kinds of syllogisms are valid among them [18-19].

In previous studies such as Łukasiewicz [20], Shushan[21], Xiaojun and Sheng [18], Xiaojun [22-23] and Beihai [24], at least two valid syllogisms were used as basic axioms when deriving all of the other valid syllogisms. While this paper only takes one syllogism (that is, *IAI-3*) as the reasoning basis in order to deduce all of the remaining 23 valid syllogisms, and then establishes a simplified formal axiom system for Aristotelian syllogistic logic.

Specifically, by making full use of the deductive rules of propositional logic [25], generalized quantifier theory [15], the transformation relations between Aristotelian quantifiers and their inner and outer negation quantifiers [18], and the symmetry of *some* and *no* [26], the remaining 23 valid syllogisms are derived from the valid syllogism *IAI-3*, so as to achieve the goal of deeply discussing the reducible relations between the syllogism *IAI-3* and the other syllogisms.

2. Formal Aristotelian Syllogism Logic

To specify a formal system we require: (1) an alphabet of symbols, called primitive symbols; (2) a set of finite strings of these symbols, called well-formed formulas; (3) a set of well-formed formulas as the basis of reasoning, called basic axioms; (4) a finite set of 'rules of deduction', used to deduce the theorems in the system.

2.1 Primitive Symbols

(1) lexical variables: x, y, z

(2) negative operator: $\neg$

(3) implication operator: $\rightarrow$

(4) quantifier: *some*

(5) brackets: (,)

The other operators such as conjunctional operator $\wedge$ and biconditional $\leftrightarrow$ can be defined by $\neg$ and $\rightarrow$ as usual, that is, $(p\wedge q) =_{\mathrm{def}} \neg(p\rightarrow\neg q)$, and $(p\leftrightarrow q) =_{\mathrm{def}} ((p\rightarrow q)\wedge(q\rightarrow p))$.

2.2 Formation Rules

(1) If Q is a quantifier, x and y are lexical variables, then $Q(x, y)$ is a well-formed formula.

(2) If p and q are well-formed formulas, then $p\rightarrow q$ are well-formed formulas.

(3) Only the formulas obtained through (1) and (2) are well-formed formulas.

For example, some*(x, y)*, and some*(x, y)→¬some(y, z)* are well-formed formulas, which read respectively as 'some *x*s are *y*', and as 'if some *x*s are *y*, then that some *y*s are *z* is false'. Others are similar. And it is can be seen that Aristotelian syllogisms merely contain the following four forms of categorical propositions, that is, *all(x, y)*, *no(x, y*), *some(x, y)* and *not all(x, y)*.

In this paper, $\vdash p$ represents a well-formed formula that can be derived from basic axioms and rules of deduction or it is a basic axiom or a fact. For example, the syllogism *IAI-3* is a basic axiom, and denoted as $\vdash$ *some(y, z)→(all(y, x)→some(x, z))*. The other notations are similar.

2.3 Basic Axioms

(1) A1: if p is a valid formula in propositional logic, then $\vdash p$.

(2) A2 (that is, the syllogism *IAI-3*): $\vdash$ *some(y, z)→(all(y, x)→some(x, z))*.

2.4 Rules of Deduction

Aristotelian syllogistic logic is obtained by extending classical propositional logic, and then the following rules of deduction in the former are also applicable in the latter. Let p, q, r and s be well-formed formulas.

(1) Rule 1 (Antecedent interchange): From $\vdash (p\rightarrow(q\rightarrow r))$ infer $\vdash (q\rightarrow(p\rightarrow r))$.

(2) Rule 2 (Subsequent weakening): From $\vdash (p\rightarrow(q\rightarrow r))$ and $\vdash (r\rightarrow s)$ infer $\vdash (p\rightarrow(q\rightarrow s))$.

(3) Rule 3 (Rule *A* of anti-syllogism): From $\vdash (p\rightarrow(q\rightarrow r))$ infer $\vdash (p\rightarrow(\neg r\rightarrow\neg q))$.

(4) Rule 4 (Rule *B* of anti-syllogism): From $\vdash (p\rightarrow (q\rightarrow r))$ infer $\vdash (q\rightarrow(\neg r\rightarrow\neg p))$.

2.5 Related Definitions and Facts

According to generalized quantifier theory ([13], [19]), the four Aristotelian quantifiers (i.e. *all*, *no*, *some* and *not all*) are special cases of type <1, 1> generalized quantifiers, and any generalized quantifier has the following three kinds of negative quantifiers: inner negative, outer negative and dual negative quantifiers. In this paper, D stands for the domain of lexical variables, and Q for a type <1, 1> generalized quantifier, $\neg Q$, $Q\neg$ $\neg Q\neg$ for the outer, inner and dual negative quantifier of Q, respectively.

Definition 1 (inner negative quantifier): $Q\neg(x, y) =_{\text{def}} \text{Q}(x, D-y)$;

By the definition, *all*¬=*no*, *no*¬=*all*, *some*¬=*not all*, *not all*¬=*some*, so we can say that *all* and *no*, *some* and *not all* are inner negations each other. From this we can get Fact 1.

Fact 1 (inner negation)

(1) $\vdash$ *all(x, y)*↔*no*¬*(x, y)*; (2) $\vdash$ *no(x, y)*↔*all*¬*(x, y)*;

(3) $\vdash$ *some(x, y)*↔*not all*¬*(x, y)*; (4) $\vdash$ *not all(x, y)*↔*some*¬*(x, y)*.

Definition 2 (outer negative quantifier): $\neg Q(x, y) =_{\text{def}}$ It is not that $Q(x, y)$.

To be specific, ¬*all*=*not all*, ¬*not all*=*all*, ¬*no*=*some*, ¬*some*=*no*, thus we can say that *all* and *not all*, *some* and *no* are outer negative each other. From this we can obtain Fact 2.

Fact 2 (outer negation):

(1) $\vdash$ *all(x, y)*↔¬*not all(x, y)*; (2) $\vdash$ *not all(x, y)*↔¬*all(x, y)*;

(3) $\vdash$ *some(x, y)*↔¬*no(x, y)*; (4) $\vdash$ *no(x, y)*↔¬*some(x, y)*.

Definition 3 (dual quantifier): $\neg Q\neg(x, y) =_{\text{def}}$ It is not that $Q(x, D-y)$.

More specifically, $\neg all\neg = some$, $\neg some\neg = all$, $\neg no\neg = not\ all$, $\neg not\ all\neg = no$, thus we can say that *all* and *some*, *no* and *not all* are dual negations each other.

By Definition 1-3, it is can be seen that any one of the Aristotelian quantifier can define the other three Aristotelian quantifiers, so any one of the four Aristotelian quantifiers (say, *some*) can be used as the initial quantifier. In this paper, the other three Aristotelian quantifiers can be respectively defined by *some* as follows : $all =_{\text{def}} \neg some\neg$, $no =_{\text{def}} \neg some$, $not\ all =_{\text{def}} some\neg$.

Definition 4 (symmetry): Let D be the domain of lexical variables, and Q be a type <1, 1> quantifiers, Q is symmetric if and only if, for all $x, y \in D$, $Q(x, y) \leftrightarrow Q(y, x)$.

For example, (a) 'Some men are gynecologists', (b) 'Some gynecologists are men', and the two sentences imply each other, and *some* is symmetric by Definition 4. Similarly, (c) 'no pig is a dog', (d) 'no dog is a pig', and the two sentences imply each other, so *no* is symmetric. And then we have Fact 3.

Fact 3 (symmetry of *some* and *no*):

(1) (symmetry of *some*): $\vdash some(x, y) \leftrightarrow some(y, x)$; (2) (symmetry of *no*): $\vdash no(x, y) \leftrightarrow no(y, x)$.

Fact 4 (assertoric subalternations): (1) $\vdash no(x, y) \rightarrow not\ all(x, y)$; (2) $\vdash all(x, y) \rightarrow some(x, y)$.

Fact 4 is a basic fact of predicate logic or generalized quantifier theory, and can be easily deduced from the above basic axioms and rules of deduction.

3. Reducible Relations between/among Syllogisms Based on the Syllogism *IAI-3*

In the following theorem 1, *IAI-3*⇒*IAI-4* means that the validity of syllogism *IAI-4* can be deduced from the validity of syllogism *IAI-3* (that is, the basic axiom A2). In other words, there is a reducible relation between the two Aristotelian syllogisms. Others are similar. The following theorem characterizes the reducible relations between the syllogism *IAI-3* and the other 23 valid syllogisms.

Theorem 1: The remaining 23 valid syllogisms can be deduced just from the syllogism *IAI-3*.

（1）*IAI-3*⇒*IAI-4*

（2）*IAI-3*⇒*AII-3*

（3）$IAI\text{-}3 \Rightarrow AII\text{-}3 \Rightarrow AII\text{-}1$

（4）$IAI\text{-}3 \Rightarrow EIO\text{-}2$

（5）$IAI\text{-}3 \Rightarrow EIO\text{-}2 \Rightarrow EIO\text{-}1$

（6）$IAI\text{-}3 \Rightarrow EIO\text{-}2 \Rightarrow EIO\text{-}4$

（7）$IAI\text{-}3 \Rightarrow EIO\text{-}2 \Rightarrow EIO\text{-}4 \Rightarrow EIO\text{-}3$

（8）$IAI\text{-}3 \Rightarrow EAE\text{-}1$

（9）$IAI\text{-}3 \Rightarrow EAE\text{-}1 \Rightarrow EAE\text{-}2$

（10）$IAI\text{-}3 \Rightarrow EAE\text{-}1 \Rightarrow AEE\text{-}4$

（11）$IAI\text{-}3 \Rightarrow EAE\text{-}1 \Rightarrow AEE\text{-}4 \Rightarrow AEE\text{-}2$

（12）$IAI\text{-}3 \Rightarrow EAE\text{-}1 \Rightarrow EAO\text{-}1$

（13）$IAI\text{-}3 \Rightarrow EAE\text{-}1 \Rightarrow EAO\text{-}1 \Rightarrow EAO\text{-}2$

（14）$IAI\text{-}3 \Rightarrow EAE\text{-}1 \Rightarrow EAO\text{-}1 \Rightarrow EAO\text{-}2 \Rightarrow AAI\text{-}3$

（15）$IAI\text{-}3 \Rightarrow OAO\text{-}3$

（16）$IAI\text{-}3 \Rightarrow OAO\text{-}3 \Rightarrow AAA\text{-}1$

（17）$IAI\text{-}3 \Rightarrow OAO\text{-}3 \Rightarrow AAA\text{-}1 \Rightarrow AAI\text{-}1$

（18）$IAI\text{-}3 \Rightarrow OAO\text{-}3 \Rightarrow AAA\text{-}1 \Rightarrow AAI\text{-}1 \Rightarrow AAI\text{-}4$

（19）$IAI\text{-}3 \Rightarrow OAO\text{-}3 \Rightarrow AAA\text{-}1 \Rightarrow AAI\text{-}1 \Rightarrow EAO\text{-}3$

（20）$IAI\text{-}3 \Rightarrow OAO\text{-}3 \Rightarrow AAA\text{-}1 \Rightarrow AAI\text{-}1 \Rightarrow EAO\text{-}3 \Rightarrow EAO\text{-}4$

（21）$IAI\text{-}3 \Rightarrow OAO\text{-}3 \Rightarrow AAA\text{-}1 \Rightarrow AAI\text{-}1 \Rightarrow AEO\text{-}2$

（22）$IAI\text{-}3 \Rightarrow OAO\text{-}3 \Rightarrow AAA\text{-}1 \Rightarrow AAI\text{-}1 \Rightarrow AEO\text{-}2 \Rightarrow AEO\text{-}4$

（23）$IAI\text{-}3 \Rightarrow OAO\text{-}3 \Rightarrow AOO\text{-}2$

Proof:

[1] $\vdash$ $some(y, z) \to (all(y, x) \to some(x, z))$ (i. e. *IAI-3*, that is axiom A2)

[2] $\vdash$ $some(y, z) \leftrightarrow some(z, y)$ (by symmetry of *some*)

[3] ⊢ *some(z, y)→(all(y, x)→some(x, z))* (i.e. *IAI-4*, by [1], [2])

[4] ⊢ *some(x, z)↔some(z, x)* (by symmetry of *some*)

[5] ⊢ *all(y, x)→(some(y, z)→some(z, x))* (i.e. *AII-3*, by [1], [4] and Rule 1)

[6] ⊢ *all(y, x)→(some(z, y)→some(z, x))* (i.e. *AII-1*, by [2], [5])

[7] ⊢ *some(y, z)→(¬some(x, z)→¬all(y, x))* (by [1] and Rule 3)

[8] ⊢ *no(x, z)→(some(y, z)→not all(y, x))* (i.e. *EIO-2*, by [7], Rule 1and Fact 2)

[9] ⊢ *no(x, z)↔no(z, x)* (by symmetry of *no*)

[10] ⊢ *no(z, x)→(some(y, z)→not all(y, x))* (i.e. *EIO-1*, by [8], [9])

[11] ⊢ *no(x, z)→(some(z, y)→not all(y, x))* (i.e. *EIO-4*, by [2], [8])

[12] ⊢ *no(z, x)→(some(z, y)→not all(y, x))* (i.e. *EIO-3*, by [9], [11])

[13] ⊢ *all(y, x)→(¬some(x, z)→¬some(y, z))* (by [1] and Rule 4)

[14] ⊢ *no(x, z)→(all(y, x)→no(y, z))* (i.e. *EAE-1*, by [13] , Rule 1 and Fact 2)

[15] ⊢ *no(z, x)→(all(y, x)→no(y, z))* (i.e. *EAE-2*, by [9], [14])

[16] ⊢ *no(y, z)↔no(z, y)* (by symmetry of *no*)

[17] ⊢ *all(y, x)→(no(x, z)→no(z, y))* (i.e. *AEE-4*, by [14], [16] and Rule 1)

[18] ⊢ *all(y, x)→(no(z, x)→no(z, y))* (i.e. *AEE-2*, by [9], [17])

[19] ⊢ *no(y, z)→not all(y, z)* (by Fact 4)

[20] ⊢ *no(x, z)→(all(y, x)→not all(y, z))* (i.e. *EAO-1*, by [14], [19] and Rule 2)

[21] ⊢ *no(z, x)→(all(y, x)→not all(y, z))* (i.e. *EAO-2*, by [9], [20])

[22] ⊢ *all(y, x)→(¬not all(y, z)→¬no(z, x))* (by [21] and Rule 4)

[23] ⊢ *all(y, x)→(all(y, z)→some(z, x))* (i.e. *AAI-3*, by [22]and Fact 2)

[24] ⊢ *not all¬(y, z)→(all(y, x)→not all¬(x, z))* (by [1] and Fact 1)

[25] ⊢ *not all(y, D–z)→(all(y, x)→not all(x, D–z))* (by [24] and Definition 1)

[26] ⊢ *not all(y, z)→(all(y, x)→not all(x, z))* (i.e. *OAO-3*, by [25])

[27] ⊢ *all(y, x)→(¬not all(x, z)→¬not all(y, z))* (by [26] and Rule 4)

[28] ⊢ *all(x, z)→(all(y, x)→all(y, z))* (i.e. *AAA-1*, by [27] and Fact 2)

[29] ⊢ *all(y, z)→some(y, z)* (by Fact 4)

[30] $\vdash$ *all(x, z)→(all(y, x)→some(y, z))* (i.e. *AAI-1*, by [28], [29] and Rule 2)

[31] $\vdash$ *all(y, x)→(all(x, z)→some(z, y))* (i.e. *AAI-4*, by [2], [30] and Rule 1)

[32] $\vdash$ *all(y, x)→(¬some(y, z)→¬all(x, z))* (by [30] and Rule 4)

[33] $\vdash$ *no(y, z)→(all(y, x)→not all(x, z))* (i.e. *EAO-3*, by [32], Rule 1 and Fact 2)

[34] $\vdash$ *no(z, y)→(all(y, x)→not all(x, z))* (i.e. *EAO-4*, by [16] and [33])

[35] $\vdash$ *all(x, z)→(¬some(y, z)→¬all(y, x))* (by [30] and Rule 3)

[36] $\vdash$ *all(x, z)→(no(y, z)→not all(y, x))* (i.e. *AEO-2*, by [35] and Fact 2)

[37] $\vdash$ *all(x, z)→(no(z, y)→not all(y, x))* (i.e. *AEO-4*, by [16], [36])

[38] $\vdash$ *not all(y, z)→(¬not all(x, z)→¬all(y, x))* (by [26] and Rule 3)

[39] $\vdash$ *all(x, z)→(not all(y, z)→not all(y, x))* (i.e. *AOO-2*, by [38], Rule 1 and Fact 2)

According to the above proof sequences and proof process, it can be concluded that different syllogisms have the above reducible relations as in theorem 1. Due to different proving paths, the reduction process between different syllogisms is not unique. These reducible relations have eloquently proved the dialectical materialism view that 'things are universally connected'.

4. Conclusion

This paper shows that the remaining 23 valid syllogisms can be deduced only just from the syllogism *IAI-3* by making the best of propositional logic and generalized quantifier theory. More specifically, on the basis of formalizing syllogisms, this paper makes full of rules of deduction in propositional logic, the definitions of outer and inner negative quantifiers of Aristotelian quantifiers, and the symmetry of Aristotelian quantifiers *no* and *some* generalized quantifier theory, then establishes a concise formalized axiom system for Aristotelian syllogistic logic.

This innovative research not only shows that formalized logic has the characteristics of structuralism, but also provides a concise and general mathematical paradigm for studying other syllogistic logics, and also provides theoretical support for knowledge and information processing in artificial intelligence and computer science.

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Reference

[1] N. Chater and M. Oaksford, the probability Heuristics model of syllogistic reasoning, Cognitive Psychology, Vol. 38, 1999, pp.191-258.

[2] G. Patzig, Aristotle's Theory of the Syllogism, J. Barnes (trans.), Dordrecht: D. Reidel, 1969.

[3] L. S. Moss, Syllogistic logics with verbs, Journal of Logic and Computation, Vol. 20, No. 4, 2010, pp.947-967.

[4] P. Murinová P and V. Novák, A formal theory of generalized intermediate syllogisms, Fuzzy Sets and Systems, Vol.186, 2012, pp. 47-80.

[5] J. Endrullis, and L. S. Moss, Syllogistic logic with 'Most', in V. de Paiva et al. (eds.), Logic, Language, Information, and Computation, 2015, pp.124-139.

[6] J. Łukasiewicz, Aristotle's Syllogistic from the Standpoint of Modern Formal Logic, Clarendon Press, Oxford, 1951.

[7] J. N. Martin, Aristotle's natural deduction reconsidered, History and Philosophy of Logic, Vol. 18, No.1 1997, pp.1-15.

[8] L. S. Moss, Completeness theorems for syllogistic fragments, in F. Hamm and S. Kepser (eds.), Logics for Linguistic Structures, Mouton de Gruyter, Berlin, 2008, pp.143–173.

[9] J. van Benthem, Questions about quantifiers, Journal of Symbol Logic, Vol. 49, No.2, 1984, pp.443-466.

[10] D. Westerståhl, Aristotelian syllogisms and generalized quantifiers, Studia Logica, Vol. XLVII, No.4, 1989, pp.577-585.

[11] F. Johnson, Models for modal syllogisms, Notre Dame Journal of Formal Logic, 1989, Vol.30, pp.271-284.

[12] F. Johnson, Aristotle's modal syllogisms, in: D. M. Gabbay and J. Woods (eds.), Handbook of the History of Logic, Amsterdam: Elsevier, Vol. I, 2004, pp.247-338.

[13] N. Ivanov, and D. Vakarelov, A system of relational syllogistic incorporating full Boolean reasoning, Journal of Logic, Language and Information, Vol. 21, 2012, pp. 433-459.

[14] S. Peters, and D. Westerståhl, Quantifiers in Language and Logic, Claredon Press, Oxford, 2006.

[15] Xiaojun Zhang, A Study of Generalized Quantifier Theory, Xiamen University Press, 2014. (in Chinese)

[16] D. Westerståhl, Quantifiers in formal and natural languages, in D. M. Gabbay and F. Guenthner (eds.), Handbook of Philosophical Logic, Vol. 14, 2007, pp. 227-242.

[17] Xiaojun Zhang, Sheng Li. Research on the formalization and axiomatization of traditional syllogisms, Journal of Hubei University (Philosophy and Social Sciences), No. 6, 2016, pp. 32-37. (in Chinese)

[18] Yijiang Hao. Formal research on discourse reasoning in natural language. Journal of Hunan University of Science and Technology (Social Sciences Edition), 2016 (1): 33-37. (in Chinese)

[19] Łukasiewicz J. Aristotle's Syllogistic: From the Standpoint of Modern Formal Logic. Second edition, Oxford: Clerndon Press, 1957.

[20] Shushan Cai. A formal system of Aristotle's syllogism different from that of Łukasiewicz. Philosophical research, 1988(4): 33- 41. (in Chinese)

[21] Mengyao Huang, Xiaojun Zhang. Assertion or rejection of Łukasiewicz's assertoric syllogism system ŁA. Journal of Chongqing University of Science and Technology (Social Sciences Edition), 2020 (2): 10-18. (in Chinese)

[22] Xiaojun Zhang. Axiomatization of Aristotelian syllogistic logic based on generalized quantifier theory. Applied and Computational Mathematics. Vol. 7, No. 3, 2018, pp. 167-172.

[23] Beihai Zhou, Qiang Wang, Zhi Zheng. Aristotle's division lattice and Aristotelian logic. Logic research, 2018 (2): 2-20. (in Chinese)

[24] A. G. Hamilton, Logic for Mathematicians, Cambridge: Cambridge University Press, 1978.

[25] Xiaojun Zhang. Research on Extended Syllogism for Natural Language Information Processing. Beijing: Science Press, 2020. (in Chinese)