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Experimental determination of normal modes and damage detection of structures using additional mass

Vasyl Redchenko [1]
[1] State Road Research Institute" ("DerzhdorNDI" SE), Ukraine
Doctor of Technical Sciences (D.Sc.), ORCID: 0000-0001-5938-1467
Email: rvp50@meta.ua

Abstract

The article presents a method of experimentally determining the ordinates of the natural forms of oscillations of bridges. The method is to add a small amount of mass to the structure and measure the change in natural frequencies. The theoretical rationale and results of a number of experiments are outlined. The accuracy of the determination of the ordinates of the natural forms of oscillations makes it possible to detect not only the appearance of a certain defect, but also its position. this method implementation requires a small number of primary transducers (sensors); so, to determine individual natural forms only one sensor is needed. You can use all types of sensors that enable obtaining vibrorecords of free vibrations of the structure.

Keywords

bridges, dynamic test, natural forms, defect, technical condition

Introduction

Keeping the bridges in a working state is an important and responsible task, especially in a situation where the number of bridges in need of repair is constantly increasing. Timely detection of defects in the construction of the bridge makes it possible to prevent an extensive development of defects and perform repairs at lower expense. Monitoring technical condition of bridges by their dynamic characteristics that are defined by dynamic tests takes important place among the other current methods of diagnostics and is increasingly spread in domestic practice [1,2,3].

Issue

Determining the actual vibrations, forms and decrements of natural vibrations of a bridge superstructure is one of the main tasks to be solved during dynamic testing. Among the listed three characteristics of natural vibrations determining, determining the form of vibrations is the most difficult task during dynamic testing. This implies a precise definition of the form of vibrations in a graph of relative amplitudes, not an approximate line of the form that shows only the relative phase position of individual parts of the structure. Precise knowledge of the forms of natural vibrations and their changes is necessary to determine the location of the defect by vibrodiagnostics [5,6]. In this case, to determine changes in the form of vibrations, the measurements of deflections of 0.001 mm dimension or less should be performed. And it is very difficult to practically implement and it involves significant error. Classical methods for determining the forms of natural vibrations by measuring the amplitude of free or forced vibrations [7] do not provide the desired degree of accuracy, especially in case of concentration of natural vibrations. To solve this issue, the studies were performed which main results are outlined in this article.

Theoretical study

A span beam is the simplest model of a bridge superstructure. Natural vibration specific of the beam is the ratio of two integrated functions, one of which depends on the function of rigidity and the other depends on the function of mass [7]:

$$\omega_i{}^2 = \frac{\int_0^L I(x)[f_i{}''(x)]^2\,dx}{\int_0^L m(x)[f_i(x)]^2\,dx}, \tag{1}$$

where ω_i – angular frequency of *i* form of natural vibrations; *I(x)* – function of rigidity; *m(x)* – function of mass; $f_i(x)$ – normalized function of *i* form of natural vibrations.

Assuming that at a small area Δl with the coordinate $x=a$ mass is changed to $M_a = \Delta m \Delta l$. Changed frequency, if ignoring the change in the form of vibrations, will be expressed as follows:

$$(\omega_i - \Delta\omega_i(a))^2 = \frac{\int_0^L I(x)[f_i{}''(x)]^2\,dx}{\int_0^L m(x)[f_i(x)]^2\,dx + M_a[f_i(a)]^2} \tag{2}$$

The ratio of changed frequency to the initial one will be derived like the ratio of functions (2) and (1)

$$\frac{(\omega_i - \Delta\omega_i(a))^2}{\omega_i{}^2} = \frac{\int_0^L m(x)[f_i(x)]^2\,dx}{\int_0^L m(x)[f_i(x)]^2\,dx + M_a[f_i(a)]^2}. \tag{3}$$

Ignoring the values of smaller order, after transformations, we obtain an expression for the relative change of frequency

$$\frac{\Delta\omega_i(a)}{\omega_i} \approx \frac{M_a[f_i(a)]^2}{2\int_0^L m(x)[f_i(x)]^2\,dx} = C\cdot[f_i(a)]^2, \tag{4}$$

where C– is a constant of the investigated object.

As you can see, the function of the relative change of frequency depending on the coordinates of additional mass is similar to the square of a form of natural vibrations function. Thus, by performing measurements of frequencies of natural forms of vibrations in the presence of additional mass on the beam and without it, you can receive modules of amplitudes of natural forms of vibrations.

It should be remembered about the simplifications which were adopted at the derivation of these dependencies, especially the fact that the results will be satisfactory only for small changes when additional mass is at least ten times less than the mass of the beam. Since at the derived dependency the amplitude of the form is squared, its phase position cannot be derived from this expression, but, as mentioned above, it can be easily solved by classical methods.

The practical essence of the method is in consistent definition of changes of natural frequencies of the structure when additional mass is placed on it at different points and corresponding definition of ordinates of normalized form of natural vibrations by the formula (4). Since defects occurrence in the structure will affect its form of natural vibrations, the location of the defect can be determined by its change [6].

Experimental studies

A) A beam with distributed mass. For a full-scale experiment, a steel box beam of 40x80 mm with wall thickness of 2 mm was selected. The length of the beam was 4 m, an average linear weight – 3,62 kg / m. The beam was installed with a wider side on one moving (Support 1) and another fixed (Support 2) bearing parts located at a distance of 5 cm from the ends of the beam, therefore, the design span made 3,9 m. The beam was conditionally divided into 10 parts (0,39 m each). Accordingly, the sections 1 ... 9 were defined in the span (Fig. 1). Excitatory pulse effect was applied at a distance of 0,49 m from Support 2 (1/8 of the beam's span length).

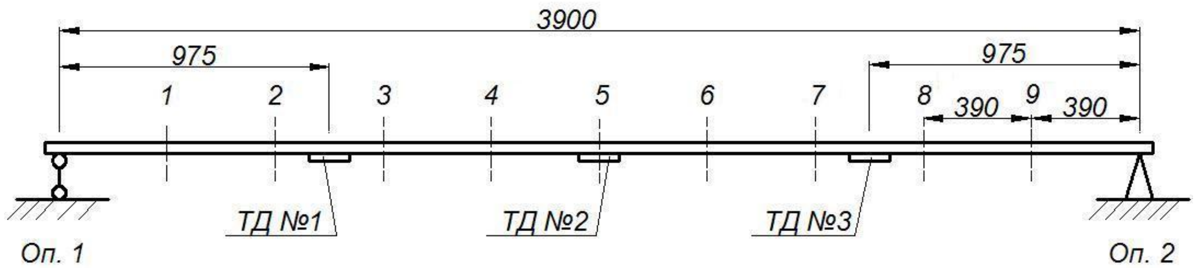


Fig.1. *Location of conditional sections and strain sensors on the beam*

Registration of the beam's response to excitation was implemented by measuring strains by strain sensors. Strain sensors were pasted along the longitudinal axis of the beam at its lower edge. Three strain sensors (SS) were used in total: №1, 2, 3 were installed in quarters along the beam's span. Registration of strains was conducted simultaneously from all three sensors using a hardware complex "Spider" with a discretization frequency of 200 Hz. The weight of 0,1 kg was used as additional mass which is about 0,7% of the beam's weight.

To determine with high accuracy the frequencies of the first three own forms of vertical vibrations of the beam, the specified algorithms of spectral analysis were used [8]. To reduce the effect of noise and other harmonics, the spectra composition method was also used [9]. This allowed determining the natural frequencies of the beam with accuracy to ± 0,02%. The results of determining the natural frequencies of the beam for the first three forms and their changes at the presence of additional mass in 9 points are presented in Table 1.

Table 1. The results of determining the natural frequencies for a series of additional points of additional mass application

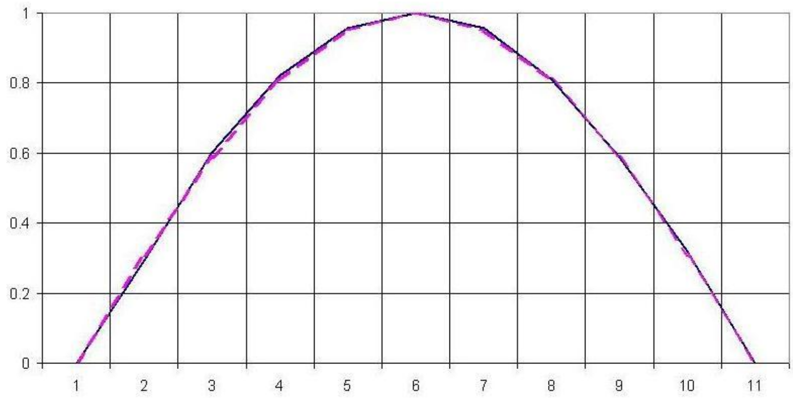
Point of application of additional mass 0,1 kg	The 1st form	Frequency change, %	The 2nd form	Frequency change, %	The 3rd form	Frequency change, %
Free vibrations	**8.811**	0.000	**34.442**	0.000	**76.365**	0.000
Point 1	8.806	0.057	34.351	0.264	76.038	0.428
Point 2	8.790	0.238	34.232	0.610	75.977	0.508
Point 3	8.772	0.443	34.238	0.592	76.349	0.021
Point 4	8.758	0.602	34.358	0.244	76.128	0.310
Point 5	8.753	0.658	34.439	0.009	75.875	0.642
Point 6	8.758	0.602	34.356	0.250	76.155	0.275
Point 7	8.773	0.431	34.235	0.601	76.341	0.031
Point 8	8.791	0.227	34.243	0.578	75.981	0.503
Point 9	8.805	0.068	34.365	0.224	75.983	0.500

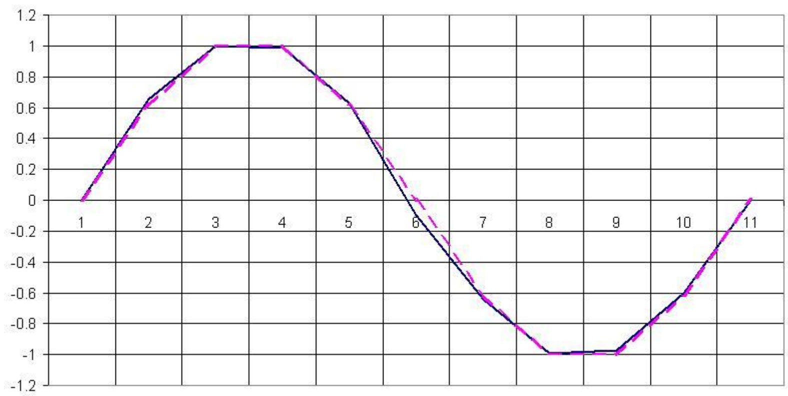
Considering the value of change in frequency of the natural form of vibrations and according to the expression (4), normalized modules of ordinates of this form of vibrations (in fractions relative to maximum ordinate of this form) were defined; the results are shown in Table 2. For comparison the ordinates of sinusoidal function modules are presented here, which are the calculated forms of natural vibrations of a mathematical model of the beam.

Table 2. Values of normalized ordinates of natural forms of vibrations basing on the results of the experiment and according to the calculation

Point on the beam	The 1st form		The 2nd form		The 3rd form	
	experimental	calculated	experimental	calculated	experimental	calculated
Point 1	0.294	0.309	0.658	0.618	0.816	0.809
Point 2	0.601	0.588	1.000	1.000	0.890	0.951
Point 3	0.821	0.809	0.985	1.000	0.181	0.309
Point 4	0.957	0.951	0.632	0.618	0.695	0.588
Point 5	1.000	1.000	0.095	0.000	1.000	1.000
Point 6	0.957	0.951	0.640	0.618	0.654	0.588
Point 7	0.809	0.809	0.993	1.000	0.220	0.309
Point 8	0.587	0.588	0.973	1.000	0.885	0.951
Point 9	0.321	0.309	0.606	0.618	0.883	0.809

Taking into account the phase position determined experimentally and for comparison, the calculated forms of natural vibrations are presented in graphs in Figure 2.





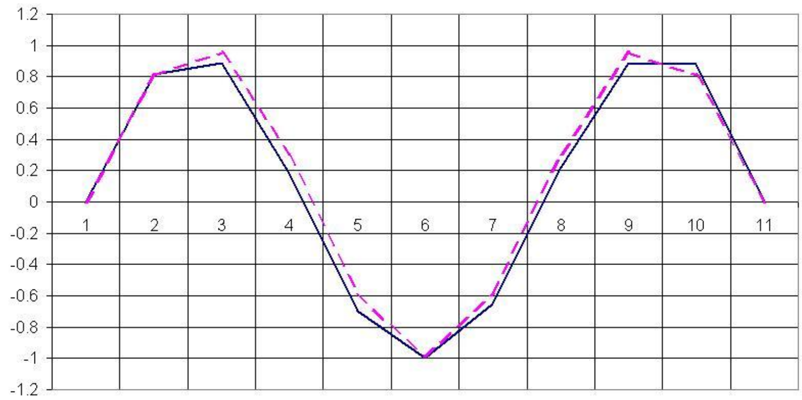


Fig.2. *The first three forms of natural vibrations: determined experimentally (solid line) and according to the calculations (dotted line)*

As you can see, the difference between the experimentally determined and calculated forms of natural vibrations is insignificant. In addition, it should be taken into account that in practice the actual forms of natural vibrations are not strictly sinusoidal; they are subject to the following impacts: presence of small consoles behind supports; resting on the bearing parts with the lower face and not along the axis of the beam; the value of ratio of the beam's height to its length (unlike mathematical lines), etc. That is why the existing differences in the lines of the highest forms of natural vibrations should not be taken fully as an inaccurate method.

B) A beam of concentrated mass in the middle of the span. For a full-scale experiment a steel box beam of 40x80 mm and wall thickness of 2 mm was used. The length of the beam was 4 m, an average linear weight - 3,2 kg / m., the design span is 3,9 m. The beam is conditionally divided into 10 parts (0,39 m each). Accordingly, the sections 1 ... 9 were defined in the span. The concentrated mass of 10,0 kg was fixed on the beam at mid-span (Fig. 3). Excitatory pulse effect was applied at a distance of 0,49 m from Support 2 (1/8 of the beam's span length).

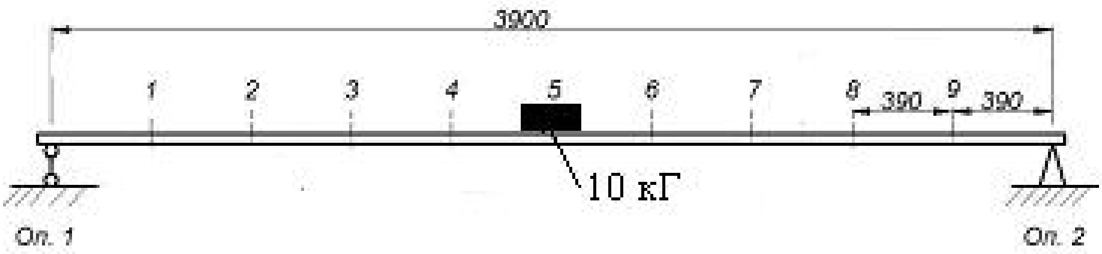


Figure 3. *Location of conditional sections and strain sensors on the beam*

Registration of the beam's response to excitation was implemented by measuring strains by strain sensors. In total, three strain sensors were used and installed in quarters along the beam's span. Registration of strains was conducted simultaneously from all three sensors using a hardware complex "Spider" with a discretization frequency of 200 Hz. The weight of 0,1 kg was used as additional mass which is about 0,4% of the beam's weight.

To determine with high accuracy the frequencies of the first three own forms of vertical vibrations of the beam, the specified algorithms of spectral analysis were used [8]. To reduce the effect of noise and other harmonics, the spectra assembly method was also used [9]. All this allowed determining the natural frequencies with accuracy to ± 0,02%. The results of determining the natural frequencies of the beam for the first three forms and their changes at the presence of additional mass in 9 points are presented in Table 3.

Table 3. The results of determining the natural frequencies for a series of points of application of additional mass

Point of application of additional mass 0,1 kg	The 1st form	Frequency change, %	The 2nd form	Frequency change, %	The 3rd form	Frequency change, %
Free vibrations	**5.613**	0.000	**34.450**	0.000	**60.633**	0.000
Point 1	5.6146	0.0293	34.5365	0.25104	60.8551	0.36635
Point 2	5.6183	0.0937	34.6636	0.62008	61.0015	0.6077
Point 3	5.6232	0.1815	34.6612	0.61302	60.7978	0.27175
Point 4	5.6265	0.24	34.5358	0.24916	60.6294	-0.006
Point 5	5.6284	0.2752	34.4568	0.0197	60.6880	0.09075
Point 6	5.6268	0.2459	34.5410	0.26404	60.6539	0.0345
Point 7	5.6229	0.1756	34.6674	0.63118	60.8352	0.33355
Point 8	5.6179	0.0878	34.6667	0.62892	60.9707	0.557
Point 9	5.6140	0.0176	34.5420	0.26706	60.8377	0.33765

Considering the value of change in frequency of the natural form of vibrations and according to the expression (5), normalized modules of ordinates of this form of vibrations were defined. Taking into account phase positions determined experimentally and for comparison, the calculated forms of natural vibrations in graphs are presented in Figure 4. Calculated forms were obtained using software "Lyra-9".

As you can see from the graphs, the difference between the experimentally determined and calculated forms of natural vibrations is insignificant. In addition, it should be taken into account that in practice the actual forms of natural vibrations are not strictly sinusoidal; they are subject to the following impacts: presence of small consoles behind supports; resting on the bearing parts with the lower face and not along the axis of the beam; the value of ratio of the beam's height to its length (unlike mathematical lines), etc. That is why the existing differences in the lines of the highest forms of natural vibrations should not be taken fully as an inaccurate method.

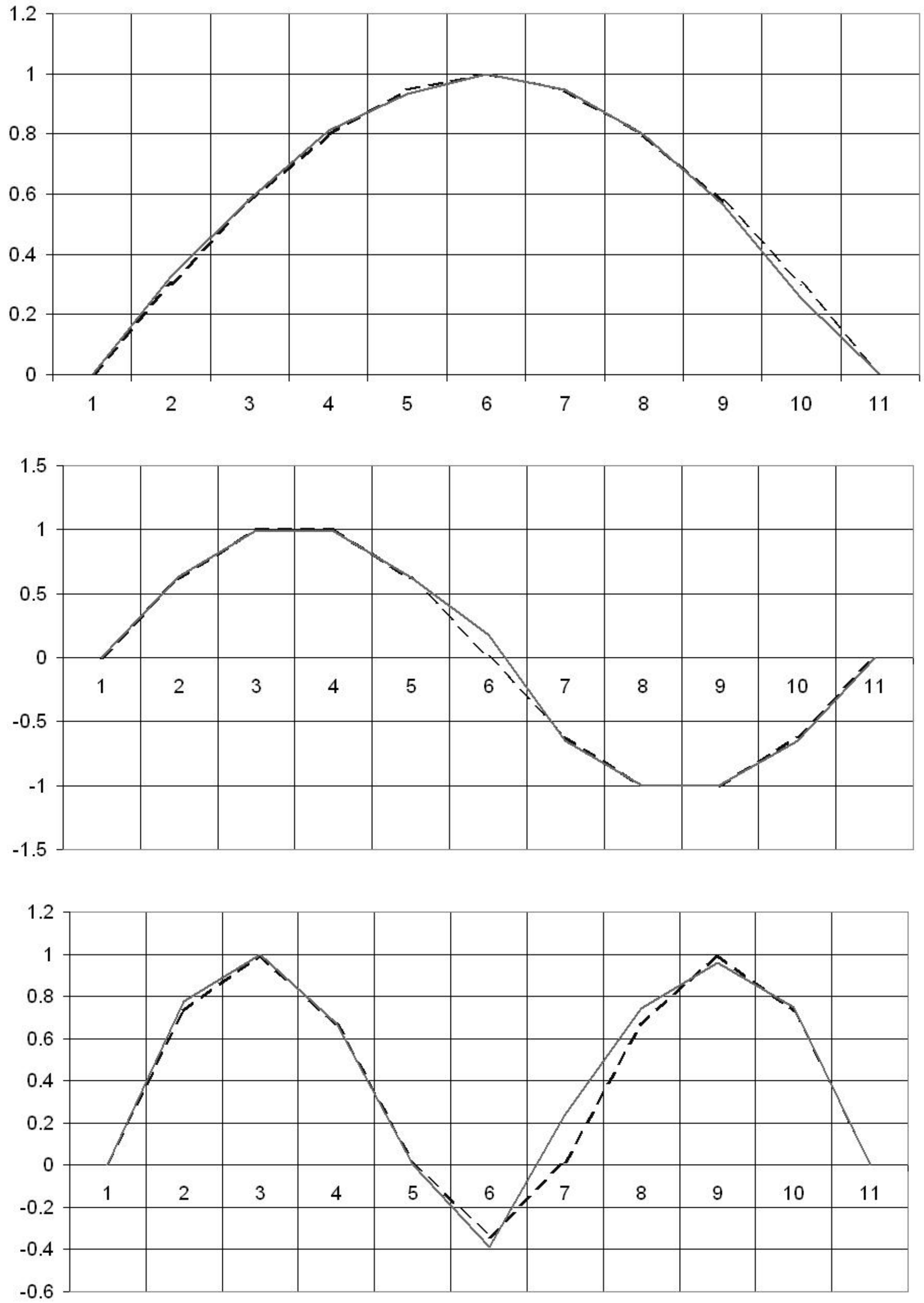


Figure 4. *The first three forms of natural vibrations: determined experimentally (solid line) and calculated (dotted line)*

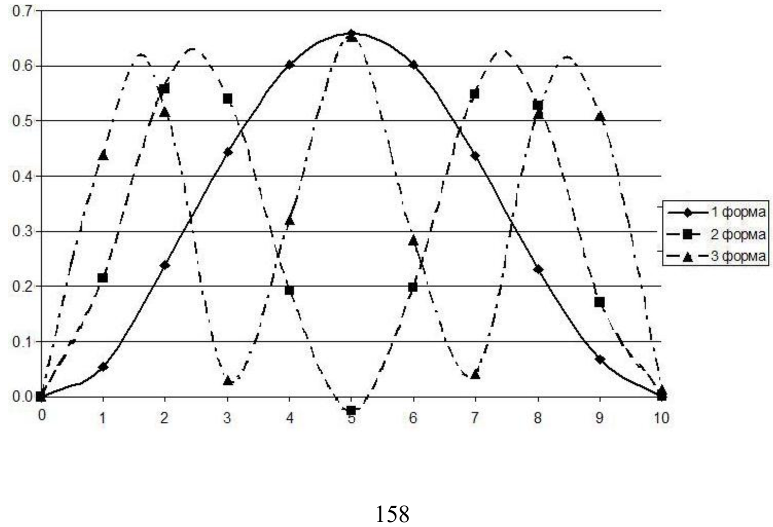
B) **Impact of defect on the forms of natural vibrations**. The experiment to determine the position of the defect consisted in the application of a real defect to the beam that was implemented in the form of a saw cut made on the top of the beam at the section 7-8. The saw cut was fulfilled in two stages: first, the saw cut of 1 mm depth was made, and then the next saw cut was made to a depth of 2 mm (at full thickness of the upper plate of the beam). Appearance of the saw cut led to a decrease in the frequency of natural vibrations of the beam and is reflected in Table 4.

Table 4. Changing the frequency of free vibrations with the appearance of the defect

Number of vibration form	Frequency before saw cut, Hz	Saw cut 1,0 mm,			Saw cut 2,0 mm,	
		Frequency after saw cut, Hz	Change in %		Frequency after saw cut, Hz	Change in %
The 1-st form	8,807	8,806	0,00011		8,694	1,28
The 2-nd form	34,502	34,477	0,00072		33,702	2,32
The 3-rd form	76,358	76,306	0,00068		75,659	0,92

As you can see, appearance of the defect led to the change of natural frequencies, but to determine the position of the defect, the change of forms of natural vibrations must be determined. To simplify the procedure of the expression (1), there is no need to determine the form of vibrations but the impact lines of additional mass on the change of natural frequencies should be used. At that, the initial impact lines should be available and the subsequent ones (as well as the initial forms of natural vibrations) which are determined during the next monitoring of the structure.

The initial impact lines of additional mass on the relative change of natural frequencies of the beam were obtained before performing a saw cut. After a series of tests on the beam with 2 mm of saw cut, the impact lines were received for the first three forms of vibrations that are together with the initial impact lines are shown in Fig. 5.



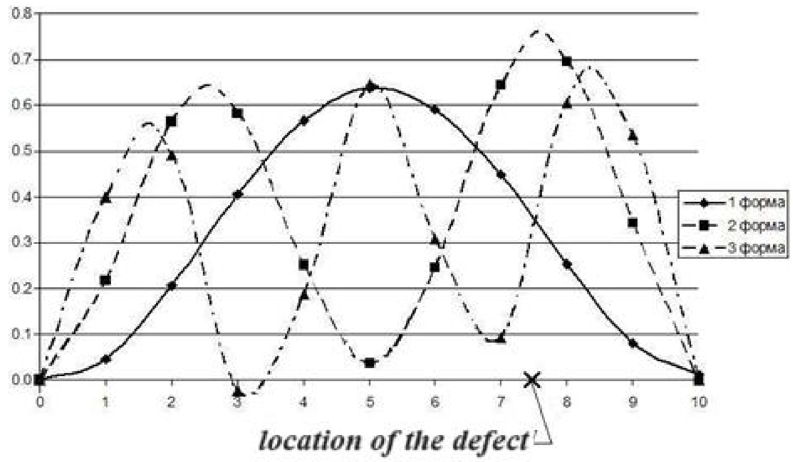


Fig. 5. *Impact lines of additional mass on changing the natural frequencies of the beam: without defect (top picture) and with the defect (bottom picture)*

As you can see, the defect led to a noticeable change of impact lines, and major changes have occurred in the place of its location. Thus, by measuring only the natural frequencies and their changes in the presence of additional mass on the structure, you can identify and track changes of natural forms of vibrations in case of defects occurrence. In this case, there is no need to measure with high precision the deflections of a series of points of the beam to determine the form of vibrations.

Conclusion

As shown by the investigations, determining the form of natural vibrations by the method of additional mass presented in this article makes it possible to obtain reliable results on the actual forms of natural vibrations of building structures which are modeled by rod elements (a beam, a frame, etc.). Also it was found that by using the analysis of changes in the impact lines of additional mass on the natural frequency of the structure, the conclusions can be drawn about the location of the defect. Implementation of the method in the procedure of bridges field diagnostics involves performance of a series of experimental (both laboratory and field) works and refining a practical technique.

I would like to note the simplicity of the method and its practical advantages, namely: this method implementation requires a small number of primary transducers (sensors); so, to determine individual natural forms only one sensor is needed. You can use all types of sensors that enable obtaining vibrorecords of free vibrations of the structure (deflectometers, accelerometers, strain sensors, etc.). So, regardless of the type, this will result in obtaining the normalized ordinates of vibrations forms like in case of using deflectometers for direct measurement of vibrations amplitude.

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