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Design of arbitrary anchor groups on concrete edge subjected to arbitrary shear load and torsional moment

Longfei Li, Yischang Li
Dr. Li Anchor Profi GmbH, Gustav-Stoll-Weg 7, 72250 Freudenstadt, Germany
Email: info@anchorprofi.de

Abstract

The design of anchor fastenings in concrete is dealt with in EN 1992-4 [1] only for simple anchor layouts (Figure 1). For fastenings with gap in the fixture near the edge under shear load towards to the edge, maximum of two anchors in a row parallel to the edge can be designed. In the case of fastenings under torsional moment, the resistance of the anchor group for the concrete edge failure mode cannot be calculated according to the current regulations [1-5] because the resulting anchor shear force in the group is equal to zero.

In order to extend the application range of fastenings beyond regulated simple anchor layouts, a design model for the failure mode of concrete edge failure for arbitrary anchor groups under arbitrary shear load and torsional moment is developed. The design model is verified by 30 test results.

In this paper, the design model is presented and explained with design examples.

Keywords: anchor design in concrete, anchor layout, clearance hole, edge, concrete edge failure

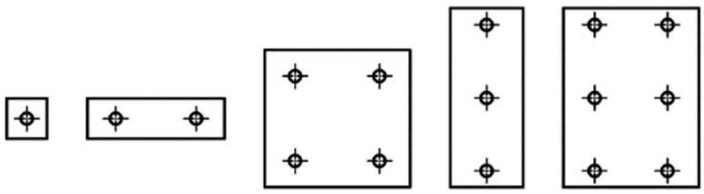
1. Introduction

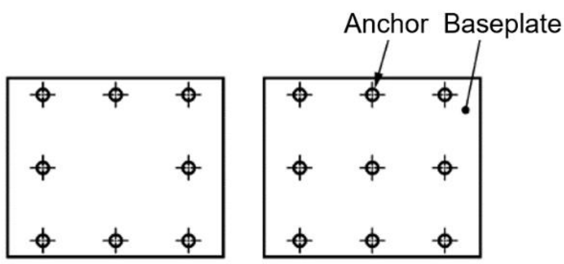
The design of anchor fastenings under shear load with a small edge distance ($c_i < \max\{10h_{ef}; 60d_{nom}\}$) is dealt with in the currently regulations [1-5] depending on the hole clearance (gap) in the fixture for a maximum of four (Figure 1a) or nine anchors (Figure 1b). In practice, however, more anchors are often required to transfer a planned shear load into concrete.

The regulated anchor layouts are matrix-like (Figure 1). For practical applications, deviating anchor layouts are necessary, such as circular anchor layouts for the anchorage of traffic light posts and robot consoles.

According to [1-5], the resistance at concrete edge failure is only calculated for the anchors close to the edge. This calculation may not be sufficient for fastenings without gap in the fixture with several rows of anchors to the edge, because the back anchors can be decisive for the verification of the concrete edge failure [6,7].

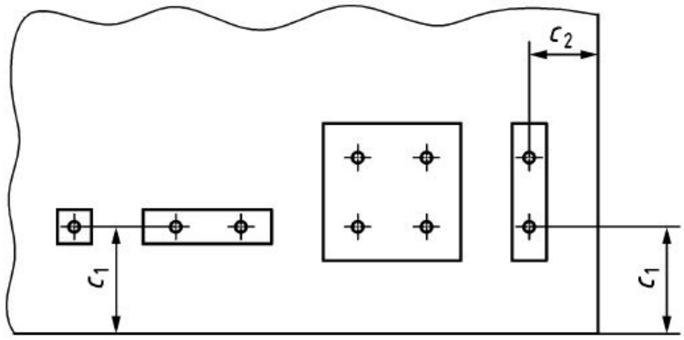
The resistance of the concrete edge failure in the back anchors cannot be calculated according to [1-5] for an anchor group under torsional moment, because the sum of the anchor shear forces in the group is equal to zero (Figure 2).





a) - Fastenings without gap for all edge distances and for all load directions and

- Fastenings with gap according to table 1 (section 2.2)
  - with a large edge distance ($c_i \geq \max\{10h_{ef}; 60d_{nom}\}$) for all load directions and
  - with a small edge distance ($c_i < \max\{10h_{ef}; 60d_{nom}\}$) exclusively for fastenings subject to tension



b) Fastenings with gap according to Table 1 with a small edge distance (c_i < max {$10h_{ef}$; $60d_{nom}$}) for all load directions

Figure 1 Anchor layout dealt with in EN 1992-4 [1].

In order to provide a possibility for the design of anchor groups with any anchor layout under any shear load and torsional moment and with any edge distance, a design model for the failure mode of concrete edge failure is developed. In this paper, this design model is presented, verified by test results and explained with design examples.

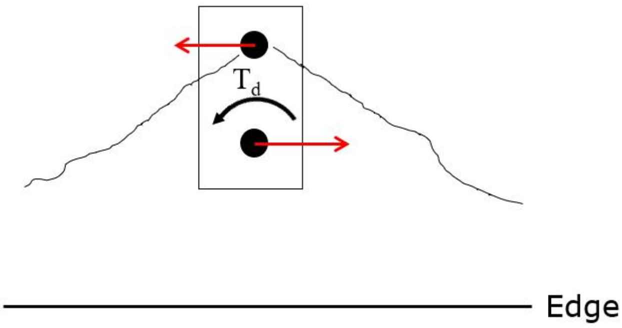


Figure 2 Anchor group loaded under torsional moment T_d

2. Distribution of the shear loads on anchors with or without gap in the fixture

2.1 Fastenings without gap

On a fixture or baseplate with n anchors under combined action load of V_{xd}, V_{yd} and T_d via the anchor center of gravity S (Figure 3), the anchor shear forces V_{xi} and V_{yi} can be derived as follows if the baseplate has sufficient rigidity [8].

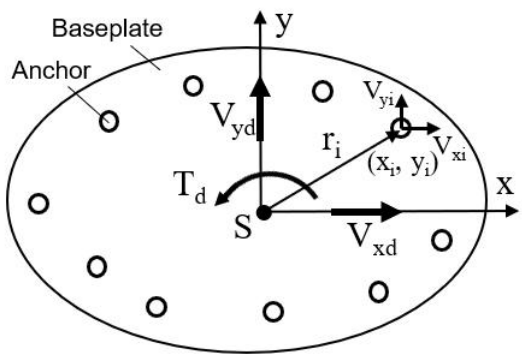


Figure 3 Anchor group under combined loading of V_{xd}, V_{yd} and T_d

According to the mechanics of materials, the following relationship between shear stress τ_i (x_i, y_i) and torsional moment T_d applies over a cross-section of a bar.

$$\tau_i\,(r_i) = \frac{T_d}{I_p} r_i \tag{2.1}$$

Putting the torsional moment of inertia I_p

$$I_p = \int r_i^2 dA = \sum_{i=1}^{n} r_i^2 \Delta A \tag{2.2}$$

with $r_i^2 = x_i^2 + y_i^2$, $\Delta A = A_s$ (anchor cross-sectional area) and $V_i = \tau_i \cdot \Delta A$

into Equation (2.1), one obtains the shear forces on the anchor i as follows:

$$V_{xi}^T = -\frac{T_d \cdot y_i}{\sum_{i=1}^{n} x_i^2 + \sum_{i=1}^{n} y_i^2}$$
$$V_{yi}^T = \frac{T_d \cdot x_i}{\sum_{i=1}^{n} x_i^2 + \sum_{i=1}^{n} y_i^2} \tag{2.3}$$

The anchor shear forces from the shear loads V_{xd} and V_{yd} through the anchor center of gravity S are:

$$V_{xi}^V = \frac{V_{xd}}{n}$$
$$V_{yi}^V = \frac{V_{yd}}{n} \tag{2.4}$$

with n: Number of anchors in the group.

Thus, the anchor shear forces from V_{xd}, V_{yd} and T_d result as follows:

$$V_{xi} = V_{xi}^{V} + V_{xi}^{T}$$
$$V_{yi} = V_{yi}^{V} + V_{yi}^{T} \qquad (2.5)$$

with x_i and y_i: coordinates of the i-th anchor in a Cartesian coordinate system whose origin is on the center of gravity S of the anchors (see Figure 3).

Based on Equation (2.5), the anchor shear forces in anchor groups with any slotted holes in the x or y direction can be determined by eliminating the anchor effect in the slotted direction. A large hole without transmission of the shear force can be simulated by two slotted holes in the x and y directions [9].

2.2 Fastenings with gap

For fastenings with normal clearance holes according to Table 1 without gap filling, the verification of the concrete edge failure is regulated for a maximum of 4 anchors [1]. It is assumed that the anchors close to the edge transfer the shear loads to the edge. The reason for this is that the concrete edge failure is brittle and, in the worst case, the concrete edge failure occurs on anchors close to the edge before the back anchors carry the shear loads.

Figure 4 shows the load-displacement behavior of shear load tests with individual anchors without gab in the fixture. With anchor size M16, the concrete edge failure occurs with a displacement of approx. 0.8 mm. However, normal gap can be up to 2 mm (Figure 5). Figure 6 shows the load displacement curves of shear load tests with anchors of size M16 with a clearance hole $d_f = 18$ mm in the fixture. The anchor bears the shear load only when the gab is compensated by shifting the fixture.

NOTE Applications where bolts are welded to the fixture or screwed into the fixture, or in the cases where any gap between the fastener and the fixture is filled with mortar of sufficient compressive strength (≥ 40 N/mm^2) or eliminated by other suitable means may be considered to have no hole clearance.

Table 1 Fasteners with normal hole clearance (gap) in the fixture according to [1]

Dimensions in millimetres

1	external diameter of fastener d^{a} or $d_{nom}{}^{b}$	6	8	10	12	14	16	18	20	22	24	27	30	> 30
2	diameter d_f of clearance hole in the fixture	7	9	12	14	16	18	20	22	24	26	30	33	d + 3 or d_{nom} + 3

a If bolt bears against the fixture.

b If sleeve bears against the fixture.

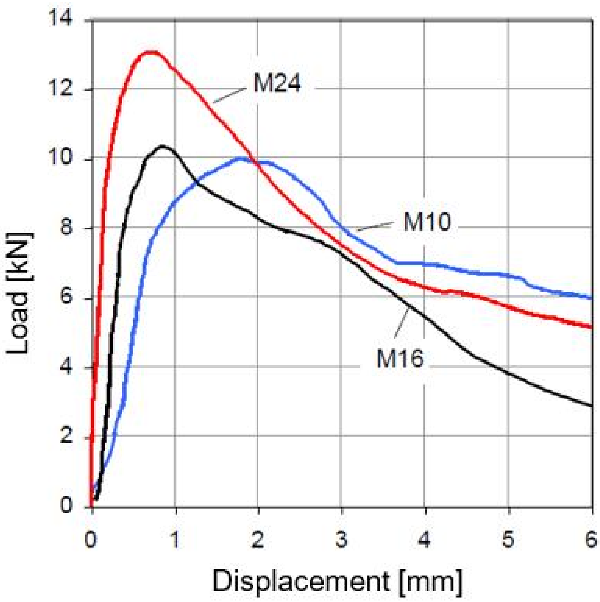


Figure 4 Load-displacement behavior of the bonded anchor without hole clearance with shear load to the edge with edge distance c = 50 mm and anchorage depth h_{ef} = 130 mm [7]

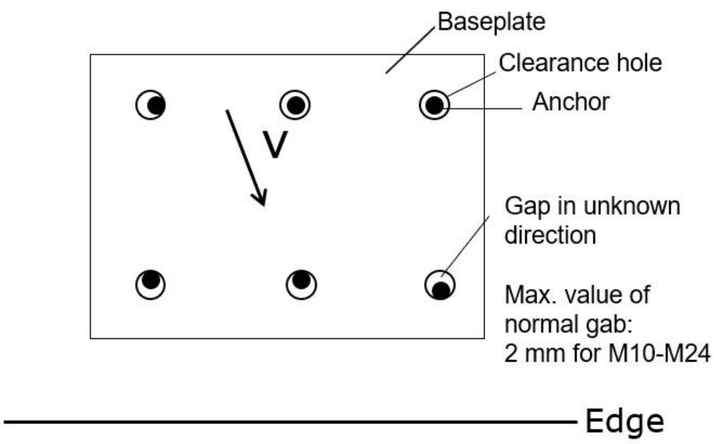


Figure 5 Schematic representation of the fastenings with normal hole clearance according to Table 1

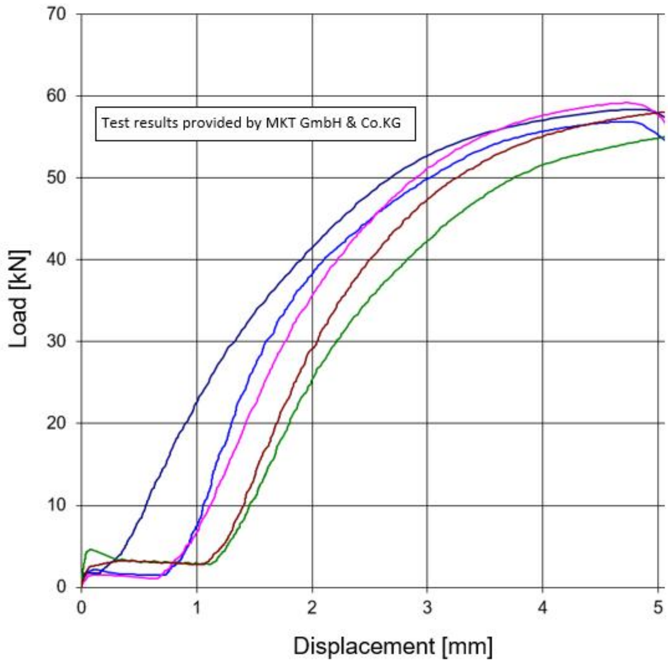


Figure 6 Load-displacement behavior of the M16 bonded anchor with normal hole clearance with shear load

In the case of fastenings with normal gap, assuming that at most the two anchors close to the edge carry the shear load to the edge, the following assumptions are made in order to determine the most unfavorable distribution of the anchor shear forces in anchor groups for the verification of the concrete edge failure.

a) In the case of anchor groups with a maximum of two anchors parallel to the edge, the shear load directed perpendicularly to the edge is taken by the anchors close to the edge. If there are more than two anchors in the first row of anchors parallel to the edge, this load is carried by a maximum of two adjacent anchors.

b) The torsional moment is carried by all anchors.

Figures 7 to 10 show the calculations based on the above assumptions explained in examples.

In the examples in Figures 7 and 8, there are a maximum of 2 anchors in the first row parallel to the edge. The most unfavorable shear force distribution is determined as follows:

1) The anchor shear forces from the torsional moment are calculated according to Equation (2.3) as in the case without gap.

2) For the calculation of the anchor shear forces from the shear load V, fictitiously slotted holes are used for the back anchors. As a result, the shear load to the edge acts only on the anchors near the edge for the calculation.

3) The shear forces on each anchor result from the sum of the forces calculated in 1) and 2).

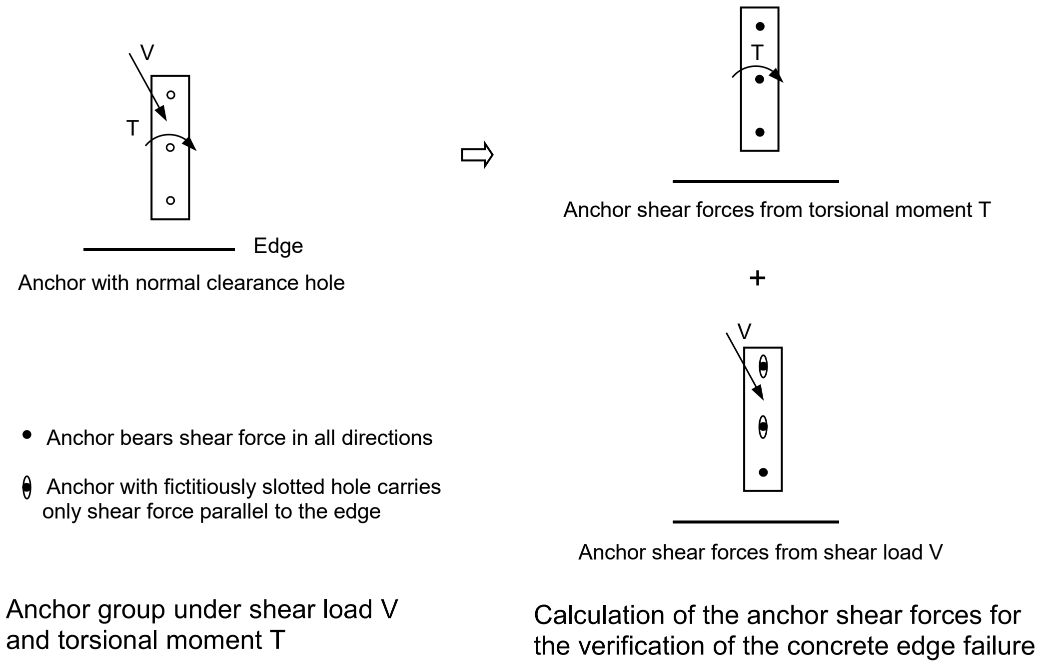


Figure 7 Example with an anchor in the first row parallel to the edge

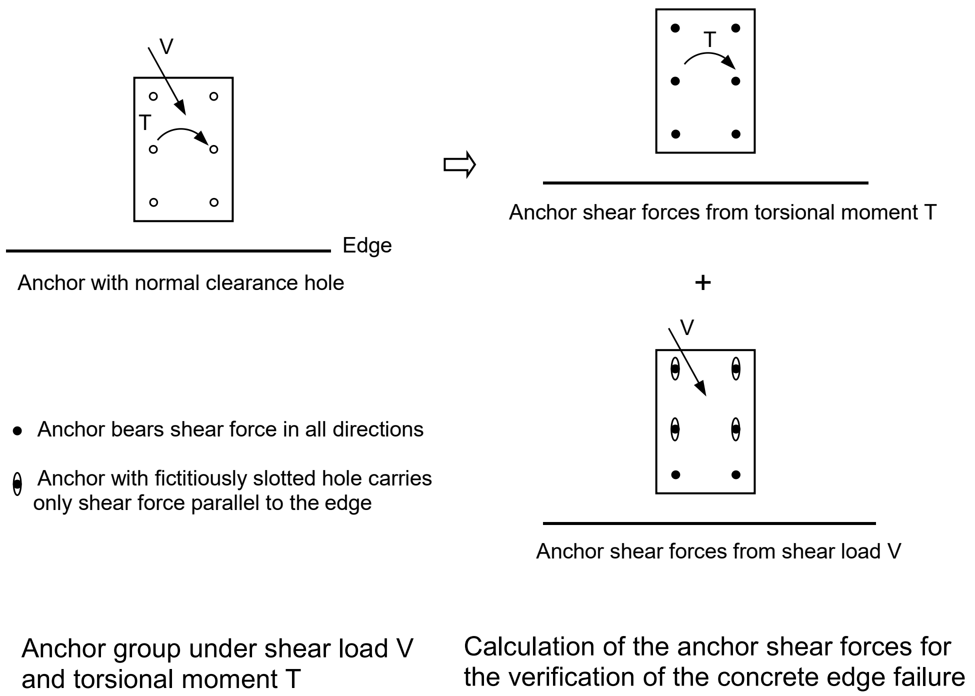


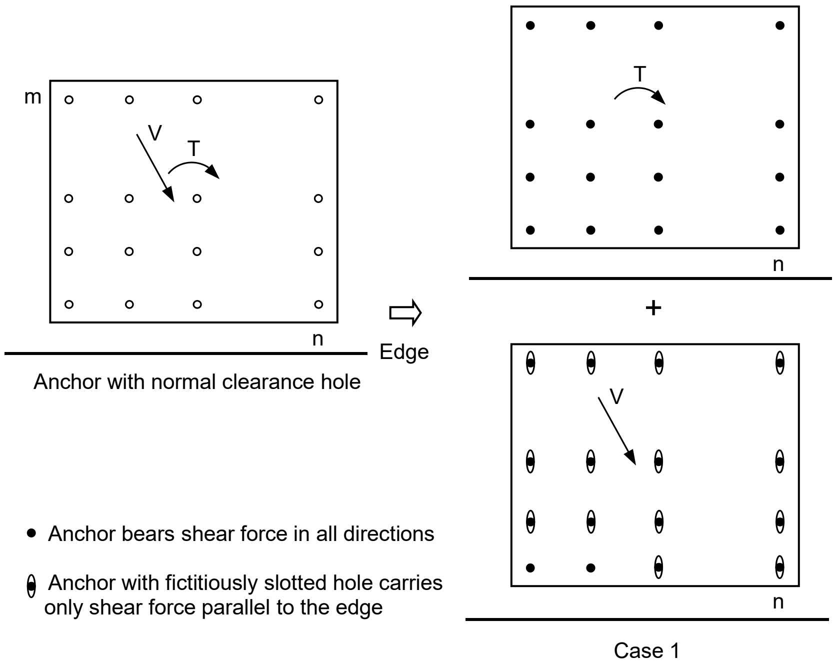
Figure 8 Example with two anchors in the first row parallel to the edge

In the examples in Figures 9 and 10, there are more than two anchors in the first row of anchors parallel to the edge. The shear force distribution is calculated as follows for cases n-1 (Figure 9) and n-2 (Figure 10):

1) The anchor shear forces from the torsional moment T are calculated according to Equation (2.3). With the slotted hole, the anchor carries only the shear force perpendicular to the direction of the slotted hole. In a large hole, the anchor does not carry any shear force.

2) Anchor shear forces from the shear load V are calculated for the different cases. In each case, only 2 adjacent anchors without a slot in the direction of the edge carry the shear force to the edge (comparison of case 1 between Figure 9 and Figure 10).

3) The shear forces in each case result from the sum of the forces calculated in 1) and 2).



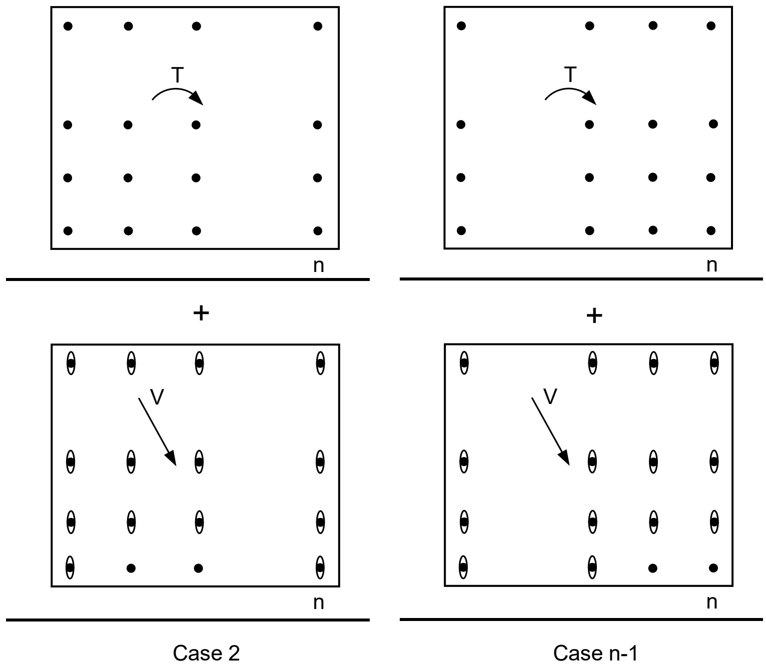
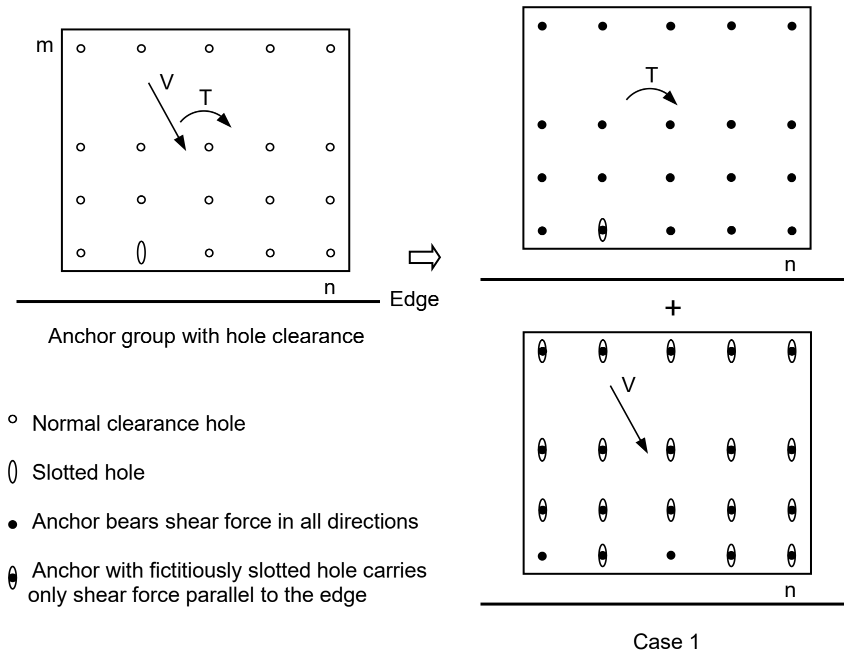


Figure 9 Example with more than two anchors in the first row parallel to the edge



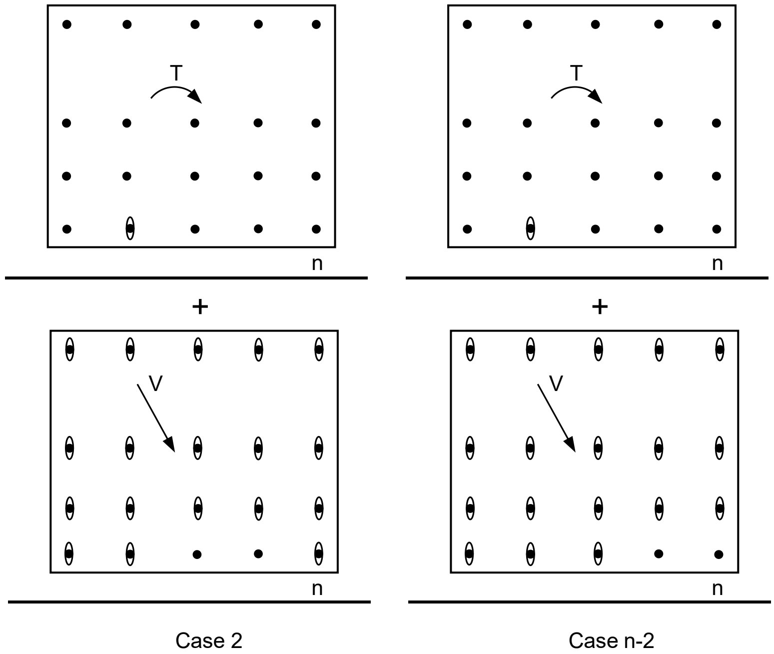


Figure 10 Example with more than two anchors and with a slotted hole in the first row parallel to the edge

3. Verification in the ultimate limit state

3.1 Fasteners without gap in the fixture

The anchor shear force distribution in the anchor group is known from the determination of shear forces in Section 2.1. The resistance of the concrete edge failure of arbitrary anchor groups can be calculated based on [1-4] with a modification explained below.

The characteristic resistance of the concrete edge failure $V_{Rk,c}$ is according to [1-4]

$$V_{Rk,c} = V^0_{Rk,c} \cdot \frac{A_{c,V}}{A^0_{c,V}} \cdot \psi_{s,V} \cdot \psi_{h,V} \cdot \psi_{a,V} \cdot \psi_{ec,V} \cdot \psi_{re,V} \quad [N] \tag{3.1}$$

The design value of the resistance results from the characteristic resistance of the concrete edge failure divided by the partial safety factor of 1.5 for concrete edge failure $V_{Rd,c} = V_{Rk,c}/1.5$.

For arbitrary anchor layout with a known distribution of the anchor shear forces, the parameters $A^0_{c,V}$, $A_{c,V}$ and $\psi_{ec,V}$ for the calculation of the resistance of the entire anchor group are extended as follows.

The verification of the concrete edge failure is carried out layer by layer for each row of anchors with the edge distance c_j. A crack line is assumed for each layer. Figures 11 to 14 show, for example, the possible crack lines of four rows of anchors parallel to the edge and the determination of the real projected areas $A_{c,V}$ (c_j) of the idealized concrete outbreak bodies with arbitrary anchor layout under shear load. The projected area of the idealized outbreak body for a single anchor on the concrete side face is also calculated with

$$A^o_{c,V}(c_j) = 4.5\ c^2_j \tag{3.2}$$

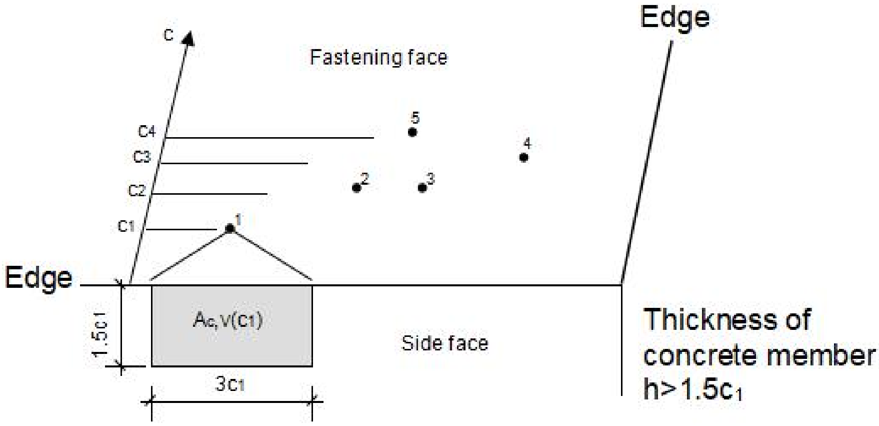


Figure 11 Example of the real projected area $A_{c,V}$ (c_1) with an anchor in the breakout body

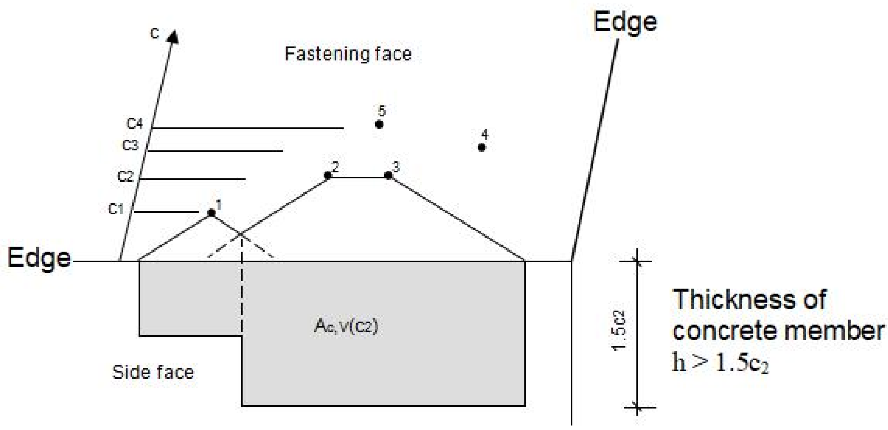


Figure 12 Example of the real projected area $A_{c,V}$ (c_2) with three anchors in the breakout body

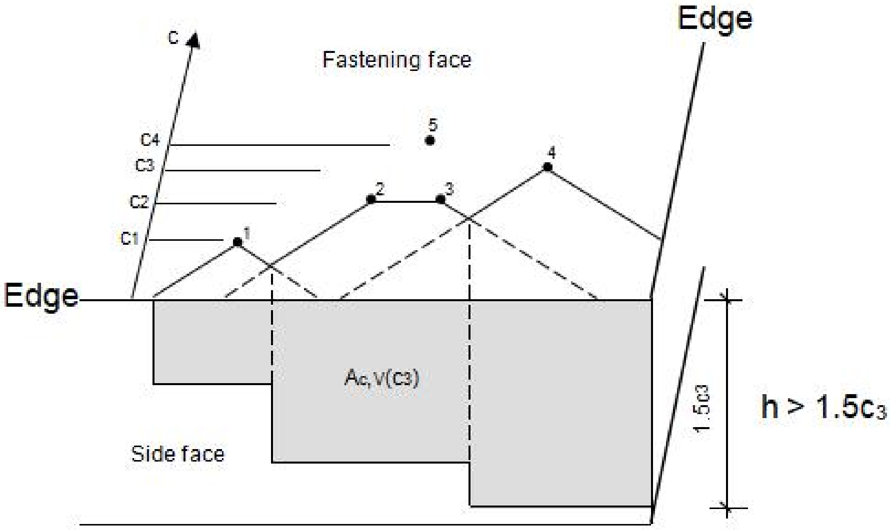


Figure 13 Example of the real projected area $A_{c,V}$ (c_3) with four anchors in the breakout body

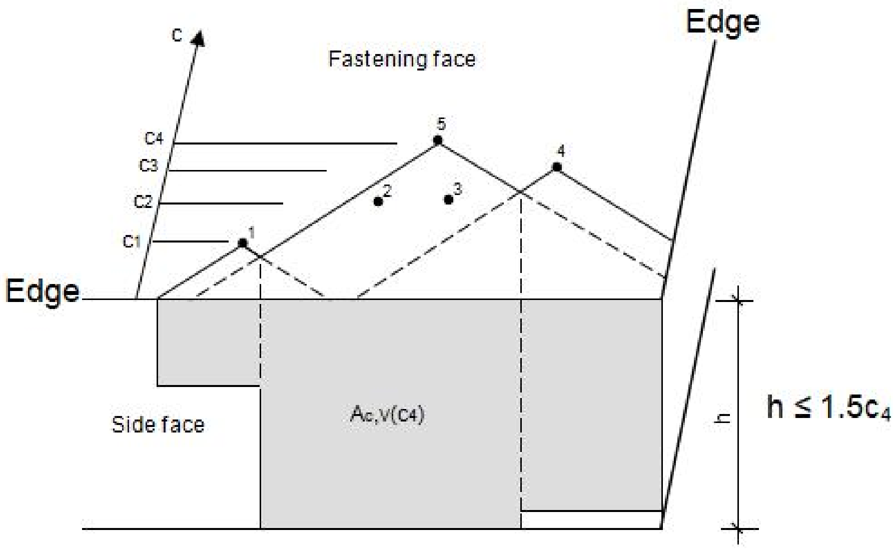
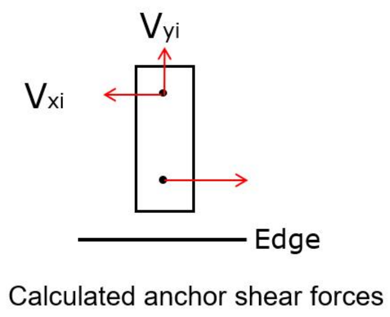


Figure 14 Example of the real projected area $A_{c,V}$ (c_4) with five anchors in the breakout body

The design value of the action V_{Ed} (c_j) on the outbreak body with the edge distance c_j is determined from the shear forces of the anchors located in the outbreak body. The force components perpendicular against the edge are neglected. If the direction of the shear force components parallel to the edge changes in the outbreak body, the smaller shear force components are converted by a factor of 0.5 according to [1] or 0.4 according to [2-4] to shear forces directed perpendicularly to the edge (Figure 15) because the factor φ in Equation (3.3) is different according to [1] and [2-4].

$$\psi_{\alpha,V} = \sqrt{\frac{1}{(\cos\alpha_V)^2 + (\varphi\sin\alpha_V)^2}} \geq 1.0, \quad 0^\circ \leq \alpha_V \leq 90^\circ \tag{3.3}$$

with φ=0.5 according to [1] and φ=0.4 according to [2-4]



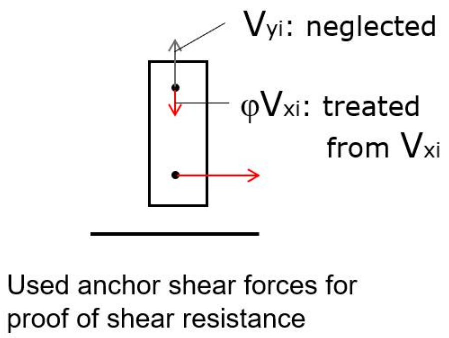


Figure 15 Treatment of the anchor shear forces acting parallel or opposite to the edge

The eccentricity factor $\psi_{ec,V}$ is calculated according to Equation (3.4):

$$\psi_{ec,V} = \frac{1}{1 + 2e_V / (3c_j)} \leq 1 \qquad (3.4)$$

with e_V : distance between the center of gravity of the anchors and the action line of the resulting shear force.

The utilization factor $\beta_{c,V}$ (c_j) results from the comparison between the action V_{Ed} (c_j) and the resistance $V_{Rd,c}$ (c).

$$\beta_{c,V}(c_j) = V_{Ed}(c_j) / V_{Rd,c}(c_j) \qquad (3.5)$$

The highest utilization of all layers is decisive for the design. The condition according to Equation (3.6) must be met.

$$\beta_{c,V} = \max(\beta_{c,V}(c_1), \beta_{c,V}(c_2), \ldots, \beta_{c,V}(c_m)) \leq 1.0 \qquad (3.6)$$

3.2 Fastenings with gap in the fixture

The anchor shear forces in different cases are known from the calculation in Section 2.2. For each case (e.g. Figure 9 for all n-1 cases), the verification of the concrete edge failure is carried out layer by layer for all rows of anchors parallel to the edge, analogous to Section 3.1.

The maximum degree of utilization of all cases is taken to control the verification of the concrete edge failure.

4. Verification of the design model by test results

In order to check the correctness of the design model, 30 tests from [8] were recalculated based on [3]. All tests were carried out in non-cracked concrete C20/25 with a member thickness of 500 mm and an edge distance of 100 mm. The anchors used had a normal gap in baseplates. The other test parameters, the calculation results and the comparison between test and calculation results are shown in Table 2.

Table 2 Compilation and comparison of test and calculation results

Group No	Tests							Calculation	Comparison
	Anchor		h_{ef}	Spacing		$f_{c,200}$	T_u 1)	T_{Rd} 2)	T_u/T_{Rd}
	Number	Size	mm	s1	s2	N/mm²	kNm	kNm	
1	4	M16	80	100	100	29.2	10.38	5.22	1.99
						29.2	8.76	5.22	1.68
						29.2	10.74	5.22	2.06
2	4	M16	80	50	50	22.5	4.16	1.62	2.56
						22.5	3.91	1.62	2.41
						22.5	3.91	1.62	2.41
3	4	M24	100	100	100	23.9	12.40	5.47	2.27
						23.9	13.35	5.47	2.44
						23.9	12.99	5.47	2.37
4	4	M16	80	50	100	23.8	4.21	2.55	1.65
						23.8	4.36	2.55	1.71
						23.8	6.14	2.55	2.41
5	4	M16	80	100	50	23.7	7.03	4.23	1.66
						23.7	8.23	4.23	1.95
						23.7	8.62	4.23	2.04
						25.2	10.79	5.24	2.06

6	4	M16	130	100	100	25.2	12.13	5.24	2.31
						26.7	13.28	5.40	2.46
7	4	M16	130	200	200	23.4	37.20	12.25	3.04
						25.2	36.63	12.71	2.88
						25.2	33.79	12.71	2.66
8	6	M16	130	100	100	22.9	20.76	7.85	2.65
						22.9	18.89	7.85	2.41
9	6	M16	130	200	100	23.6	36.34	11.03	3.30
						22.9	32.10	10.86	2.96
10	6	M16	130	100	200	26.7	36.66	16.35	2.24
						23.6	40.56	15.37	2.64
11	9	M16	130	100	100	23.4	36.79	16.14	2.28
						23.4	39.66	16.14	2.46
						23.4	42.94	16.14	2.66
Test number n= 30						Mean value			2.353
						Scatter V [%]			0.174
						5%-Fractile			1.500

1) T_u : Fracture torsional moment from tests

2) T_{Rd} : Design value of torsional moment at utilization of 100%

In Figure 16, the fracture torsional moments T_u measured in the tests are compared with the calculated design values of the torsional moments T_{Rd}. From the evaluation of 30 values of T_u/T_{Rd} with unknown standard deviation, the 5% fractile value is 1.5. This corresponds to the required partial safety factor of concrete edge failure.

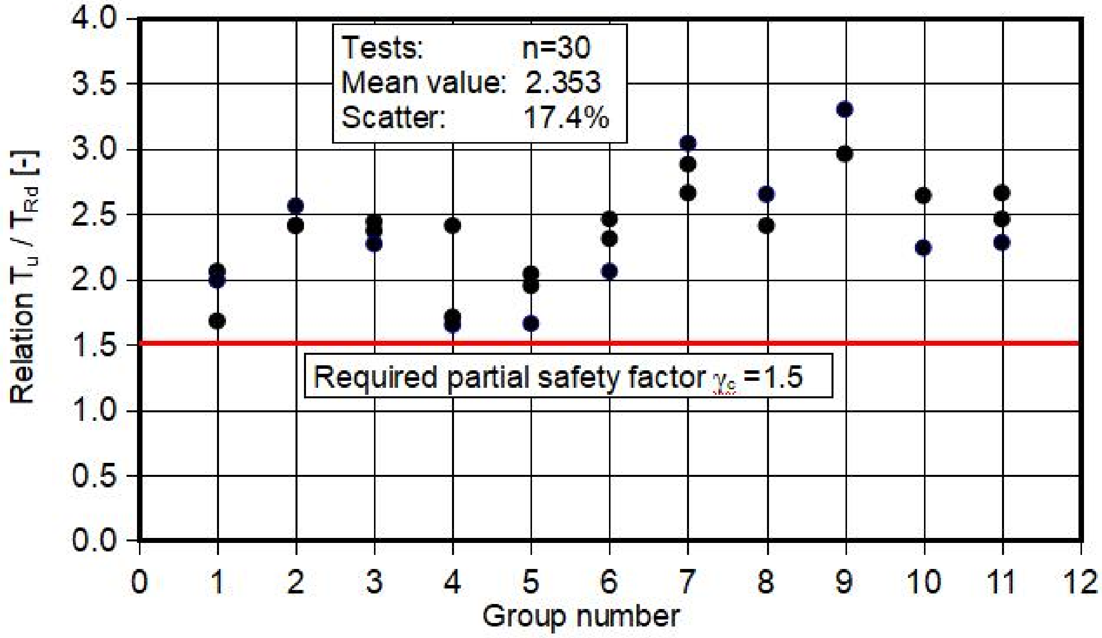


Figure 16 Comparison between the fracture torsional moments T_u from the test and the design values of the torsional moments T_{Rd} from the calculation

5. Design examples

5.1 Fastenings without gap in the fixture

In the case of fastenings with several rows of anchors parallel to the edge, the verification of the concrete edge failure using the described design model is quite complex. Therefore, the calculations were implemented into a software [9]. The design examples are based on EN 1992-4 [1] and explained using the results from [9].

Figure 17 shows the anchor layout. The edge distance of anchor no. 3 is 200 mm. The other boundary conditions of the fastening as well as the determined anchor shear forces are given in panel 1 and 2 and shown in Figure 17.

Input data

Base material:

- Cracked concrete, Thickness of base material h=500mm, Strength class C20/25, f_{ck} =20.0N/mm²
- No edge and stirrup reinforcement

Action loads:

- Predominantly static and quasi-static design loads

Shear load: $V_{x,Ed}$=20 kN, $V_{y,Ed}$=4 kN

Torsional moment: T_{Ed}=3 kNm

Steel profile center or shear load acting point (0, 0)

Installation:

- With gap filling

Coordinates of anchors [mm]:

No.	x	y
1	-52.5	-90.9
2	52.5	-90.9
3	105.0	0.0
4	52.5	90.9
5	-52.5	90.9
6	-105.0	0.0

Panel 1 Boundary conditions of fastening [9]

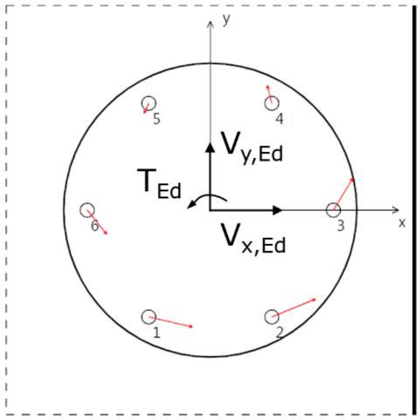


Figure 17 Anchor layout and shear forces [9]

Anchor internal forces [kN]

Anchor No.	Shear Vi	Shear x	Shear y
1	7.652	7.457	-1.714
2	8.056	7.457	3.048
3	6.370	3.333	5.429
4	3.148	-0.791	3.048
5	1.888	-0.791	-1.714
6	5.280	3.333	-4.095

Panel 2 calculated shear forces according to Equation (2.5)

Panel 3 shows that the verification was carried out layer by layer for all rows of anchors parallel to the edge. When verifying groups (group no.) 3 and 4, the smaller force component in the -y direction (II to edge) of the shear forces directed parallel to the edge was converted by a factor of 0.4 to the +x direction (II → ⊥). In this example group no. 4 with all six anchors is decisive for the design. The highest degree of utilization results from Equation (3.6) as follows:

$$\beta_{c,V} = \max(\beta_{c,V}(c_1), \beta_{c,V}(c_2), \ldots, \beta_{c,V}(c_m)) = \max(0.203, 0.382, 0.451, \mathbf{0.556}) = 0.556 < 1.0$$

Thus, the fastening for concrete edge failure is verified.

Concrete edge failure

$V_{Rk,c} = V^0_{Rk,c} \cdot \psi_{A,V} \cdot \psi_{s,V} \cdot \psi_{h,V} \cdot \psi_{\alpha,V} \cdot \psi_{ec,V} \cdot \psi_{re,V}$ $\quad V^0_{Rk,c} = k_9 \cdot d_{nom}^{\alpha} \cdot l_f^{\beta} \cdot (f_{ck})^{0.5} \cdot c_1^{1.5}$ [N] $\quad \psi_{A,V} = A_{c,V}/A^0_{c,V}$ $\quad V_{Rd,c} = V_{Rk,c}/\gamma_{Mc}$

$\alpha = 0.1 \cdot (l_f / c_1)^{0.5}$ $\quad \beta = 0.1 \cdot (d / c_1)^{0.2}$

Basic data for verification of concrete edge failure:

h_{ef} [mm]	l_f [mm]	d_{nom} [mm]	k_9	f_{ck} [N/mm²]	$\psi_{re,V}$	γ_{Mc}
70.0	70.0	12.0	1.7	20.0	1.0	1.5

There is gap filling between anchor and base plate.

Concrete edge x+ :

Gr-No.	Anchor-No.	Failure-Line	c_1 [mm]	c'_1 [mm]	α	β	$V^0_{Rk,c}$ [kN]	$A_{c,V}$ [mm²]	$A^0_{c,V}$ [mm²]	$\psi_{A,V}$
1	3	3	200.0	-	0.059	0.057	31.730	180000	180000	1.000
2	3,2,4	2,4,3	252.5	-	0.053	0.054	43.803	354825	286903	1.237
3	3,2,4,1,5	1,5	357.5	-	0.044	0.051	71.157	627183	575128	1.091
4	3,2,4,1,5,6	6,5,1	410.0	-	0.041	0.049	86.255	627183	756450	0.829

Gr-No.	Anchor-No.	$\psi_{s,V}$	$\psi_{h,V}$	$\psi_{\alpha,V}$	e_V [mm]	$\psi_{ec,V}$	$\psi_{re,V}$	$V_{Rk,c}$ [kN]	$V_{Rd,c}$ [kN]	V_{Ed} [kN]	$\beta_{V,c}$
1	3	1.000	1.000	1.482	0.0	1.000	1.000	47.021	31.347	6.370	0.203
2	3,2,4	1.000	1.000	1.291	48.2	0.887	1.000	62.017	41.345	15.787	0.382
3	3,2,4,1,5	1.000	1.036	1.109	87.0	0.860	1.000	76.712	51.141	23.050	0.451
4	3,2,4,1,5,6	1.000	1.109	1.071	80.7	0.884	1.000	75.107	50.072	27.840	**0.556**

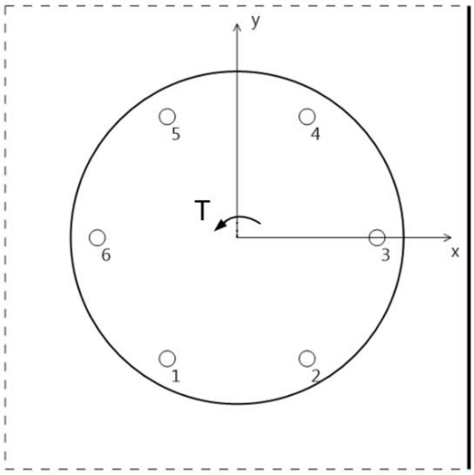
Intermediate data for calculation of load excentricities e_V and of angles α_V:

Gr-No.	Anchor-No.	Forces ⇄ ‖ to edge	Treated ‖ → ⊥	$\sum(Q_x)$ [kN]	$\sum(Q_y)$ [kN]	$\sum(Q_x \cdot y)$ [kNmm]	$\sum(Q_y \cdot x)$ [kNmm]	x_{s_Qy} [mm]	y_{s_Qx} [mm]	x_{s_D} [mm]	y_{s_D} [mm]	α_V [°]
1	3	-	-	3.333	5.429	0.0	570.0	105.0	0.0	105.0	0.0	58.4
2	3,2,4	-	-	10.791	11.524	-678.1	890.0	77.2	-62.8	70.0	0.0	46.9
3	3,2,4,1,5	√	y-	19.962	11.524	-1356.2	890.0	77.2	-67.9	21.0	0.0	30.0
4	3,2,4,1,5,6	√	y-	25.343	11.524	-1356.2	890.0	77.2	-53.5	0.0	0.0	24.5

Panel 3 Layer-by-layer verification of concrete edge failure based on EN 1992-4 [1]

5.2 Fastenings with normal gap with an anchor in the first row parallel to the edge

For the anchor group from Example 1 in Section 5.1, a design with gap or without annular gap filling was carried out. The anchor shear forces result from the sum of the force components from the torsional moment T and the shear load V (Figure 18).



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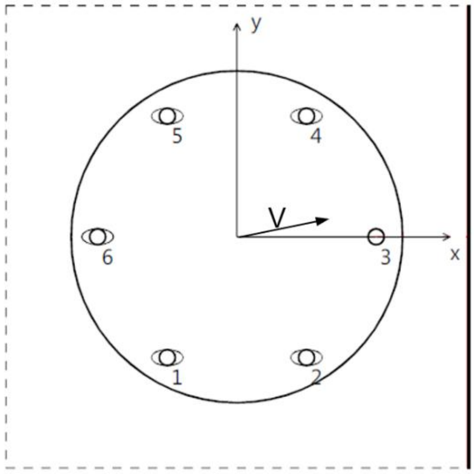
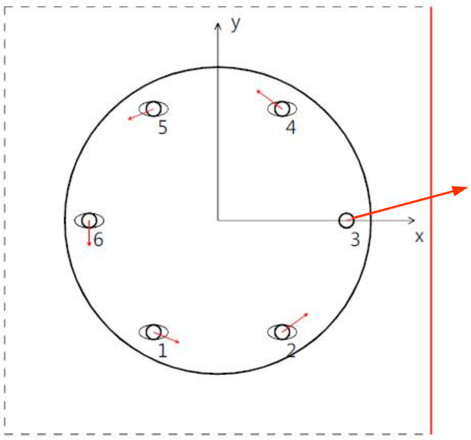


a) Anchor with normal hole clearance for determining the shear forces from T

b) Back anchors with fictitious slotted holes for determining the shear forces



c) Resulting shear shear forces

Figure 18 Representation of the separate determination of the force components Qx_T and Qy_T from the torsional moment T and Qx_V and Qy_V from the shear load V (Panel 4)

Anchor No.	Q	Qx	Qy	Qx_V	Qy_V	Qx_T	Qy_T
1	4.466	4.124	-1.714	0.000	0.667	4.124	-2.381
2	5.128	4.124	3.048	0.000	0.667	4.124	2.381
3	20.724	20.000	5.429	20.000	0.667	0.000	4.762
4	5.128	-4.124	3.048	0.000	0.667	-4.124	2.381
5	4.466	-4.124	-1.714	0.000	0.667	-4.124	-2.381
6	4.095	0.000	-4.095	0.000	0.667	0.000	-4.762

Panel 4 Calculated anchor shear forces from the torsional moment T and the shear load V

Verification of resistances:

Gr-No.	Anchor-No.	Failure-Line	c_1 [mm]	c'_1 [mm]	α	β	$V^0_{Rk,c}$ [kN]	$A_{c,V}$ [mm²]	$A^0_{c,V}$ [mm²]	$\psi_{A,V}$
1	3	3	200.0	-	0.059	0.057	31.730	180000	180000	1.000
2	3,2,4	2,4,3	252.5	-	0.053	0.054	43.803	354825	286903	1.237
3	3,2,4,1,5	1,5	357.5	-	0.044	0.051	71.157	627183	575128	1.091
4	3,2,4,1,5,6	6,5,1	410.0	-	0.041	0.049	86.255	627183	756450	0.829

Gr-No.	Anchor-No.	$\psi_{s,V}$	$\psi_{h,V}$	$\psi_{\alpha,V}$	e_V [mm]	$\psi_{ec,V}$	$\psi_{re,V}$	$V_{Rk,c}$ [kN]	$V_{Rd,c}$ [kN]	V_{Ed} [kN]	$\beta_{V,c}$
1	3	1.000	1.000	1.027	0.0	1.000	1.000	32.579	21.719	20.724	**0.954**
2	3,2,4	1.000	1.000	1.078	17.1	0.957	1.000	55.865	37.243	26.735	0.718
3	3,2,4,1,5	1.000	1.036	1.052	43.5	0.925	1.000	78.200	52.133	32.102	0.616
4	3,2,4,1,5,6	1.000	1.109	1.046	48.2	0.927	1.000	76.934	51.289	34.021	0.663

Intermediate data for calculation of load excentricities e_V and of angles α_V:

Gr-No.	Anchor-No.	Forces ⇄ || to edge	Treated || → ⊥	$\sum(Q_x)$ [kN]	$\sum(Q_y)$ [kN]	$\sum(Q_x \cdot y)$ [kNmm]	$\sum(Q_y \cdot x)$ [kNmm]	x_{s_Qy} [mm]	y_{s_Qx} [mm]	x_{s_D} [mm]	y_{s_D} [mm]	α_V [°]
1	3	-	-	20.000	5.429	0.0	570.0	105.0	0.0	105.0	0.0	15.2
2	3,2,4	-	-	24.124	11.524	-375.0	890.0	77.2	-15.5	70.0	0.0	25.5
3	3,2,4,1,5	√	y-	29.962	11.524	-750.0	890.0	77.2	-25.0	21.0	0.0	21.0
4	3,2,4,1,5,6	√	y-	32.010	11.524	-750.0	890.0	77.2	-23.4	0.0	0.0	19.8

Panel 5 Layer-by-layer verification of concrete edge failure based on EN 1992-4 [1]

The concrete edge failure verifications for each row of anchors parallel to the edge are summarized in Panel 5. In this example, the possible crack line of anchor no. 3 is decisive for the design. The highest degree of utilization results from Panel 5 according to Equation (3.6) as follows:

$$\beta_{c,V} = \max(\beta_{c,V}(c_1), \beta_{c,V}(c_2), \ldots, \beta_{c,V}(c_m)) = \max(\mathbf{0.954}, 0.718, 0.616, 0.663) = 0.954 < 1.0$$

Thus, the load-bearing capacity of the anchorage is proven despite no annular gap filling for concrete edge failure.

From Panel 4, it can be seen that the calculated shear force on the anchor no 3 near the edge can exceed the steel shear capacity of the anchor. A special verification for this is not required, since with the actual steel failure load, the shear force is redistributed to the other anchors due to large displacement. The increased shear force of anchor no. 3 used for the verification of the concrete edge failure is therefore conservative.

5.3 Fastenings with normal gap with three anchors in the first row parallel to the edge

The boundary conditions of the example are given in Panel 6. In this example there are more than two anchors in the first row parallel to the edge. The concrete edge failure check is performed for two cases.

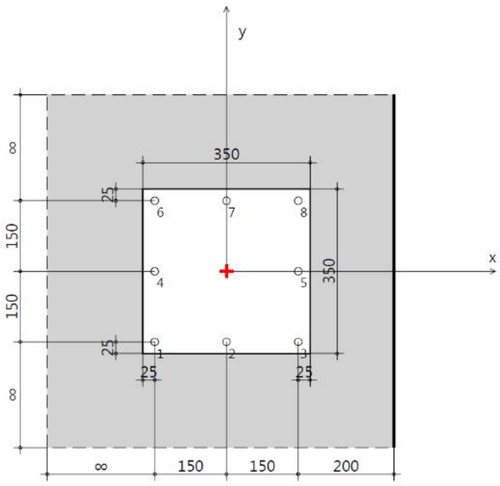
Input data

Base material:

- Cracked concrete, Thickness of base material h=250 mm,

  Strength class C20/25, f_{ck} =20.0N/mm²

- No edge and stirrup reinforcement



Action loads:

- Predominantly static and quasi-static design loads

  Shear load: $V_{x,Ed}$=8 kN

  Torsional moment: T_{Ed}=22 kNm

Steel profile center or shear load acting point (0, 0)

Concrete edge failure

$V_{Rk,c}=V^0_{Rk,c} \cdot \psi_{A,V} \cdot \psi_{s,V} \cdot \psi_{h,V} \cdot \psi_{\alpha,V} \cdot \psi_{ec,V} \cdot \psi_{re,V}$ $V^0_{Rk,c} = k_9 \cdot d_{nom}^{\alpha} \cdot l_f^{\beta} \cdot (f_{ck})^{0.5} \cdot c_1^{1.5}$ [N] $\psi_{A,V} = A_{c,V}/A^0_{c,V}$ $V_{Rd,c} = V_{Rk,c}/\gamma_{Mc}$

$\alpha = 0.1 \cdot (l_f / c_1)^{0.5}$ $\beta = 0.1 \cdot (d / c_1)^{0.2}$

Basic data for verification of concrete edge failure:

h_{ef} [mm]	l_f [mm]	d_{nom} [mm]	k_9	f_{ck} [N/mm²]	$\psi_{re,V}$	γ_{Mc}
85.0	85.0	16.0	1.7	20.0	1.0	1.5

There is no gap filling between anchor and base plate.

Panel 6 Anchorage base, geometry and anchor basic parameters

For case 1, the anchor shear forces are determined as follows:

- The shear forces from the torsional moment T_{Ed} = 22 kNm are calculated according to Equation (2.3) (see Q_{X_T} and Q_{Y_T} in Panel 7).

- To calculate the shear forces from the shear load $V_{X,Ed}$ = 8 kN, it is assumed that the shear load is only carried by anchors no. 3 and no. 5 (see Q_{X_V} and Q_{Y_V} in Panel 7). The small force components Q_{Y_V} (e.g. 0.615 kN for anchor 1) are caused by an additional torsional moment, which arises from the load eccentricity between the shear load $V_{X,Ed}$ and the new center of gravity of the anchor. This new anchor center of gravity is under the above assumption redefined.

With the resulting anchor shear forces Q_X and Q_Y, the verification of the concrete edge failure is carried out layer by layer (Panel 7). When verifying anchor group (group no.) 4, the force components parallel to the edge (V ǁ to edge) were converted by a factor of 0.5 to shear forces acting perpendicular to the edge (ǁ → ⊥).

For case 2, it is assumed that the shear load $V_{X,Ed}$ = 8 kN is carried only by anchors no. 5 and no. 8 (see Q_{X_V} and Q_{Y_V} in Panel 8). The further verification steps are carried out in the same way as in case 1.

In this example, the possible crack line through anchors no. 1, 4 and 6 in case 1 is decisive for the design. The highest degree of utilization according to Equation (3.6) results from Panel 7:

$$\beta_{c,V} = \max(\beta_{c,V}(c_1), \beta_{c,V}(c_2), \ldots, \beta_{c,V}(c_m)) = 1.546 > 1.0$$

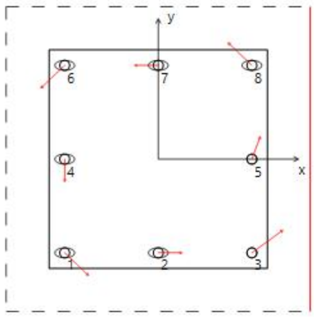
The utilization therefore exceeds the load-bearing capacity of the anchor group by approx. 20%.

Concrete edge x+ : 2 nearby anchors (except the anchor(s) with slotted hole(s) in x-direction) in the first row are assumed to bear the shear load perpendicular to the edge, if there are more than 2 anchors in the row.

Case 1: The anchors 3 and 5 bear the shear load perpandicular to the edge (x+). The torsional moment is carried by all anchors.

Shear forces [kN]:

Anchor No.	Q	Qx	Qy	Qx_V	Qy_V	Qx_T	Qy_T
1	16.855	12.222	-11.607	0.000	0.615	12.222	-12.222
2	12.222	12.222	0.000	0.000	0.000	12.222	0.000
3	19.697	15.915	11.607	3.692	-0.615	12.222	12.222
4	11.607	0.000	-11.607	0.000	0.615	0.000	-12.222
5	12.380	4.308	11.607	4.308	-0.615	0.000	12.222
6	16.855	-12.222	-11.607	0.000	0.615	-12.222	-12.222
7	12.222	-12.222	0.000	0.000	0.000	-12.222	0.000
8	16.855	-12.222	11.607	0.000	-0.615	-12.222	12.222



Explanation:
1. Qx_V, Qy_V are the x- and y-components of Anchor forces from the shear loads.
2. Qx_T, Qy_T are the x- and y-components of Anchor forces from the torsional moment.
3. The assumed slotted holes showed in the figure are not active for the calculation of shear force components Qx_T and Qy_T from the torsional moment. They serve as only for the calculation of shear load components from Qx_V and Qy_V.
4. Edge distance is not to scale.

Verification of resistances with this assumption:

Gr-No.	Anchor-No.	Failure-Line	c_1 [mm]	c'_1 [mm]	α	β	$V^0_{Rk,c}$ [kN]	$A_{c,V}$ [mm²]	$A^0_{c,V}$ [mm²]	$\psi_{A,V}$
1	3,5	3,5	155.0	-	0.074	0.063	23.886	142988	108113	1.323
2	3,5,8	3,5,8	155.0	-	0.074	0.063	23.886	177863	108113	1.645
3	3,5,8,2,7	2,7	305.0	-	0.053	0.055	59.977	303750	418613	0.726
4	3,5,8,2,7,1,4,6	1,4,6	455.0	-	0.043	0.051	104.424	416250	931613	0.447

Gr-No.	Anchor-No.	$\psi_{s,V}$	$\psi_{h,V}$	$\psi_{\alpha,V}$	e_V [mm]	$\psi_{ec,V}$	$\psi_{re,V}$	$V_{Rk,c}$ [kN]	$V_{Rd,c}$ [kN]	V_{Ed} [kN]	$\beta_{V,c}$
1	3,5	1.000	1.000	1.320	28.3	0.892	1.000	37.190	24.793	30.787	1.242
2	3,5,8	1.000	1.000	1.509	59.3	0.797	1.000	47.250	31.500	40.267	1.278
3	3,5,8,2,7	1.000	1.353	1.293	132.6	0.775	1.000	58.999	39.333	47.593	1.210
4	3,5,8,2,7,1,4,6	1.000	1.652	1.104	158.4	0.812	1.000	69.072	46.048	71.176	**1.546**

Intermediate data for calculation of load excentricities e_V and of angles α_V:

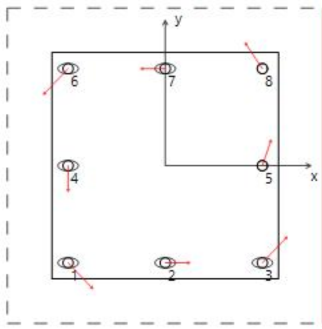
Gr-No.	Anchor-No.	Forces ⇄ ǁ to edge	Treated ǁ → ⊥	$\sum(Q_x)$ [kN]	$\sum(Q_y)$ [kN]	$\sum(Q_x \cdot y)$ [kNmm]	$\sum(Q_y \cdot x)$ [kNmm]	x_{s_Qy} [mm]	y_{s_Qx} [mm]	x_{s_D} [mm]	y_{s_D} [mm]	α_V [°]
1	3,5	-	-	20.222	23.214	-2387.2	3482.1	150.0	-118.0	150.0	-75.0	48.9
2	3,5,8	-	-	20.222	34.821	-2387.2	5223.1	150.0	-118.0	150.0	0.0	59.9
3	3,5,8,2,7	-	-	32.444	34.821	-4220.5	5223.1	150.0	-130.1	90.0	0.0	47.0
4	3,5,8,2,7,1,4,6	√	y+	62.077	-34.821	-6053.8	5223.1	-150.0	-97.5	0.0	0.0	29.3

Panel 7 Anchor shear forces and layer-by-layer verification of the concrete edge failure in case 1 based on EN 1992-4 [1]

Case 2: The anchors 5 and 8 bear the shear load perpandicular to the edge (x+). The torsional moment is carried by all anchors.

Shear forces [kN]:

Anchor No.	Q	Qx	Qy	Qx_V	Qy_V	Qx_T	Qy_T
1	17.725	12.222	-12.838	0.000	-0.615	12.222	-12.222
2	12.222	12.222	0.000	0.000	0.000	12.222	0.000
3	17.725	12.222	12.838	0.000	0.615	12.222	12.222
4	12.838	0.000	-12.838	0.000	-0.615	0.000	-12.222
5	13.541	4.308	12.838	4.308	0.615	0.000	12.222
6	17.725	-12.222	-12.838	0.000	-0.615	-12.222	-12.222
7	12.222	-12.222	0.000	0.000	0.000	-12.222	0.000
8	15.413	-8.530	12.838	3.692	0.615	-12.222	12.222



Explanation:
1. Qx_V, Qy_V are the x- and y-components of Anchor forces from the shear loads.
2. Qx_T, Qy_T are the x- and y-components of Anchor forces from the torsional moment.
3. The assumed slotted holes showed in the figure are not active for the calculation of shear force components Qx_T and Qy_T from the torsional moment. They serve as only for the calculation of shear load components from Qx_V and Qy_V.
4. Edge distance is not to scale.

Verification of resistances with this assumption:

Gr-No.	Anchor-No.	Failure-Line	c_1 [mm]	c'_1 [mm]	α	β	$V^0_{Rk,c}$ [kN]	$A_{c,V}$ [mm²]	$A^0_{c,V}$ [mm²]	$\psi_{A,V}$
1	5,8	5,8	155.0	-	0.074	0.063	23.886	142988	108113	1.323
2	3,5,8	3,5,8	155.0	-	0.074	0.063	23.886	177863	108113	1.645
3	3,5,8,2,7	2,7	305.0	-	0.053	0.055	59.977	303750	418613	0.726
4	3,5,8,2,7,1,4,6	1,4,6	455.0	-	0.043	0.051	104.424	416250	931613	0.447

Gr-No.	Anchor-No.	$\psi_{s,V}$	$\psi_{h,V}$	$\psi_{\alpha,V}$	e_V [mm]	$\psi_{ec,V}$	$\psi_{re,V}$	$V_{Rk,c}$ [kN]	$V_{Rd,c}$ [kN]	V_{Ed} [kN]	$\beta_{V,c}$
1	5,8	1.000	1.000	1.923	12.4	0.949	1.000	57.660	38.440	26.034	0.677
2	3,5,8	1.000	1.000	1.651	43.7	0.842	1.000	54.619	36.413	41.910	1.151
3	3,5,8,2,7	1.000	1.353	1.389	124.4	0.786	1.000	64.289	42.860	48.062	1.121
4	3,5,8,2,7,1,4,6	1.000	1.652	1.131	157.7	0.812	1.000	70.795	47.197	71.491	1.515

Intermediate data for calculation of load excentricities e_V and of angles α_V:

Gr-No.	Anchor-No.	Forces ⇄ || to edge	Treated || → ⊥	$\sum(Q_x)$ [kN]	$\sum(Q_y)$ [kN]	$\sum(Q_x \cdot y)$ [kNmm]	$\sum(Q_y \cdot x)$ [kNmm]	x_{s_Qy} [mm]	y_{s_Qx} [mm]	x_{s_D} [mm]	y_{s_D} [mm]	α_V [°]
1	5,8	-	-	4.308	25.675	0.0	3851.3	150.0	0.0	150.0	75.0	80.5
2	3,5,8	-	-	16.530	38.513	-1833.3	5776.9	150.0	-110.9	150.0	0.0	66.8
3	3,5,8,2,7	-	-	28.752	38.513	-3666.7	5776.9	150.0	-127.5	90.0	0.0	53.3
4	3,5,8,2,7,1,4,6	√	y+	60.231	-38.513	-5500.0	5776.9	-150.0	-91.3	0.0	0.0	32.6

Panel 8 Anchor shear forces and layer-by-layer verification of the concrete edge failure in case 2 based on EN 1992-4 [1]

6. Summary

For the design of anchor groups near concrete edge under shear loads, the concrete edge failure mode plays a decisive role. This applies in particular to fastenings with gap. Since there are often fastenings with gap in practice, the relevant shear force distribution on anchors near the concrete edge cannot be calculated precisely for arbitrary anchor groups. So far, the shear force distribution on anchors close to the edge for the verification of the concrete edge failure is limited for a maximum of four anchors using a conservative assumption that only the two anchors near the edge take the shear load to the edge [1]. If the anchor groups near the concrete edge are mainly loaded by torsional moment, it cannot be ruled out that the back anchors can be decisive for the concrete edge failure.

In order to extend the scope of fastenings to arbitrary anchor group under arbitrary load, a design model for verification of the concrete edge failure of arbitrary anchor groups with or without gap in the fixture under arbitrary shear load is developed. In the case of fastenings with gap, it is assumed that the shear force perpendicular to the edge is taken up only by two adjacent anchors in the first row parallel to the edge. The torsional moment is carried by all anchors in the group.

If there are more than two anchors in anchor groups with gap in the first row of anchors parallel to the edge, the verification of the concrete edge failure is carried out for different cases, with only two adjacent anchors carrying the shear load to the edge in each case. For each case, all rows of anchors parallel to the edge are checked for concrete edge failure. The calculation is implemented into a software [9] for research and design purpose.

To verify the developed design model, 30 tests were recalculated. The calculation results show that the design model has sufficient safety. The design model is therefore suitable for the investigation and design of anchor groups with arbitrary anchor layout under arbitrary shear load and torsional momennt.

Finally, the developed design model is explained in details using three design examples.

Literatur

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