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The enigma of the UFO named *Tic Tac*

Jean Varnier

Senior Research Engineer (i.r.) at The French Aerospace Research Center (ONERA, 92320 Châtillon, France)

Senior Member of The French Association of Aeronautics and Astronautics (3AF, 6 rue Galilée, 75116 Paris, France)

Abstract

On November 14, 2004 off the coast of California, a major incident involved a US Navy fleet, comprising the aircraft carrier USS *Nimitz* and the cruiser USS *Princeton*, and a "*Tic Tac* shaped" aerial vehicle which *Nimitz*'s fighters intercepted two times, the first not far from the surface of the sea, the second at medium altitude. The case is known through testimonies from pilots and sailors, an authenticated video and an exhaustive study carried out in the USA by the Scientific Coalition for Ufology. The unknown vehicle appears capable of standing stationary, being animated with seemingly random movements, maneuvering to keep aircraft at bay, and escaping at very high speed. What is remarkable is the intentional appearance of the behavior of the vehicle, underlined by the witnesses and seeming to emanate from the analysis of the facts. Rather than examining performances that seem to escape what our knowledge of physics allows us to conceive, we hypothesize that the simple geometric shape of this strange vehicle could also convey an intention. The description of the object, the interpretations that have been made of it and the examination of the video allow us to define its shape in a plausible way. Theoretical calculations of surface, volume and compactness can be made from a minimum number of parameters, in particular the compactness with respect to the sphere which only involves the diameter to length ratio, the value of which can be deduced from the analysis of the video pictures with a reasonable uncertainty. Moreover, it seems that a somewhat forgotten universal constant may emanate from the proportions of the vehicle and is consistent with the previous hypothesis. This approach leads in any case to a surprising simplification of the established equations, and to the synthetic expression of their results according to this constant, namely the Golden Ratio phi, key to a geometric enigma as old as the one that led to the number pi.

Keywords: Unidentified Flying Object - USS *Nimitz* UFO Incident 2004 - Analytical Geometry - Golden Ratio

L'énigme de l'OVNI nommé *Tic Tac*

Résumé - Le 14 novembre 2004 au large de la Californie, un incident majeur implique une escadre de l'US Navy, comportant le porte-avions USS *Nimitz* et le croiseur USS *Princeton*, et un aéronef "en forme de *Tic Tac*" que des avions du *Nimitz* interceptent à deux reprises, la première non loin de la surface de la mer, la seconde en altitude. L'affaire est connue par des témoignages de pilotes et de marins, une vidéo authentifiée et une étude très complète effectuée aux USA par la Scientific Coalition for Ufology. L'aéronef inconnu apparaît capable de rester stationnaire, d'être animé de mouvements d'apparence aléatoire, de manœuvrer pour tenir les avions à distance et de s'échapper à très grande vitesse. Ce qui est remarquable, c'est l'apparence intentionnelle du comportement de l'aéronef, souligné par les témoins et semblant émaner de l'analyse des faits. Plutôt que d'examiner des performances qui semblent échapper à ce que nos connaissances en physique nous permettent de concevoir, nous faisons l'hypothèse que la forme géométrique simple de cet étrange aéronef pourrait également être porteuse d'une intention. La description de l'objet, les interprétations qui en ont été faites et l'examen de la vidéo nous permettent de définir cette forme de façon plausible. Des calculs théoriques de surface, de volume et de compacité peuvent être effectués à partir d'un nombre minimal de paramètres, en particulier la compacité par rapport à la sphère qui ne fait intervenir que le rapport diamètre sur longueur, rapport dont la valeur peut être déduite de l'analyse des images vidéo avec une incertitude raisonnable. De plus, il semble qu'une constante universelle quelque peu oubliée puisse émaner des proportions de l'aéronef et soit compatible avec notre hypothèse. Cette approche aboutit en tout cas à une surprenante simplification des équations établies et à l’expression synthétique de leurs résultats en fonction de cette constante, à savoir le Nombre d’Or phi, clé d’une énigme géométrique aussi ancienne que celle qui a abouti au nombre pi.

Mots clés : Objet Volant Non Identifié - Incident OVNI USS *Nimitz* 2004 - Géométrie Analytique - Nombre d’Or

I - Introduction

From 10 to 14 November 2004, during an exercise off the coast of California (USA), a fleet of warships made up mainly of the aircraft carrier USS *Nimitz* and the guided missile cruiser USS *Princeton* detected numerous radar echoes not identified at high altitude (above 24,000 m). On November 14 around 2 p.m., a patrol of two two-seater F/A-18 E/F *Super-Hornet* interceptors having taken off following a low altitude detection flew over an area of restricted diameter where the sea was curiously rough and finally spotted a smooth white "*Tic Tac* shaped” object moving erratically above the waves. The patrol leader then decides to go down to observe him. After imitating the maneuvers of the plane circling around it, the unknown vehicle escapes at high speed as soon as the plane points towards its direction.

Two minutes later, the USS *Princeton* reports that the object remains stationary at the control point planned for the mission (CAP point), located at an altitude of 6,000 m and, surprisingly, at 60 nautical miles from the point of interception. Two hours later, the same object is spotted by an E-2 *Hawkeye* aerial lookout plane about 30 nautical miles from the CAP point and at the same altitude. Two other *Super-Hornets* take off from the aircraft carrier, one of them managing to follow the object and film it during a minute using its infrared camera.

These facts, sometimes referred to as the “USS *Nimitz* incident”, are known from the testimonies of aircraft and ship crews, official documents [1] and the well-documented study made by the Scientific Coalition for Ufology (USA), published in March 2019 [2]. The US Navy video, called *FLIR 1* [3], however, was only officially recognized by the Pentagon (Department of Defense) in April 2020 [4], along with two other videos of unidentified flying objects. The event has now passed into the public domain with the usual extrapolations [5]. The shape of cylinder with hemispherical bases is generally adopted for the *Tic Tac* (Figure 1), this one being sometimes presented in autorotation in the simulations carried out for the television.

Figure 1 Artist's rendering of the *Tic tac* and of an F-18 *Super-Hornet*

It should be noted if need be that the performance of the object in flight clearly surpasses that of our most modern aircraft and seems to defy our current scientific knowledge (apparent ignorance of the effects of gravity, air resistance and inertia). The picture in Figure 2 (a), taken from *FLIR 1* video, shows the object fleeing ahead of the aircraft flying at Mach 0.55 at an altitude of 20,000 feet [3]. At this speed of approximately 350 knots, the object pivots a quarter turn and thus flies for several seconds without modifying its trajectory - Figure 2 (b), then escapes instantly perpendicular to it - Figure 2 (c).

(a) **(b)** **(c)**

Figure 2 Footage taken from the *FLIR 1* video

Our purpose is obviously not to try to find explanations for this astonishing mobility. On the other hand it seems possible, in spite of the uncertain definition of the pictures (halo effect), to conjecture on the shape of the object and to determine its proportions in an approximate way to try to draw usable information from it, according to the principle adopted in Reference [6]. This is the direction in which our analysis is oriented.

2 – Experimental approach

We propose us to examine from a geometric point of view the object named *Tic Tac* by a pilot because of its shape reminiscent of that of the eponymous candy, namely a cylinder whose bases are completed by two hemispheres. In the absence of a common geometric name, although the term “capsule” is sometimes used [7], we will agree to call it spherocylindrical shape, since such an object would be a sphere if the length of its cylindrical part was zero.

Some petroleum products and liquid gas tanks have a similar shape. The spherical-ended cistern is in fact a special case of the standard elliptical-ended cistern (Figure 3), namely that the end ellipsoids are replaced by two hemispheres (Figure 4). This shape is taken up in an approximate way by certain tank wagons (Figure 5), by cigar cases or medicine capsules (Figure 6).

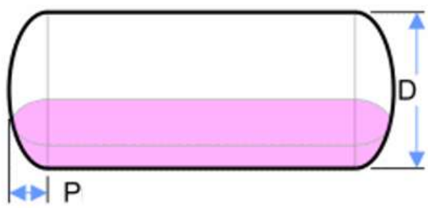


Figure 3

Commonly shaped cistern

Figure 4

Spherocylindrical cistern

It should be noted that the spherocylindrical shape of Figure 4 is entirely described by the dimensionless parameter D / L, where L is the total length of the shape and D the common diameter of the cylinder and of the two hemispheres. This ratio being easy to measure on a photograph, it is to him that we will be interested. But first let us say a few words about the size of the unknown vehicle.

The leader of the first patrol which observed it evolving above the waves indicated that the object was perhaps a little shorter than the Super-Hornet, the length of which is 18 m. A pilot from the second patrol estimated its length at 14 or 15 m, with a diameter of 4 to 5 m. That is why, probably, one of the authors of Reference [2] implicitly takes in a numerical table a diameter to length ratio of 0.34, but it is nowhere mentioned in the text. In fact, it is curious that none of the testimonies quoted in this study make at least mention of the ratio length to diameter, which seems much easier to estimate by sight than metric dimensions. About the image universally accepted to name the object, it can be observed that the D / L ratio of the current *Tic Tac* candy is about 0.6.

Figure 5

Tank wagon

Figure 6

Objects of related form

Due to the incompleteness and uncertainties linked to the testimonies as to the geometry of the object, it seemed to us preferable to stick to the images of the *FLIR 1* video. These images are not of good quality but have the advantage to often show the profile of the vehicle, the shape of which is more reminiscent of a cylinder than the slightly convex body represented in [2]. We therefore modeled it as shown in Figure 7, taking a D / L ratio equal to 0.38. This proportion was determined based on a statistical analysis of the most readable images such as that in Figure 8, a statistic whose result is, on average and standard deviation:

$$D \;/\; L \approx 0{,}38 \;\pm\; 0{,}03 \qquad (1)$$

The average differs slightly from the value 0.4 adopted in Reference [6] at the same time as a length of around 12 m according to the estimate of one of the witnesses quoted in Reference [2].

Figure 7

Geometric shape adopted

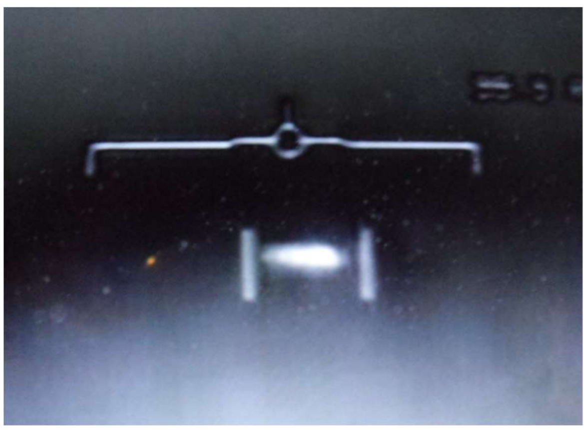


Figure 8

Detail of a picture from the *FLIR 1* video

The only immediately possible calculations are on the one hand that of the surfaces and volumes of the shape, whether it is a question of the quantities concerning its globality, its cylindrical part or its spherical part formed by the fictitious assembly of the two end hemispheres, on the other hand that of its compactness. This one stems from notions of geometry used to compare the interior volume V of a container to its exterior surface S. The surface to volume ratio S / V is generally accepted as a compactness criterion, which one seeks to reduce in architecture, to increase in thermodynamics. In fact, it seems just as natural to consider the inverse ratio V / S which increases the compactness when the surface decreases. In any case, this is the choice made in the study [6], which we will adopt here.

Noting that a characteristic length of the shape we are considering appears in the 1st degree in the numerator of the ratio V / S, we will name it "metric compactness". This one having the disadvantage of depending not only on the shape, but on the dimension of the solid considered, we will introduce to free ourselves from it the notion of compactness in relation to that of the sphere, which we will call "absolute compactness" (cf. § 3). The one of the perfect form that is the sphere being defined as equal to 1, we know that the absolute compactness of any other form will be between 0 and 1.

First, it should be noted that L, D and H, where H denotes the cylindrical length of the object (Figure 9), are linked by the relation

$$L = H + D \qquad (2)$$

since the joined hemispheres give a sphere of diameter D. We can therefore write, by dividing this equality by L, the sum of the reduced quantities w = H / L and t = D / L, namely:

$$\frac{H}{L} + \frac{D}{L} = w + t = 1 \qquad (3)$$

Taking D / L = 0.38 from (1), we get H / L = 0.62, which implies the inequality L > H > D.

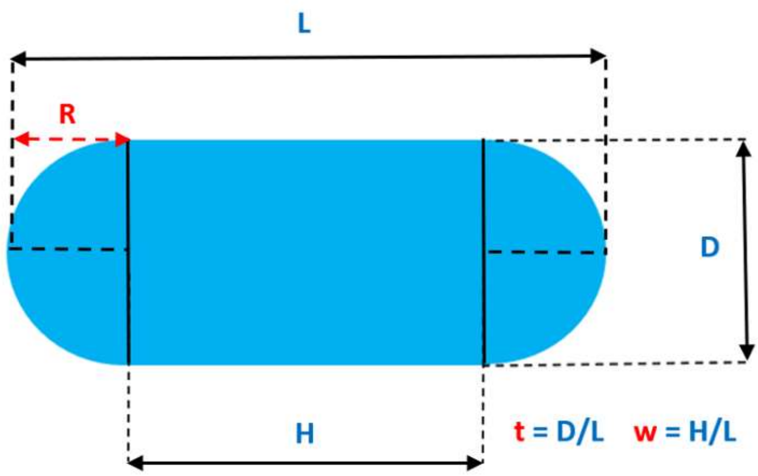


Figure 9 Geometric model and notations used

From a hypothesis made in parallel in the hope that the characteristic lengths of the object can be correlated by a relation less trivial than a sum and an inequality (cf. § 4), we have observed, from of the numerical values found, that the ratios L / H = 1 / 0.62 ≈ 1.613 and H / D = 0.618 / 0.382 ≈ 1.632 were very close. This result suggested us that the equality

$$\frac{L}{H} = \frac{H}{D} \qquad (4)$$

could be the relationship sought. This discovery seemed in any case to confirm our hypothesis, the consequences of which are the subject of § 5.

We can also notice that only the shape of the object intervenes in the identities (3) and (4), its real metric dimensions being indifferent: this has the effect of eluding numerical data whose value cannot be objectively specified. All these findings encouraged us to deepen the study of a mathematical model which seems to be compatible, on first analysis, with the experimental data that we have determined.

3 – Geometric study of the shape

In a first step, we sought to know if the spherocylindrical shape illustrated by Figures 4 to 7 had specific properties. We can first notice that this shape is not particularly aerodynamic, airships, even old ones, being better profiled (Figure 10), modern airships tending towards shapes of fish or of drop of water (Figure 11). Regarding hydrodynamics, we have to go back very far to find quasi-hemispherical bows of submersibles, although some modern submarines now have them [7].

Figure 10

Airship of a classic shape

Figure 11

Airship of a recent type

Curiously, we find related natural forms in biology, such as those of the bacteria studied in Reference [8], which range from the sphere to the elongated cylinder with hemispherical bases, passing through an intermediate form very close to the one we are studying. It can be seen the authors are particularly interested in the ratio of the surface area to the volume of the living forms considered.

As already said, it is clear that the ratio D / L entirely defines our object, the length H of the cylindrical part being equal to the difference L - D. In the practical applications cited in § 2, this ratio does not seem to have any preferred value, ranging for example from 0.2 to 0.35 in Figures 4 to 6. In Reference [9] a 2 D (two-dimensional) cavity, whose ratio D / L is equal to 0.5, is considered to study chaotic acoustic resonances. This proportion is interesting insofar as it implies H = D, which makes it possible, in the case of the corresponding 3 D shape of Figure 12 (a) drawn from Reference [7], to slide the two hemispheres in a way so that they form a sphere inscribed in the cylinder, as shown in Figure 12 (b).

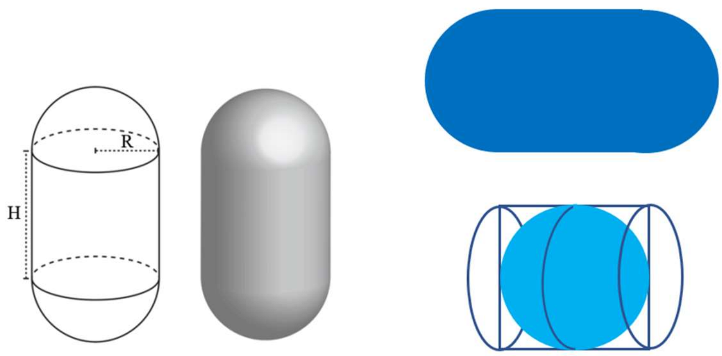
The cylindrical part of length H = D having lateral surface S_C and volume V_C , the two joined hemispheres forming a sphere of diameter D, surface S_S and volume V_S , the classical formulas which give the surface and the volume of the cylinder and of the sphere lead to:

$$S_C = S_S = \pi\, D^2 \qquad (5)$$

$$4\, V_C = 6\, V_S = \pi\, D^3 \qquad (6)$$

$$\frac{S_C}{S_S} = \frac{2}{3}\,\frac{V_C}{V_S} \qquad (7)$$

In the general case of any D / L ratio, we can distinguish in the same way a cylindrical part and a spherical part of the shape to examine the properties before defining those of the global shape.



(a) **(b)**

Figure 12 Case D / L = 0.5: 3 D and 2 D representations, sphere and equivalent cylindrical surface

The remarkable point is that the equation (7), established in the case H = D, remains verified for any value of H different from D. This is on the one hand linked to the fact that for the cylindrical part, S_C and V_C are directly proportional to H, on the other hand to the observation that by virtue of Figure 12 (b) and equation (5), one can always replace the two hemispheres by two cylinders of the same surface. Another consequence is that the total area S_T of the shape can be written

$$S_T = \pi \, D \, L \qquad (8)$$

as if it were a simple cylinder of length L, whatever the value of H ≥ 0. We check that if H = 0 we have L = D, the expression (8) then giving the surface πD^2 of the residual sphere of diameter D.

The formula giving the total volume V_T of the shape is more complex. We find in fact:

$$V_T = \pi \, D^2 \left(\frac{D}{6} + \frac{H}{4} \right) \qquad (9)$$

We can now calculate the metric compactness of the shape in the general case. Let us first note that the metric compactness C_S of a sphere of radius R is:

$$C_S = \frac{V_S}{S_S} = \frac{R}{3} \qquad (10)$$

The metric compactness C_T of the spherocylindrical shape is obtained in the same way by the ratio of expressions (9) and (8), knowing that L = H + D:

$$C_T = \frac{V_T}{S_T} = \frac{D}{12} \left(3 - \frac{D}{L} \right) \qquad (11)$$

We see that if L = D, we find the metric compactness D / 6 of the sphere. At this stage, it is convenient to introduce the dimensionless reduced variable t = D / L of Figure 9. We then obtain the expression:

$$C_T = \frac{L}{12} \, t \, (3 - t) \qquad (12)$$

We will name “shape factor” the dimensionless factor t (3 – t).

Knowing that for a given surface area, the sphere is the shape that encompasses the largest volume, we now consider the compactness of a geometric body compared to that of the sphere. This so-called absolute compactness C_{abs} will be a dimensionless number depending only on the shape of the considered body. As indicated in § 2, the absolute compactness of the sphere is the maximum compactness and is obviously equal to 1.

The simplest approach to calculating this quantity is as follows: first, a "reference sphere" is defined whose surface S_{ref} is equal to the surface S_T of the considered body. The absolute compactness, equal to the ratio C_T / C_{Sref}, will then be equal to the ratio of volumes V_T / V_{ref}, the term S_{ref} / S_T in factor being by hypothesis equal to 1.

The equality $S_T = S_{ref}$ makes it possible to find, for purpose of substitution, the relation between the diameter D_{ref} of the reference sphere and the diameter D of the spherocylindrical body:

$$D_{ref} = D\sqrt{\frac{L}{D}} = \frac{D}{\sqrt{t}} \tag{13}$$

Note that the cylindrical length H is implicitly taken into account in the total length L and therefore in the ratio t. It is verified that if t = 1 therefore H = 0 and L = D, there remains D_{ref} = D, the residual sphere and the reference sphere then being merged.

We can now calculate the absolute compactness of the body from formula (9) and the volume V_{ref} of the reference sphere, which by applying formula (13) is equal to:

$$V_{ref} = \frac{\pi}{6}\, {D_{ref}}^3 = \frac{\pi}{6}\left[\frac{D}{\sqrt{t}}\right]^3 \tag{14}$$

The cylindrical length H appearing in (9), we use the reduced variables w = H / L and t = D / L for the calculation, knowing that w + t = 1 as indicated by the relation (3). After a few manipulations, we arrive at the following form of absolute compactness:

$$C_{abs} = \frac{V_T}{V_{ref}} = \frac{\sqrt{t}}{2}\,(\,3 - t\,) \tag{15}$$

We check that if t = 1, which corresponds to L = D, we have C_{abs} = 1, the reference compactness of the sphere.

It is not surprising to find that the absolute compactness which only characterizes the shape of the object is expressed only according to the dimensionless ratio t = D / L.

4 – The Golden Ratio phi

Curiously enough, the idea that the Golden Ratio phi could intervene in the proportions of the Tic Tac in the form of the relation L / H ≈ φ was suggested to us by its aesthetics and distant memories of reading, not by the relation (4) of § 2 which we had not yet found.

Present in the nature and, since Antiquity, in works of art, the Golden Ratio is indeed a proportion that intervenes in the balance and beauty that we subjectively attribute to forms. An example that seems to attest to a very ancient knowledge of this number - although this is disputed by some - is given by the proportions of the Great Pyramid of Cheops. Its slope of 14/11 in fact, already determined by Thales of Miletus from its shadow cast (around 600 B.C.), gives to its apothem, with the conventions of Figure 13, the value 1.61859.

Moreover, its base perimeter is close enough to the circumference of the circle whose radius is the height h of the pyramid so that its division by the diameter of this circle gives 3.14286 (Figure 14). Knowing that the rounded value of the Golden Ratio phi is $\varphi \approx 1.61803$ and that of the number pi is $\pi \approx 3.14159$, we are struck by the precision of the approximate values of these two irrational numbers from a slope expressed by the ratio of two small integers which resembles a rounding. In fact, the relative errors committed are respectively 0.035 % for phi and 0.040 % for pi.

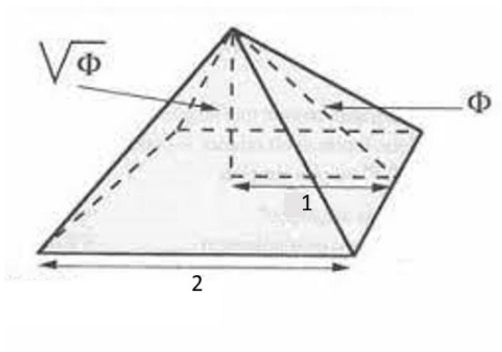


Figure 13

Ratios of the Pyramid of Cheops

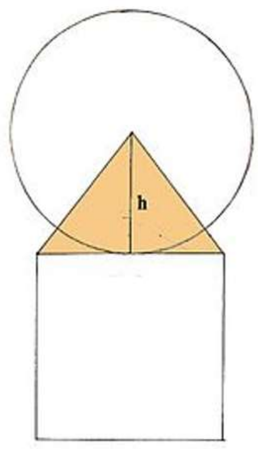


Figure 14

Perimeter and equivalent circumference

We can notice that the relationships suggested by Figures 13 and 14 can be summarized in a single formula:

$$\pi \sqrt{\varphi} \approx 4 \qquad (16)$$

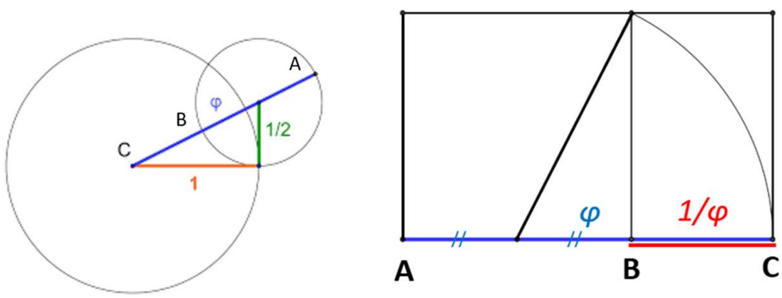
where 4 is the semi-perimeter of the pyramid according to the conventions of Figure 13. In fact, the exact values of π and φ injected into this equation give 3.99627, i.e. a default deviation of 0.1 % compared to the expected result. Surprisingly, this difference is greater than the 0.04 % error that would result from introducing into equation (14) the approximate values of the constants π and φ obtained from the slope 14/11. Anyway, this pretty relationship is unfortunately only an approximation.

A geometric definition of the Golden Ratio due to Euclid considers three points A, B, C carried by the same line and such that AB > BC (see figure 15). If the AC / AB and AB / BC ratios are equal, the common value of these ratios is the Golden Ratio. This leads us to the identity:

$$\frac{AC}{AB} = \frac{AB}{BC} = \varphi \qquad (17)$$

Figure 15 shows two geometric constructions of the Golden Ratio derived from more complex constructions proposed by Euclid [10]. The radius of the great circle (a) or the side of the square (b) being taken as a unit, we have AB = 1, AC = φ and BC = 1 / φ. Note that the application of the Pythagorean theorem to the right triangle of Figures 15 (a) and 15 (b) makes it possible to find by simple addition the length AC, therefore the numerical value of φ according to the unit chosen. Note that Figure 15 (b) also has two "golden rectangles" whose aspect ratio is equal to φ.

We can also see that if it was the AC segment that was taken as a reference instead of AB, we would obtain AC = 1, AB = 1 / φ and BC = 1 / φ^2. We can make the connection with relation (3) which amounts to taking the length L of the object as a unit, and with relation (4) whose ratios necessarily have the value φ. Of course, the analogy between relation (4) and relation (17) is only effective if we consider D as the sum of the two radii R collinear with L and H (Figure 9).



(a) **(b)**

Figure 15 Geometric constructions of the Golden Ratio phi

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It is probably worth emphasizing that the geometric relationship considered gives mathematical legitimacy to the Golden Ratio which could, failing that, pass for a view of the mind. The unique solution φ of the equality AC / AB = AB / BC, "proportion of extreme and average ratio" of Euclid(1) sometimes called continuous harmonic proportion(2), therefore has a worth of a universal constant, at least in the Euclidean space, just like the ratio π of the circumference and the diameter of a circle.

A rather singular resurgence appears in the Middle Ages with the Fibonacci sequence [11], solution of a counting problem whose terms Ui are defined by [$U_1 = 1$; $U_2 = 2$; $U_n = U_{n-1} + U_{n-2}$], i.e. [1 ; 2 ; 3 ; 5 ; 8 ; 13 ; 21 ; 34 ; 55 ; ...]. It turns out that the ratio of two consecutive terms U_n / U_{n-1} tends, by excess then by default alternately, towards the Golden Ratio. We can see here, from the last three terms expressed, that we have 34 / 21 ≈ 1,61905 > φ et 55 / 34 ≈ 1,61765 < φ.

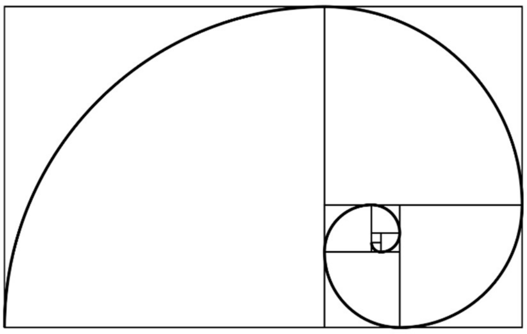

Figure 16 The golden spiral, asymptote of the Fibonacci spiral

(1) Probably alluding to the form AC. BC = AB^2, where AC and BC are the largest and the smallest segment, AB the medium length segment. It has remained, in the vocabulary of the algebraic proportions, "the product of the extreme and mean terms".

(2) The term "continuous" probably distinguishes it from the classic harmonic division AI / AJ = - BI / BJ, which involves two pairs of points (A, B) and (I, J) aligned in the order A, I, B, J on an oriented line. It would seem to us more precise to name it "harmonic division to three points", the principle considered in the division to four points being the same.

The Fibonacci sequence can be represented in two dimensions by a curve connecting its terms which tends asymptotically towards the “golden spiral”, a logarithmic spiral passing through the homologous points of nested golden rectangles as shown in Figure 15 (b). The outline of this spiral is shown in Figure 16. It can be extended ad infinitum in both directions: it is therefore a fractal object, which explains why it is found in various forms in the animal or plant kingdom.

The Golden Ratio enjoyed great popularity among scholars - mathematicians, philosophers and artists - during the Renaissance [12], the 19th Century [13] and the beginning of the 20th Century [14]. But because of alleged mystical or esoteric aspects and in defiance of its obvious mathematical properties [15], it seems to have subsequently fallen into disuse, if not disgrace: after 1950, one could follow an entire schooling in France without ever hearing about it, whether in history, drawing, natural sciences, algebra or even geometry. At the same time appeared, under the pressure of scientific rationalism, the relatively new idea that man could be, in the universe, the only being endowed with conscience and reason [16].

The Golden Ratio φ is also defined algebraically as the positive solution of the equation

$$x^2 = x + 1 \qquad (18)$$

result that can also be found from the geometric constructions of Figure 15, namely:

$$\varphi = \frac{1 + \sqrt{5}}{2} \qquad (19)$$

Note that the passage from the geometric definition (17) to the algebraic expression (18) is simple: it suffices to consider its first two terms and to set AC = x, AB = 1, BC = x – 1, the equality obtained identifying with equation (18) whose solutions are $x_1 = \varphi$ and $x_2 = - 1 / \varphi$.

The numerical calculation of (19) makes it possible to fix the approximate value $\varphi \approx 1.6180340$ with a relative error of less than 10^{-8}. It is interesting to remark that this precise value already results from the ratio of the 20th and 19th terms of the sequence of Fibonacci, a mathematician who also became aware of the relationship of the Golden Ratio with the 2nd degree equation mentioned afore.

It can also be noted that the identity

$$\varphi^2 = \varphi + 1 \qquad (20)$$

is the expression of the Pythagorean theorem in Figure 13. It is in fact to Euclid that we owe the first demonstration of this ancient theorem that has come down to us. Moreover, the algebraic properties of the number φ resulting from equation (20) induce relations such as:

$$\frac{1}{\varphi} = \varphi - 1 \quad (21)$$

$$\frac{1}{\varphi} + \frac{1}{\varphi^2} = 1 \quad (22)$$

$$\frac{1}{\varphi^2} = 2 - \varphi \quad (23)$$

$$\varphi^2 + \frac{1}{\varphi^2} = 3 \quad (24)$$

Curiously, these identities make it possible to simplify algebraic expressions already apparently reduced.

In the following we will not detail, thanks to these formulas, calculations whose results are easily verifiable with a calculator, taking $\varphi \approx 1.618034$.

5 - Geometric shape and Golden Ratio

We will call "ideal geometric shape" the spherocylindrical shape defined by a ratio of total length to cylindrical length equal to the Golden Ratio. This is the working hypothesis that we made in parallel with the experimental approach described in § 2.

The reduced variables w = H / L and t = D / L are linked according to (3) by the relation

$$w + t = 1 \quad (25)$$

the length L of the object being equal to 1 in its reduced form.

Moreover, equation (4), which in fact characterizes the ideal shape in question, is identical to equation (17). It follows that

$$\frac{L}{H} = \frac{H}{D} = \varphi \quad (26)$$

phi being the Golden Ratio whose rounded value is $\varphi \approx 1.618034$. This equation is written again:

$$\frac{1}{w} = \frac{w}{t} = \varphi \quad (27)$$

which leads immediately to the solutions

$$w = \frac{1}{\varphi} \qquad t = \frac{1}{\varphi^2} \qquad (28)$$

Equation (25) is then identified with equation (22).

Let us express the shape factor t (3 – t) of equation (12) as a function of φ and apply the identity (24), it comes:

$$t\,(3 - t) = \frac{1}{\varphi^2}\left(3 - \frac{1}{\varphi^2}\right) = \frac{\varphi^2}{\varphi^2} = 1 \qquad (29)$$

This remarkable result characterizes the ideal geometric shape independently of its dimensions, its metric compactness being written for a given length L, according to equation (12):

$$C_T = \frac{V_T}{S_T} = \frac{L}{12} \qquad (30)$$

We see that the ratio D / L adopted here, i.e. t = 1 / φ^2, leads to the value 1 of the shape factor whatever the real length of the object, which therefore remains indeterminate - curiously, the choice L = 12 m made in [6] leads the author to assume $C_T = 1$, which would immediately follow from identity (30).

The numerical calculation of the relations (28) gives:

$$t = \frac{1}{\varphi^2} = 0{,}381966 \approx\ 0{,}382 \qquad (31)$$

$$w = \frac{1}{\varphi} = 0{,}618034 \approx\ 0{,}618 \qquad (32)$$

These values are very close to those we determined in § 2 from the pictures of the video footage [3], namely t ≈ 0.38 and w ≈ 0.62. Chance or necessity, this is obviously a strong argument in favor of our starting hypothesis L / H = φ.

We now consider the expression (15) of the absolute compactness of the body C_{abs} calculated here with respect to a sphere of the same surface and we substitute the variable t = 1 / φ^2:

$$\frac{\sqrt{t}}{2}(3 - t) = \frac{1}{2\,\varphi}\left(3 - \frac{1}{\varphi^2}\right) \qquad (33)$$

The identity (22) makes it possible to simplify this expression and to obtain:

$$C_{abs} = \frac{\varphi^2}{2\,\varphi} = \frac{\varphi}{2} \qquad (34)$$

Starting from a 3rd degree expression in $\sqrt{t}$, this result is of an unexpected simplicity. It is interesting to note that its value ($C_{abs} \approx 0.809$) remains very close to that of the compactness of an ellipsoid which would have the same length and the same diameter ($C_{abs(1)} \approx 0.819$, see Appendix A). The volume of this ellipsoid is approximately 3/4 of the volume of the ideal geometric shape. Figure 17 (a) illustrates this comparison.

It should be noted that the ellipsoid generated here by the rotation of the ellipse around its major axis is entirely included in the shape. In fact, this same 2 D figure could just as well represent a wheel-shaped ellipsoid generated by the ellipse rotating around its minor axis: its compactness $C_{abs(2)} \approx 0.768$ would then be lower than that of the ideal geometric shape, but with a volume twice larger. The calculations relating to the compactness and the volume of the ellipsoids are given in Appendix A, those concerning the isoperimetric approach of the problem are the subject of Appendix B.

We notice otherwise that the expressions (33) and (34) lead to the equality

$$\sqrt{t}\,(3 - t) = \varphi \qquad (35)$$

and that the equations (29) and (35), respectively related to a particular value of the shape factor and of the absolute compactness, lead after simplification to:

$$\sqrt{t} = \frac{1}{\varphi} \qquad (36)$$

This identity in fact directly restores the common solution of the system $t = 1 / \varphi^2$.

Figure 17 (a)

Ideal geometric shape and ellipsoid

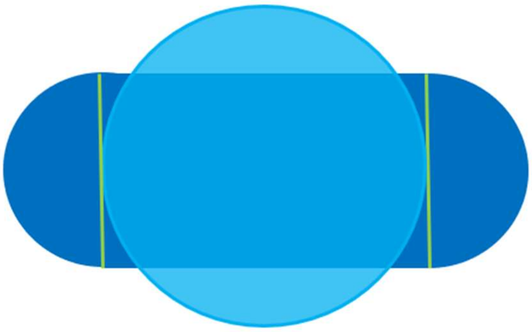

Figure 17 (b)

Ideal geometric shape and reference sphere

We now consider the ratio of the formal expressions (12) and (15) giving respectively the metric compactness C_T and the absolute compactness C_{abs} of the form in the general case. We obtain:

$$\frac{C_T}{C_{abs}} = \frac{L}{6}\sqrt{t} = \frac{1}{6}\sqrt{L\,D} \qquad (37)$$

or again, by applying equation (13) of § 3 without using a reduced variable to express the diameter of the reference sphere, i.e. $D_{ref} = \sqrt{L\,D}$:

$$\frac{C_T}{C_{abs}} = \frac{1}{6}\,D_{ref} \qquad (38)$$

We verify that if D = L, we obtain D_{ref} = D, C_{abs} = 1 and C_T = D / 6, the metric compactness of the sphere.

Let us finally substitute the value t = 1 / φ^2 in the equation (13) which makes it possible to equalize the total surface of the shape and that of the reference sphere. The diameter of the sphere is then written:

$$D_{ref} = \varphi\,D = H \qquad (39)$$

This result, illustrated by Figure 17 (b), is a remarkable consequence of the application of the Golden Ratio to the proportions of the form.

Regarding the surfaces and geometric volumes that constitute it, we can notice that the ratio of the surfaces of the cylinder and the sphere resulting from the two joined hemispheres is equal to:

$$\frac{S_C}{S_S} = \varphi \qquad (40)$$

This result is in fact a direct consequence of relations (8) and (26): knowing that the sphere can always be replaced by a cylinder of the same length and the same surface - Figure 12 (b), it is obvious that the ratio of surfaces considered will be equal to the ratio of their lengths H / D, therefore to φ. We obtain from equation (9) a similar relationship for the ratio of the volumes:

$$\frac{V_C}{V_S} = \frac{3}{2}\,\varphi \qquad (41)$$

Finally, equation (8) giving the total area of the shape becomes:

$$S_T = \frac{\pi\,L^2}{\varphi^2} \qquad (42)$$

an elegant formula that results both from the properties of the shape itself and from those linked to the introduction of the Golden Ratio.

Unexpectedly, equation (9) giving the total volume of the shape simplifies through identities (21) to (24) and resolves to:

$$V_T = \frac{\pi\, L^3}{12\, \varphi^2} \qquad (43)$$

In fact, this result could be predicted immediately by substituting S_T between equation (42) and equation (30) giving the metric compactness of the shape.

If we consider, as we did to deal with the dimensionless problem, that the length L is unitary, we obtain the following expressions of the reduced surface and volume:

$$S_T = \frac{\pi}{\varphi^2} \approx \frac{6}{5} \qquad V_T = \frac{\pi}{12\, \varphi^2} \approx \frac{1}{10} \qquad (44)$$

The numerical calculation gives more precisely $S_T = 1.1999816 \approx 1.2$ and $V_T = 0.0999985 \approx 0.1$ with a default relative deviation of $-1.5.10^{-5}$ compared to the rounded values. This difference, much smaller than in the case of equation (16) taken from figures 13 and 14 of § 4, tends to suggest that there is a mysterious sympathy between the ancient geometric ratios pi and phi. Starting from the geometric properties of the pentagon whose diagonal is φ if the side is 1 [15], we can in any case demonstrate the following formal relation:

$$2\, \varphi \cos\frac{\pi}{5} = 1 \qquad (45)$$

Even disregarding these numerical particularities, it is remarkable that the reduced surface and volume of the unknown aerial vehicle, or at least of the model that it inspired, are expressed in such a direct way from two universal constants, namely "fixed dimensionless quantities intervening in the equations of physics".

6 - Conclusion

We have arrived at the end of this itinerary which looks like a mathematical game as there are so many surprises. One cannot help comparing equation (16) with equalities (44) or Figure 14 with Figure 17 (b), testifying 45 centuries apart to the curious properties that result from the intervention of the Golden Ratio in geometry and other fields. Regarding our study, we see that algebraic relations that seem irreducible transform in mysterious ways and give results of unexpected simplicity, this being also linked to the specific properties of the shape attributed to the aerial vehicle.

It is obviously possible that all this is the result of chance and that the shape and the real proportions of the observed object are a little different from those that we have determined based on the video pictures. But the fact that they have inspired us with a study that gives such consistent results is in itself quite surprising. The question is obviously whether or not this is the consequence of an intention, a sign of intelligence in some way, as strange as that may seem.

In this regard, the testimonies reported about the incident of the USS *Nimitz* and the unknown aerial vehicle suggest the following observations.

1) The appearance of the vehicle is not a surprise because it was preceded by numerous detections, spread over time, of unidentified radar echoes at very high altitude.
2) The positioning of the vehicle over a foamy area of restricted extent, whatever the cause of this aquatic disturbance, clearly favored its visual detection by a patrol of fighters.
3) The movements of the vehicle copying those of the patrol leader's plane, then its sudden escape as soon as an apparently hostile act was triggered, testify to the awareness of the presence of the plane and the adequacy of reactions to its maneuvers.
4) The subsequent station of the vehicle at the air control point of the mission, anticipating the passage of the patrol, unexpectedly implies the knowledge of confidential information, since the passage by this point signals friendly aircraft. For this reason, this fact which evokes a clear intrusion could also be interpreted as an act of appeasement.
5) The late reappearance of the vehicle in the surrounding and at the altitude of this control point, and its apparent complacency in presenting itself in profile for observation while continuing to fly in a perpendicular direction, has look like a demonstration but could proceed from any other intention, for example allow the observer to detail this profile.

Even if delivering a message without giving the recipient the opportunity to respond to it seems futile, our general impression is that everything happened as if a willingness directing the unknown vehicle had knowingly conducted a significant scenario. Analogue opinions in fact emanate from certain testimonies [2]. Starting from this hypothesis, it could be that the particular shape of the vehicle and its remarkable geometric properties are also the expression of an original mode of communication requiring only the common knowledge of constants of universal worth. We remember on this subject the symbolism used in the message of Carl Sagan et al. sent towards the stars in 1977 [17].

Special greetings

This essay is dedicated to my colleagues and friends of The French Association of Aeronautics and Astronautics and to all the witnesses, authors and researchers who, despite received ideas and current trends, have the curiosity and the courage to face the Unknown to advance the Knowledge.

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8 - Appendix **A: ellipsoids of revolution**

The ellipsoid is the volume generated by the rotation around one of its axes of a generating ellipse represented by Figure A 1. This ellipse comprises a semi-major axis *a*, a semi-minor axis *b*, two foci F' and F located at distance *c* of the intersection of the axes. This distance is defined geometrically as seen in Figure A 1 and scaled with respect to the semi-major axis a to define the eccentricity e:

$$e = \frac{1}{a}\sqrt{a^2 - b^2} \qquad (a1)$$

We see that $0 \leq e \leq 1$, the ellipse being a circle of radius $a = b = R$ when the eccentricity is zero.

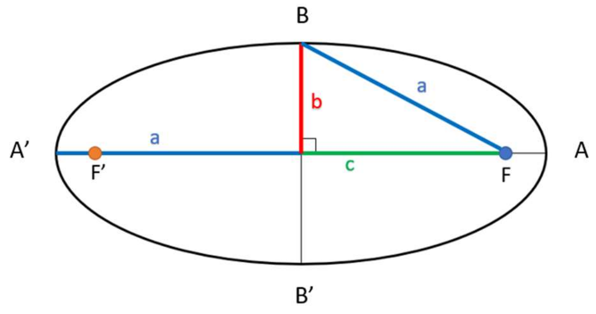
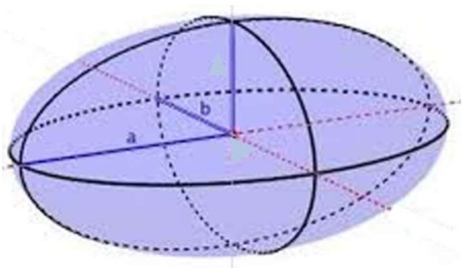


Figure A 1

Generating ellipse



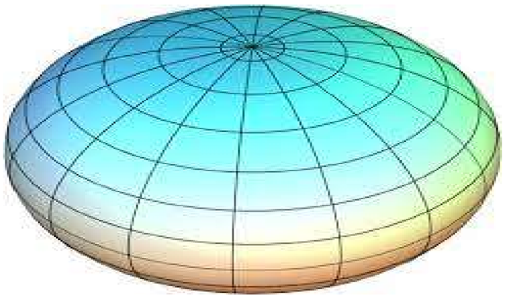


Figure A 2

Figure A 3

Rotation of the ellipse around major axis AA'

Rotation of the ellipse around minor axis BB'

Two axes of rotation are possible to define a symmetry of revolution: the major axis AA' and the minor axis BB'. In the first case (1) an elongated shape (E1) reminiscent of a rugby ball (Figure A 2) is obtained, in the second case (2) a flattened shape (E2) reminiscent of a rounded pebble (Figure A 3).

The interest of these forms lies in the fact that they appear in numerous testimonies of unidentified flying objects. The elongated ellipsoid appears for example in the representation of the *Tic Tac* given in Reference [2], while the wheel-shaped ellipsoid seems appropriate to describe the object appearing in the video taken on April 26, 2013, from a US Coast Guard aircraft from Aguadilla Airport in Puerto Rico [18].

The formulas giving the volume $V_{(x)}$ of the ellipsoid (E_X) are different depending on whether we are in case (1) or (2), but logically related:

$$V_{(1)} = \frac{4}{3} \pi \, a \, b^2 \qquad (a2)$$

$$V_{(2)} = \frac{4}{3} \pi \, a^2 \, b \qquad (a3)$$

We find of course the formula for the volume of the sphere $V = 4 \pi R^3 / 3$ when $a = b = R$.

On the other hand, the formulas giving the surface $S_{(x)}$ of the ellipsoid as a function of the eccentricity e are notoriously different:

$$S_{(1)} = 2\pi \left(b^2 + \frac{a\,b}{e}\ arc\ sinus\ e\right) \qquad (a4)$$

$$S_{(2)} = 2\pi \left(a^2 + \frac{b^2}{2\,e}\ Ln\ \frac{1+e}{1-e}\right) \qquad (a5)$$

It is obvious that $S_{(1)}$ tends towards the formula of the surface of the sphere $S = 4 \pi R^2$ when $a = b = R$, but to find this result for $S_{(2)}$, it is necessary to use the limited development of the logarithm at the neighborhood of 1, which ultimately leads to $Ln\ (1 + e) - Ln\ (1 - e) \approx 2\ e$.

Apart from this particular case, we are dealing with two transcendent equations, which makes it impossible to separate the variables. We will therefore have to go through the numerical calculation.

Let us now approach the calculation of the compactness of the ellipsoids in the case $a = 1$, $b = 1 / \varphi^2$. We see that this choice doubles for convenience the size of the geometric shapes of length 1 considered in § 5.

The procedure is identical to that which is indicated in § 3: we calculate the diameter of a reference sphere having the same surface as the ellipsoid considered, then we calculate the ratio of the ellipsoid volume $V_{(x)}$ to the volume $V_{S\,(x)}$ of the reference sphere which gives the absolute compactness $C_{abs\,(x)}$ of the ellipsoid. In fact, it is more convenient here to introduce the radius $R(x)$ of the reference sphere to operate the simplifications. The relationships found are as follows:

$$a = 1 \qquad b = \frac{1}{\varphi^2} \qquad e = \sqrt{3\,\varphi - 4} \qquad (a6)$$

$$R(1)^2 = \frac{1}{2}\left(\frac{1}{\varphi^4} + \frac{1}{e\,\varphi^2}\ arc\ sinus\ e\right) \qquad (a7)$$

$$R(2)^2 = \frac{1}{2}\left(1 + \frac{1}{2\,e\,\varphi^4}\ Ln\ \frac{1+e}{1-e}\right. \qquad (a8)$$

$$V_{S\,(1)} = \frac{4}{3}\pi\, R(1)^3 \qquad V_{S\,(2)} = \frac{4}{3}\pi R(2)^3 \qquad (a9)$$

$$V_{(1)} = \frac{4}{3}\ \frac{\pi}{\varphi^4} \qquad V_{(2)} = \frac{4}{3}\ \frac{\pi}{\varphi^2} \qquad (a10)$$

$$C_{abs(1)} = \frac{V_{(1)}}{V_{S\,(1)}} = \frac{1}{R(1)^3\ \varphi^4} \qquad (a11)$$

$$C_{abs(2)} = \frac{V_{(2)}}{V_{S\,(2)}} = \frac{1}{R(2)^3\ \varphi^2} \qquad (a12)$$

The reduced form of the eccentricity e is allowed by the algebraic properties of the Golden Ratio. We have in fact $1 - 1 / \varphi^4 = 3 / \varphi - 1 = 3\ \varphi - 4$. The numerical application gives the results:

$e = 0{,}924176$

$R_{(1)} = 0{,}562642$

$R_{(2)} = 0{,}792231$

$C_{\mathrm{abs}(1)} = 0{,}819131 \approx 0{,}819$

$C_{\mathrm{abs}(2)} = 0{,}768191 \approx 0{,}768$

The ratio of the volumes of the ellipsoids to the spherocylindrical shape is a calculation independent of the previous one. It is necessary to take care with the fact we have set $a = 1$, it results from it that $L = 2\ a = 2$, and the formula (43) giving the volume of the form becomes:

$$V_T = \frac{2}{3}\ \frac{\pi}{\varphi^2} \qquad (a13)$$

It is now enough to compare this result and the expressions (a10) giving the volume of the ellipsoids to obtain after simplification:

$$\frac{V_{(1)}}{V_T} = \frac{2}{\varphi^2} \approx 0{,}764 \qquad \frac{V_{(2)}}{V_T} = 2 \qquad (a14)$$

Of course, the result

$$\frac{V_{(2)}}{V_{(1)}} = \frac{a}{b} = \varphi^2 \qquad (a15)$$

follows directly from expressions (a2) and (a3) and the assumption concerning the values of a and b.

9 - Appendix B: isoperimetry and compactness

Resulting from an ancient problem concerning the area included in the perimeter of a given plane figure, the isoperimetric quotient q is a shape descriptor which allows us to evaluate the sphericity of a geometric shape of surface area S and volume V. Expressed at the origin under the form of the ratio

$$q = \frac{V^2}{S^3} \qquad (b1)$$

or of its inverse S^3 / V^2, it is a dimensionless quantity independent of the size of the object that we consider. Knowing that the isoperimetric quotient of the sphere is q_S = 1 / 36 π, we will preferably use the isoperimetric quotient reduced with respect to q_S [19] which is written

$$Q = 36\ \pi \frac{V^2}{S^3} \qquad (b2)$$

an expression whose maximum possible value, corresponding to the case of the sphere, is Q = Q_S = 1. We see the analogy with the absolute compactness C_{abs} defined in § 2. Starting from this definition, the isoperimetric quotient Q_T of the ideal shape described in § 5 can be deduced from equations (44):

$$Q_T = 36\,\pi \frac{{V_T}^2}{{S_T}^3} = \frac{\varphi^2}{4} \approx 0{,}654 \qquad (b3)$$

We obtain in the same way the quotients Q_1 and Q_2 of ellipsoids (E_1) and (E_2) defined in Appendix A

$$Q_{1,2} = 36\,\pi \frac{{V_{(1,2)}}^2}{{S_{(1,2)}}^3} \qquad (b4)$$

$S_{(1)}$ and $S_{(2)}$ being the surfaces of the reference spheres of (E_1) and (E_2) the radii of which have been calculated. The numerical application gives $Q_1 \approx 0.671$ and $Q_2 \approx 0.590$. Logically, we find the same order of values $Q_2 < Q_T < Q_1 < 1$ as in the case of the absolute compactness, i.e. $C_{abs(2)} < C_{absT} < C_{abs(1)} < 1$.

In fact, the comparison of the numerical values found for these quantities shows that the isoperimetric quotient Q and the absolute compactness C_{abs} of these shapes are linked by the equality:

$$Q = {C_{abs}}^2 \qquad (b5)$$

This relation, obvious for the sphere, arises from the literal calculation in the case of the ideal shape ($C_{absT} = \varphi / 2$, $Q_T = \varphi^2 / 4$), and from the numerical calculation in the case of the ellipsoids, but is of a general manner demonstrable through the artifice of the reference sphere that we defined in § 3.

Consider a solid of area S and volume V, S_{ref} and V_{ref} being area and volume of its reference sphere. We have then $S_{ref} = S$ and $C_{abs} = V / V_{ref}$. The reduced isoperimetric quotient of the solid can be written:

$$Q = \frac{V^2}{S^3} \frac{{S_{ref}}^3}{{V_{ref}}^2} \qquad (b6)$$

knowing that the second term of this expression is worth 36 π as indicated by equation (b2), which would be the case for any other sphere. But if we are careful not to make this substitution, the surfaces S_{ref} and S, being equal, simplify and there remains the identity (b5), which had to be demonstrated.

What may look like a writing game is in fact an essential point because it dispenses, in practice, with having to calculate the diameter and the volume of the sphere of reference as in Appendix A: it suffices to know the surface and the volume of a solid to obtain, by means of equations (b2) and (b5), its absolute compactness, a quantity more interesting than the isoperimetric quotient because it has a precise geometric meaning, namely a ratio of volumes with equal surface areas.

We can take the example of two regular polyhedra, the cube and the tetrahedron. The volume V of the cube with edge a is equal to a^3, but we can take $a = 1$, from which $V = 1$. Its surface area S equals 6 a^2, therefore $S = 6$. The arithmetic calculation of equation (b2) and of equation (b5) reverse then gives:

$$Q = \frac{\pi}{6} \qquad C_{abs} = \sqrt{\frac{\pi}{6}} \approx 0{,}724 \qquad (b7)$$

The volume V of the regular tetrahedron with edge $a = 1$ is $\sqrt{2} / 12$, its surface area S is $\sqrt{3}$. We obtain:

$$Q = \frac{\pi \sqrt{3}}{18} \qquad C_{abs} = \sqrt{Q} \approx 0{,}550 \qquad (b8)$$

The implicit use of a non-specified reference sphere is the whole point of this approach.