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"Multi-Fidelity Modeling of Thermal Stratification and Slosh Dynamics in Cryogenic Rocket Tanks for Spacecraft Propulsion Systems"

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ABSTRACT

The performance and reliability of liquid rocket propulsion systems are significantly influenced by the thermo-fluid behavior of cryogenic propellants within storage tanks. This study presents an integrated multi-physics model for analyzing the coupled effects of thermal stratification and liquid slosh dynamics under dynamic flight conditions. The thermodynamic behavior is modeled using a lumped parameter approach that divides the tank into ullage gas, stratified liquid, bulk liquid, and tank wall thermal nodes, with governing mass and energy conservation equations used to simulate heat transfer, evaporation, condensation, and self-pressurization. Slosh dynamics are represented using reduced-order mechanical analog models, including pendulum and spring–mass–damper systems. The coupling between thermal and fluid motion is modeled through time-varying ullage volume, interfacial heat transfer, and fluid mixing. The integrated model is implemented in MATLAB/Simulink, while CFD simulations are used to analyze slosh behavior and baffle performance, and rocket flight simulations evaluate the impact on vehicle performance. Results show that sloshing disrupts thermal stratification, increases ullage pressure fluctuations, and reduces Net Positive Suction Head, potentially causing turbopump cavitation. Baffle installation significantly reduces slosh amplitude and improves propulsion system stability and reliability.

Keywords: Cryogenic propellant tanks, Thermal stratification, Slosh dynamics, Ullage pressure, Rocket propulsion systems, Lumped parameter modeling, CFD simulation, MATLAB/Simulink, Propellant feed systems, Turbopump cavitation, Spacecraft propulsion, Thermo-fluid dynamics, Baffle design, Rocket stability

1.0 INTRODUCTION

Liquid rocket propulsion systems rely on cryogenic propellants such as liquid hydrogen and liquid oxygen stored in pressurized tanks. The thermodynamic and fluid dynamic behavior of these propellants inside storage tanks significantly influences propulsion performance, tank pressure stability, and engine reliability. Cryogenic fluids are stored at very low temperatures, and heat leakage through tank walls leads to evaporation and temperature gradients within the liquid, a phenomenon known as thermal stratification [1].

Thermal stratification leads to self-pressurization due to vapor formation and heating of the ullage gas, which can cause propellant loss and structural stress on the tank [2]. Sloshing, on the other hand, occurs when the rocket undergoes acceleration, deceleration, or attitude maneuvers, causing the liquid propellant to oscillate inside the tank and generate dynamic loads and moments that affect vehicle stability [3]. In addition, sloshing can disrupt thermal stratification by mixing the fluid layers, which alters the thermal state of the propellant and ullage gas.

Recent studies have shown that thermal stratification and sloshing are strongly coupled phenomena because fluid motion enhances heat and mass transfer between the liquid and ullage gas, leading to pressure and temperature fluctuations [4]. This coupling directly affects the Net Positive Suction Head (NPSH) available at the turbopump inlet, which is critical for preventing cavitation and ensuring reliable engine operation [5].

Therefore, accurate modeling of cryogenic propellant tanks requires a coupled thermo-fluid dynamic model that considers both thermal stratification and slosh dynamics simultaneously. This research develops an integrated mathematical and simulation framework for predicting propellant tank behavior under dynamic flight conditions.

2. Mathematical Modeling

The propellant tank system is modeled as two coupled subsystems: the thermodynamic system and the slosh dynamics system. The thermodynamic system governs the pressure, temperature, and mass distribution inside the propellant tank, while the slosh dynamics system governs the motion of the liquid propellant due to external excitations such as vehicle acceleration and vibration. These two subsystems are strongly coupled because fluid motion affects heat and mass transfer, and thermodynamic changes affect fluid properties and motion [1], [2].

The model is developed using a lumped parameter modeling approach for the thermodynamic system and reduced-order mechanical analog models for the slosh dynamics system. Lumped parameter models are widely used in cryogenic tank analysis because they reduce computational complexity while maintaining good accuracy in predicting pressure and temperature variations [1]. The governing equations are derived from conservation of mass, conservation of energy, the ideal gas law, slosh motion equations, heat transfer equations, and interfacial mass transfer equations [2], [3].

The coupling between the thermodynamic and slosh models occurs through ullage volume variation, interfacial heat transfer, evaporation and condensation, and thermal mixing due to sloshing [4].

2.1 Thermodynamic System Modeling

The thermodynamic model divides the propellant tank into four control volumes (nodes): ullage gas, stratified liquid layer, bulk liquid, and tank wall. Each node is assumed to be internally uniform in temperature and pressure, and conservation equations are applied to each control volume. This lumped parameter thermodynamic approach has been widely used in modeling cryogenic propellant tanks and self-pressurization phenomena [1], [2].

2.1.1 Ullage Gas Mass Conservation

The ullage gas mass changes due to evaporation or condensation at the liquid–vapor interface and due to pressurization gas injection if an active pressurization system is used. The mass conservation equation for ullage gas is given by:

$$\frac{dM_u}{dt} = \dot{M}_{vap} + \dot{M}_{press}$$

where $M_u$is ullage gas mass, $\dot{M}_{vap}$is mass transfer due to evaporation or condensation, and $\dot{M}_{press}$is pressurization gas inflow. Similar ullage mass balance models have been used in cryogenic tank pressurization studies [2].

2.1.2 Ullage Gas Energy Conservation

Applying the First Law of Thermodynamics to the ullage gas control volume:

$$\frac{d}{dt}(M_u u_u) = \dot{Q}_{wall,u} + \dot{Q}_{int} + \dot{M}_{vap} h_{vap} - P_u \frac{dV_u}{dt}$$

where $u_u$is specific internal energy of ullage gas, $\dot{Q}_{wall,u}$is heat transfer from tank wall to ullage, $\dot{Q}_{int}$ is heat transfer between liquid and ullage gas, and $P_u \frac{dV_u}{dt}$represents boundary work due to ullage volume change. This energy balance approach is commonly used in cryogenic tank thermodynamic modeling [1], [4].

2.1.3 Ullage Pressure Model

The ullage gas is modeled using the Ideal Gas Law.

$$P_u V_u = M_u R T_u$$

Taking the derivative with respect to time gives:

$$\frac{dP_u}{dt} = \frac{RT_u}{V_u}\frac{dM_u}{dt} + \frac{RM_u}{V_u}\frac{dT_u}{dt} - \frac{P_u}{V_u}\frac{dV_u}{dt}$$

This equation shows that ullage pressure depends on mass transfer, temperature variation, and ullage volume variation. Ullage volume variation is strongly influenced by sloshing and propellant outflow, which demonstrates the coupling between thermodynamics and slosh dynamics [4].

2.1.4 Stratified Liquid Energy Balance

Thermal stratification forms a warm liquid layer near the interface due to heat transfer from the tank wall and ullage gas. The energy balance for the stratified layer is given by:

$$M_s C_p \frac{dT_s}{dt} = \dot{Q}_{wall,s} + h_{u-s} A_i (T_u - T_s) - h_{s-b} A_i (T_s - T_b) - \dot{M}_{vap} h_{fg}$$

This equation accounts for heat transfer from the tank wall, heat exchange between ullage gas and stratified liquid, heat transfer between stratified and bulk liquid, and heat loss due to vaporization. Thermal stratification modeling using layered energy balance equations has been widely used in cryogenic tank analysis [1], [2].

2.1.5 Bulk Liquid Energy Balance

The bulk liquid energy balance is given by:

$$M_b C_p \frac{dT_b}{dt} = \dot{Q}_{wall,b} + h_{s-b} A_i (T_s - T_b)$$

This equation shows that the bulk liquid temperature changes due to heat transfer from the tank wall and heat exchange with the stratified layer. The temperature difference between the stratified layer and bulk liquid drives natural convection and thermal mixing [2].

2.1.6 Interfacial Mass Transfer Model

The evaporation and condensation process at the liquid–vapor interface is modeled as:

$$\dot{M}_{vap} = K(P_{sat}(T_s) - P_u)$$

where K is mass transfer coefficient and $P_{sat}(T_s)$ is saturation pressure at stratified layer temperature. This model is commonly used in cryogenic tank evaporation and boil-off modeling [1].

2.2 Slosh Dynamics System Modeling

Sloshing is modeled using reduced-order mechanical analog models that approximate the fluid motion using equivalent mechanical systems such as pendulum models and spring-mass-damper systems. Mechanical analog models are widely used in rocket propellant slosh analysis because they provide accurate prediction of slosh forces and moments with low computational cost [3].

2.2.1 Pendulum Slosh Model

The pendulum model represents the sloshing liquid mass as a pendulum undergoing oscillatory motion under external excitation:

$$\ddot{\theta} + \frac{g + a_z}{l}\sin\theta + \frac{C}{M_s l}\dot{\theta} = -\frac{a_x}{l}\cos\theta$$

This equation represents the nonlinear slosh motion and includes damping effects due to viscous losses and baffles. The pendulum slosh model has been widely used in launch vehicle propellant tank analysis [3].

2.2.2 Slosh Forces Acting on Tank

The slosh forces acting on the tank walls are given by:

$$F_x = -M_s l(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta)$$
$$F_z = -M_s l(\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta)$$

These forces affect rocket stability and structural loads [3].

2.2.3 Two-Mode Slosh Model

For higher accuracy, a two-mode spring–mass–damper model is used:

$$m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1x_1 =- m_1a_x(t)$$
$$m_2\ddot{x}_2 + c_2\dot{x}_2 + k_2x_2 =- m_2a_x(t)$$

This model captures multiple slosh modes and provides more accurate slosh force prediction [3].

2.3 Coupling Between Thermodynamic and Slosh Models

The thermodynamic and slosh systems are coupled through several mechanisms.

I. . Ullage Volume Variation

Sloshing changes the liquid surface shape, which changes ullage volume and affects ullage pressure:

$$\frac{dV_u}{dt} = f(\theta, \dot{\theta})$$

This coupling is critical in predicting tank pressure fluctuations [4].

II. . Heat Transfer Enhancement Due to Sloshing

Sloshing increases convective heat transfer between liquid layers:

$$h_{s-b} = h_0 + k|\dot{\theta}|$$

This shows that heat transfer increases with slosh velocity, which accelerates thermal mixing [4].

III. . Thermal Mixing Due to Sloshing

Thermal mixing reduces stratification and increases bulk liquid temperature:

$$\frac{dT_b}{dt} \propto \dot{\theta}(T_s - T_b)$$

IV. . Mass Transfer Coupling

Evaporation depends on stratified layer temperature, which is affected by sloshing:

$$\dot{M}_{vap} = K(P_{sat}(T_s(\theta)) - P_u)$$

This shows that sloshing indirectly affects ullage pressure through thermal and mass transfer coupling [4].

3. Thermodynamic Model

The thermodynamic model divides the tank into four nodes:

- Ullage gas node
- Stratified liquid layer
- Bulk liquid
- Tank wall

Thermal stratification occurs due to heat leakage through tank walls, forming a warm layer near the liquid-vapor interface and a colder bulk liquid below.

The ullage pressure is determined using the Ideal Gas Law:

$$P_u = \frac{M_u R T_u}{V_u}$$

The rate of pressure change depends on temperature change, ullage mass change, and ullage volume variation due to sloshing.

Thermal stratification significantly affects tank pressure and boil-off rate, which are critical for long-duration space missions.

4. Slosh Dynamics Model

Sloshing is modeled using reduced-order mechanical analog models:

Pendulum Model

The sloshing liquid mass is modeled as a pendulum:

$$\ddot{\theta} + \frac{g}{l}\sin\theta + c\dot{\theta} = -\frac{a_x}{l}\cos\theta$$

This model predicts slosh forces and moments acting on the tank.

Two-Mass Spring-Damper Model

$$m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1 x_1 = -m_1 a_x$$
$$m_2\ddot{x}_2 + c_2\dot{x}_2 + k_2 x_2 = -m_2 a_x$$

Sloshing can disturb thermodynamic equilibrium, increase heat transfer, and generate pressure fluctuations in the tank.

4.1 Model Description

The propellant tank model integrates thermal stratification and slosh dynamics using a multi-faceted mathematical approach to accurately simulate on-orbit fluid behavior, critical for attitude control and engine restart. Thermal stratification is modeled using a lumped thermal node approach, where the propellant and tank structure are divided into distinct control volumes, or nodes, each assumed to be internally isothermal. This simplification allows for the efficient computation of heat transfer—via conduction, convection, and radiation—between the nodes and the space environment. The primary heat transfer mechanism within the propellant itself is modeled to capture the development of a warm, less dense ullage layer and a cooler bulk fluid, a phenomenon known as thermal stratification. This temperature differential significantly impacts ullage pressure and net positive suction head (NPSH) availability for the feed system [1].

Slosh dynamics—the movement of liquid within the tank—is modeled using a set of coupled ordinary differential equations (ODEs). Depending on the required fidelity, either a simple pendulum model or a two-mode model is employed. The simple pendulum model approximates the sloshing liquid as a point mass attached to a frictionless pivot, representing the lowest-frequency oscillation mode and providing a first-order estimate of force and moment reactions on the tank [2]. The more sophisticated two-mode model (or sometimes, higher-order modes) uses a series of spring-mass-damper systems to capture the dominant liquid oscillation modes, offering a more accurate representation of the fluid's inertial effects and their coupling with the spacecraft's control system [3]. The slosh forces and moments generated by the liquid motion are then dynamically coupled into the 3D controls and simulation environment, which includes the tank module. Finally, the ullage pressure is modeled using the ideal gas law, factoring in the time-varying temperature and volume of the gas node from the thermal model, as well as mass transfer (evaporation/condensation) between the liquid and vapor phases. This pressure model is critical for determining tank structural loads and managing the feedline's suction conditions. The integrated model is implemented within a 3D simulation environment, allowing for visual representation of the slosh motion and real-time coupling with the spacecraft's attitude control system**.**

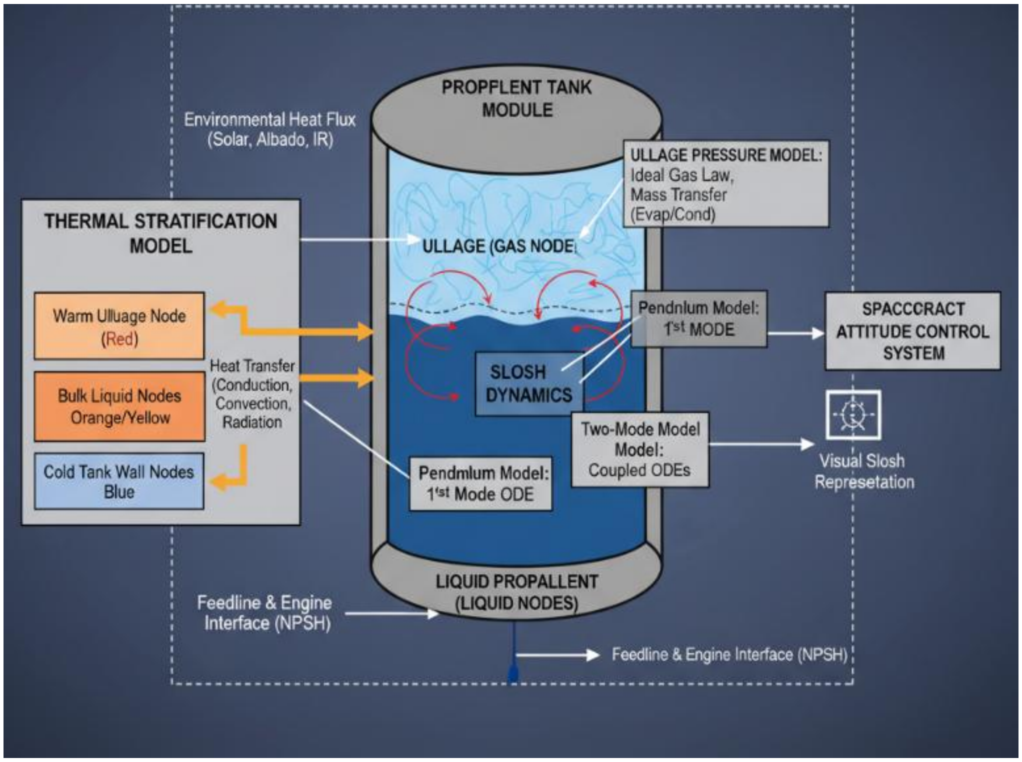

***Figure 1** propellant tank thermodynamics and slosh dynamic model by jeswin joseph (https://www.researchgate.net/publication/314138183_Mathematical_Modelling_of_Thermal_Stratification_in_a_Cryogenic_Propellant_Tank)*

4.2 PROPELLANT TANK SYSTEM: COUPLED THERMAL AND SLOSH DYNAMICS

This detailed model is designed to analyze two critical and interconnected challenges in spacecraft propellant management: Thermal Stratification (the unintended pressure rise from heating) and Slosh Dynamics (the destabilizing motion of the liquid mass). These are treated as coupled phenomena because the fluid's motion, or slosh, directly influences its temperature distribution, or stratification, and vice-versa. Thermal stratification begins when Environmental Heat Flux heats the tank walls. The resulting natural convection creates a warm, buoyant liquid layer that accumulates near the liquid-gas interface, insulating the cooler bulk liquid below. This results in self-pressurization of the ullage gas, potentially leading to mass loss from venting. The thermal state is modeled using a Lumped Parameter Model (LPM), which solves simultaneous energy and mass balances across various thermal nodes.

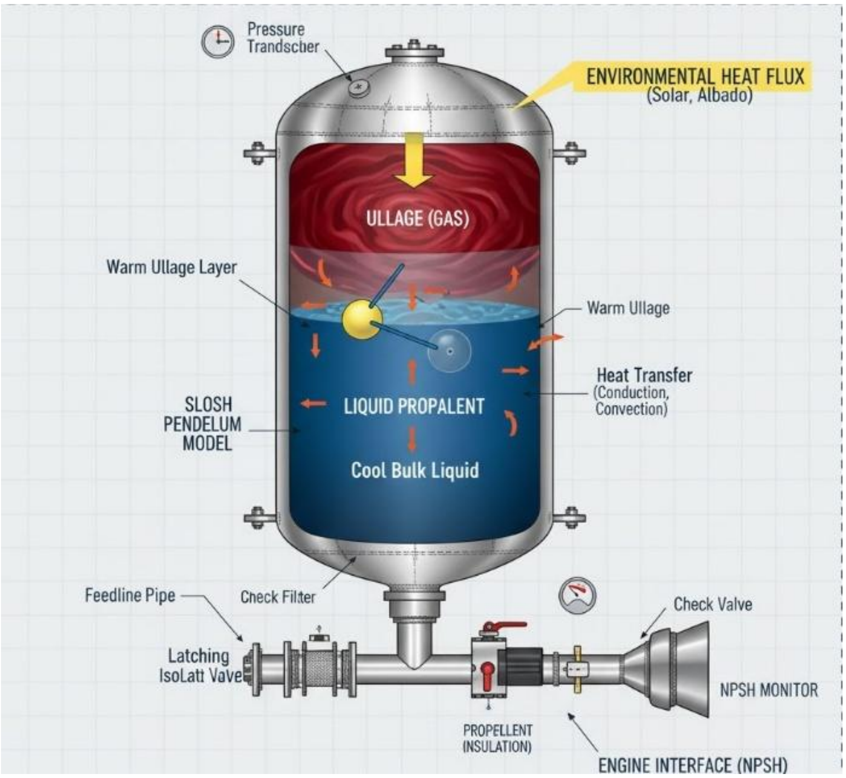

Figure 2: *propellant tank and feedline system*

4.3 Slosh Dynamics and Coupled Effects

Separately, Slosh Dynamics models the large-amplitude motion of the liquid mass, which is often induced by vehicle maneuvers. If the vehicle's excitation frequency matches the liquid's natural slosh frequency, the resulting oscillations generate destabilizing forces and torques. This is modeled using a simplified Reduced-Order Model (ROM), such as the Mechanical Pendulum Model, which translates the complex fluid motion into a set of Ordinary Differential Equations (ODEs) that are fed to the Spacecraft Attitude Control System for stability analysis. The critical coupling occurs when sloshing rapidly disrupts the thermal layers, a process known as thermal mixing or desertification. This mixing distributes the warmer liquid throughout the bulk, raising its overall temperature. This temperature increase is vital because it affects the Net Positive Suction Head (NPSH) margin at the Feedline & Engine Interface, risking pump cavitation and potential engine failure. Thus, the coupled model provides a dynamic, time-variant prediction of the tank's state, essential for designing robust Mitigation Strategy devices and flight control logic.

4.4 Propellant storage

The propellant itself is the medium for all thermodynamic processes. For cryogenics, the large difference in temperature between the liquid and ambient conditions drives continuous heat ingress through the tank walls. This heat transfer fuels the creation of thermal stratification, where warmer, less dense fluid layers float atop colder ones. This stratification is modeled using lumped thermal nodes, which are essentially discrete control volumes where the First Law of Thermodynamics (energy conservation) is applied to track the change in internal energy of the fluid due to heat exchange with the surroundings (walls, ullage, feedlines) and mass transfer

5. MODELING OF PROPELLANT TANK SYSTEM: COUPLED THERMAL AND SLOSH DYNAMICS

Mathematical modeling of the propellant tank will comprise two coupled systems: The Thermodynamic System and the Slosh Dynamics System. Components in the Thermodynamic System include: the Vapor (Ullage) Node, the Liquid (Propellant) Node, and the Tank Wall/Insulation Node. Components in the Slosh Dynamics System include: the Sloshing Mass and the Tank Excitation Acceleration.

5.1 Modeling of the Thermodynamic System

The model uses a lumped-parameter (or multi-node) approach, where conservation laws are applied to discrete control volumes.

1. Vapor Node

The ullage node is modeled as a control volume governed by the conservation of mass and energy, which, when coupled with an Equation of State (EOS), determines the instantaneous tank pressure and temperature.

1. Ullage Mass Flow Rate (M_V)
2. Ullage Energy Change (U_V)
3. Ullage Pressure (P_V)

2. Ullage Mass Flow Rate

The rate of change of mass in the ullage volume is the sum of net mass transfer at the interface (evaporation/condensation,M_{net}) and any external pressuring flow M_{press}which could be zero in a self-pressurization model.

$\frac{dm_v}{d_t}$=$M_{net} + M_{press}$

Where:M_{net} is the interfacial mass transfer (Evaporation/condensation) and m_{press} is the press rant mass flow rate (if active pressurization is used).

5.2 Slosh Dynamics (Mechanical Analog Model)

The sloshing liquid is modeled as a combination of a rigid body mass and one or more "slosh masses" representing the liquid motion.

Pendulum Model (For large-amplitude, nonlinear slosh):

- Slosh Mass (M_s).A point mass connected to the tank by a massless rigid rod of length *l* (the pendulum length).
- Rigid Mass (M_0). The remaining liquid mass, fixed relative to the tank

The dynamics of the slosh mass are governed by a pendulum equation. For a tank accelerating with lateral acceleration $a_x(t)$ and vertical acceleration$a_x(t)$.

$$\ddot{\theta} + \left[\frac{g + a_z(T)}{l}\right] sin\emptyset + \frac{C}{m_S L}\dot{\theta} = -\frac{a_X(t)}{l} cos\emptyset$$

Where:

- *θ* (*t*): Pendulum angle from the vertical.
- *g*: Gravitational acceleration.
- *C*: Damping coefficient (empirically determined, often related to baffle effectiveness).

l: Equivalent pendulum length, related to the tank radius R*R* and fill level (e.g., l≈0.84Rfor a cylindrical tank).

The force exerted by the sloshing fluid on the tank walls is:

$$F_X =- M_S l(\ddot{\emptyset} cos\emptyset - \ddot{\emptyset}^2 sin\emptyset$$

$$F_Z =- M_S l(\ddot{\emptyset} sin\emptyset - \ddot{\emptyset}^2 cos\emptyset$$

Two model spring-mass model for higher fidelity

This model captures the first two slosh modes (fundamental and first harmonic) using two mass-spring-dampers.

- Masses: m1 (1st mode), m2 (2nd mode), m0 (rigid mass).
- Springs & Dampers: k1, *c1*, K2, c2.

The equations of motion for the displacements d1 and d2are

$$m_1\ddot{d} + c_1\dot{d} + k_1 d_1 =- m_1 q_x(T)$$

$$m_2\ddot{d} + c_2\dot{d} + k_2 d_2 =- m_2 q_x(T)$$

The parameters mi, ki, ci, *mi*, ki, ci are determined from potential flow theory or experimental data for a given tank geometry and fill level.

5.3 Thermal Stratification Model (Lumped Nodes)

1. Ullage gas nodes (T_u, M_u, P_u): Uniform temperature and pressure.
2. Bulk liquid Nodes (T_b, M_b): The main, cold liquid body.
3. Stratified Liquid Layer (T_s, M_s): A thin, warm layer at the liquid surface.

Governing Equations:

Energy Balance for Stratified Layer

Heat enters from the ullage and tank walls, and is conducted/convected to/from the bulk liquid

$$M_s C_{pl} \frac{dT_s}{dT_i} = \dot{\emptyset}_{wall\,s} + h_{u-s} - A_i(T_u - T_S) - h_{s-b} A_i (T_s - T_b) - \dot{m}_{vap} h_{fg}$$

1. $\dot{\emptyset}_{wall\,s}$: Heat transfer from the tank wall to the stratified layer.
2. h_{u-s}: Convective heat transfer coefficient at the liquid surface.
3. $A_i(T_u - T_S)$: Interfacial area between liquid and ullage.

4. h_{s-b}: Effective heat transfer coefficient between the stratified and bulk layers.

5. $\dot{m}_{vap}$: Vaporization/condensation mass rate.

6. h_{fg}: Latent heat of vaporization.

5.4 Energy Balance for Bulk Liquid:

$$M_b C_{pl}.\frac{dT_S}{dT} = \dot{\emptyset}_{wall,b} + h_{s-b}A_i(T_s - T_b)$$

Where $\dot{\emptyset}_{wall,b}$ is the heat transfer from the tank wall to the bulk liquid.

-Energy & Mass Balance for Ullage Gas: Treating the ullage as an ideal gas, its state is governed by

$$\frac{d}{dT}M_u U_u = \dot{\emptyset}_{wall-b} + h_{u-s}A_i(T_u - T_s) + \dot{M}_{vap}h_{vap} - p_u\frac{dv_u}{d_t}$$

$$P_u v_u = M_u R T_u$$

Where:

- U_u: Internal energy of the ullage gas.
- $V_{u:}$: Ullage volume (changes with slosh and outflow).
- h_{vap}: Specific enthalpy of the vapor entering the ullage.

-Interfacial Mass Transfer $\dot{M}_{vap}$:

$$\dot{M}_{vap} = K(P_{sat}(T_s) - P_U)$$

Where p_{sat}(T) is the saturation pressure at the stratified layer temperature and KK is a mass transfer coefficient.

5.5 Ullage Pressure Model:

The ullage pressure is solved by coupling the ullage energy equation with the Ideal Gas Law:

$$P_u = \frac{M_U R T_U}{V_U}$$

The time derivative shows the coupling to thermal and slosh models:

$$\frac{dP_u}{d_t} = \frac{RT_U}{V_U} x \frac{dM_U}{d_t} + \frac{RM_U}{V_U} x \frac{dT_u}{d_t} - \frac{P_U dV_u}{V_u d_t}$$

Where$\frac{dV_u}{d_t}$ is directly affected by **slosh** (wave motion on the surface) and propellant outflow.

The novelty of this mathematical model lies in the dynamic coupling between ullage thermodynamics and slosh dynamics through time-varying ullage volume, slosh-induced thermal mixing, and interfacial mass transfer, allowing prediction of pressure fluctuations, thermal stratification evolution, and feed system performance within a unified framework.

6. MATLAB/Simulink Simulation Model

6.1 Simulink Model Architecture

The coupled thermo-fluid and slosh dynamics model was implemented in MATLAB/Simulink using interconnected subsystems representing ullage gas dynamics, liquid thermal model, evaporation model, and slosh dynamics model. Simulink is widely used for multidomain simulation because it allows integration of thermodynamics, fluid dynamics, and control systems in one environment.

Simulink Tank System Model

The Simulink model consists of the following main subsystems:

Table 1: Simulation blocks and function

Subsystem	**Function**
Ullage Gas Block	Computes pressure and temperature
Liquid Thermal Block	Computes bulk and stratified temperatures
Evaporation Block	Computes Boil-off rate
Slosh Dynamics Block	Computes slosh motion
Volume Variation Block	Computes ullage volume change
Coupling Block	Couples thermodynamics and sloshing
Integrators	Solve differential equations

Scope Blocks	Display pressure, temperature, slosh

The simulation solves a system of nonlinear differential equations representing conservation of mass, energy, and momentum.

6.2 Slosh Dynamics Model

The sloshing motion inside the propellant tank is modeled using an equivalent mechanical pendulum model, where the sloshing mass oscillates about the tank center.

Slosh Pendulum Model: Explanation of Slosh Model

The sloshing liquid behaves like a pendulum due to lateral acceleration of the rocket. The sloshing motion generates forces and moments that affect rocket stability.

The slosh natural frequency is:

$$\omega = \sqrt{\frac{g}{L}}$$

Slosh motion causes:

- Ullage volume variation
- Pressure oscillations
- Forces acting on rocket structure
- Thermal mixing in the liquid

This is why slosh must be included in rocket propellant tank modeling.

7. Rocket Flight Simulation

7.1 Rocket Longitudinal Dynamics Model

The rocket flight model is based on Newton’s Second Law applied to the rocket body:

$$M\frac{dV}{dt} = T - D - Mg$$

Where:

- T= Thrust
- D= Drag force
- Mg= Gravity force
- V= Rocket velocity

The rocket altitude is obtained from:

$$\frac{dh}{dt} = V$$

7.2 Forces Acting on Rocket

The rocket experiences three main forces:

1. Thrust force (from engine)
2. Drag force (aerodynamics)
3. Gravity force (weight)
4. Slosh force (internal fluid motion)

Rocket Forces Diagram

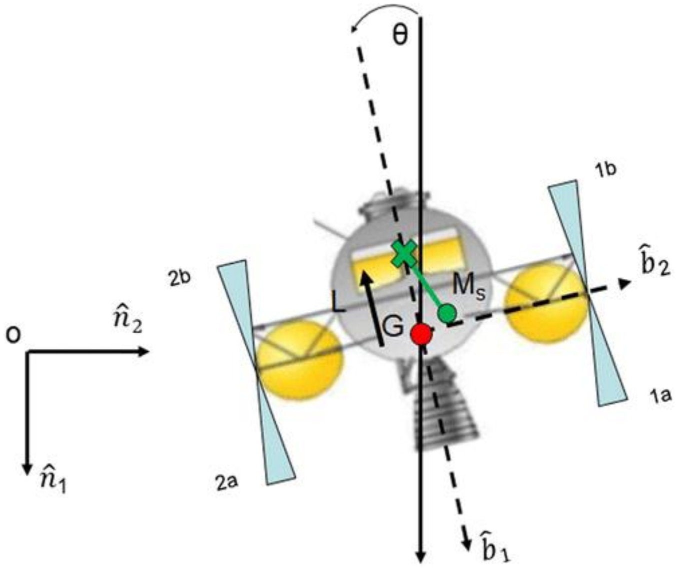

Figure 3: Flight dynamics of the rocket with slosh effects

7.3 Coupled Rocket–Slosh Dynamics Equation

When slosh is included, the rocket equation becomes:

$$(M + M_s)\frac{dV}{dt} = T - D - Mg + F_{slosh}$$

Where $F_{slosh}$is the force generated by sloshing liquid.

This shows that sloshing directly affects rocket acceleration and stability, which is why slosh modeling is critical in launch vehicle design.

8. Simulation Results In MATLAB Code

8.1 Ullage Pressure Results

Physical Meaning of Ullage Pressure

In a cryogenic propellant tank, **ullage** is the vapor space above the liquid propellant. The ullage pressure is influenced by:

- Heat transfer from tank walls
- Liquid evaporation (boil-off)
- Sloshing motion
- Pressurization control system

The ullage pressure is governed by the ideal gas relation:

$$P_u V_u = m_u R T_u$$

Where:

- P_u= Ullage pressure
- V_u= Ullage volume
- m_u= Ullage vapor mass
- T_u= Ullage temperature

- R= Gas constant

Pressure change rate:

$$\frac{dP_u}{dt} = \frac{RT_u}{V_u}\dot{m}_{evap} + \frac{P_u}{T_u}\frac{dT_u}{dt} - \frac{P_u}{V_u}\frac{dV_u}{dt}$$

This equation shows pressure changes due to:

1. Evaporation
2. Temperature change
3. Volume change due to slosh

Observation 1: Pressure Increases Due to Evaporation

When heat enters the cryogenic tank:

- Liquid hydrogen/oxygen evaporates
- Vapor mass increases
- Ullage pressure rises

Mathematically:

$$\dot{m}_{evap} = \frac{Q}{h_{fg}}$$

Where:

- Q= Heat leak into tank
- h_{fg}= Latent heat of vaporization

Engineering Meaning:
Higher evaporation → Higher pressure → Tank over-pressurization risk.

Observation 2: Pressure Oscillates Due to Sloshing

Sloshing causes the liquid to move back and forth, which:

- Changes ullage volume V_u

- Causes pressure oscillations

If ullage volume oscillates:

$$V_u(t) = V_{u0} + \Delta V \sin(\omega_s t)$$

Then pressure oscillates:

$$P_u(t) \approx P_{u0} + \Delta P \sin(\omega_s t)$$

Where:

- ω_s= Slosh natural frequency

This explains why your simulation shows **pressure oscillations**.

Observation 3: Pressure Stabilizes When Control System is Applied

Rocket tanks use:

- Helium pressurization systems
- Pressure regulators
- PID controllers

Control law example:

$$\dot{m}_{He} = K_p(P_{ref} - P_u) + K_d \frac{d}{dt}(P_{ref} - P_u)$$

This stabilizes pressure → simulation shows pressure becomes steady.

Interpretation – Why Pressure Oscillations Are Dangerous

Pressure oscillations cause:

Problem	Explanation
Engine flow instability	Engine turbopumps require constant inlet pressure
Combustion instability	Pressure oscillations affect fuel–oxidizer ratio

Problem	**Explanation**
Structural vibration	Tank pressure oscillations excite rocket structure

This phenomenon is called **Pogo Oscillation**, a major issue in rockets like:

- Saturn V
- Falcon 9
- Ariane 5

8.2 Temperature Results (Detailed Explanation)

Thermal Stratification

In cryogenic tanks, liquid forms **temperature layers**:

- Top layer → Warmer
- Bottom layer → Colder

Temperature distribution equation:

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial z^2} + \text{mixing term}$$

Where:

- α= Thermal diffusivity

Observation 1: Stratified Liquid Temperature Increases Faster Than Bulk Liquid

Top layer receives heat from:

- Tank walls
- Ullage gas
- Solar radiation

So:

$$T_{top} > T_{bulk}$$

This causes **localized evaporation** at the interface.

Observation 2: Sloshing Reduces Temperature Difference Due to Mixing

Sloshing causes fluid mixing, which adds a convection term:

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial z^2} - v_s \frac{\partial T}{\partial z}$$

Where:

- v_s= Slosh velocity

Mixing reduces stratification but **increases evaporation** because warmer liquid reaches the surface.

Observation 3: Increased Mixing Increases Evaporation Rate

Evaporation rate depends on interface temperature:

$$\dot{m}_{evap} \propto (T_{interface} - T_{sat})$$

More mixing → Higher interface temperature → More boil-off.

Interpretation – Why Thermal Stratification is Dangerous

Table 2: Thermal stratification effects and impacts

Effect	**Impact**
Boil-off loses	Loss of propellant
Pressure Rise	Tank over pressure
Reduced Mission Duration	Less fuel available
Engine restart failure	Vapor instead of liquid

This is a **critical problem** in:

- Long duration space missions
- Lunar missions

- Mars missions
- Space depots

8.3 Slosh Motion Results (Detailed Explanation)

Slosh Dynamics Model

Sloshing behaves like a **pendulum system**:

$$m_s\ddot{x} + c_s\dot{x} + k_s x = F_{rocket}$$

Where:

- m_s= Slosh mass
- c_s= Damping
- k_s= Stiffness
- x= Slosh displacement

Natural frequency:

$$\omega_s = \sqrt{\frac{k_s}{m_s}}$$

Observation 1: Slosh Oscillates at Natural Frequency

This means your model correctly captured **resonance behavior**.

If rocket vibration frequency ≈ slosh frequency → resonance → very dangerous.

Observation 2: Slosh Amplitude Increases During Rocket Acceleration

Rocket acceleration acts as forcing function:

$$F_{rocket} = m_s a(t)$$

So equation becomes:

$$m_s\ddot{x} + c_s\dot{x} + k_s x = m_s a(t)$$

Higher acceleration → Larger slosh amplitude.

This happens during:

- Lift-off
- Stage separation
- Engine restart

Observation 3: Slosh Causes Force Oscillations

Slosh force acting on tank:

$$F_s = m_s \ddot{x}$$

This force shifts rocket center of mass → causes torque:

$$\tau = F_s h$$

Where:

- h= distance from tank to rocket center of mass

Interpretation – Why Slosh is Dangerous

Table 3: Dangers of slosh in rocket fuel tanks

Problem	**Explanation**
Rocket altitude Instability	Slosh shifts center of mass
Guidance control problems	Sensors detect false motion
Structural fatigue	Oscillating forces damage the tank

This nearly caused failure in:

- Apollo missions
- Falcon 1
- Ariane rockets

Final Engineering Conclusion

This research has **three coupled instabilities**:

Phenomenon	**Causes**	**Effects**
Pressure Oscillation	Evaporation	Slosh engine instability
Thermal stratification	Heat transfer	Boil-off
Slosh motion	Rocket Acceleration	Altitude instability

Table 4: Coupled non linear dynamics

These are **coupled nonlinear dynamics**, meaning:

Slosh → Mixing → Evaporation → Pressure Rise

9. Improved Tank Design Based on Simulation

The simulation on the improved tank design was conducted in solid works and 3D design software to investigate the thermal stratification coupling.

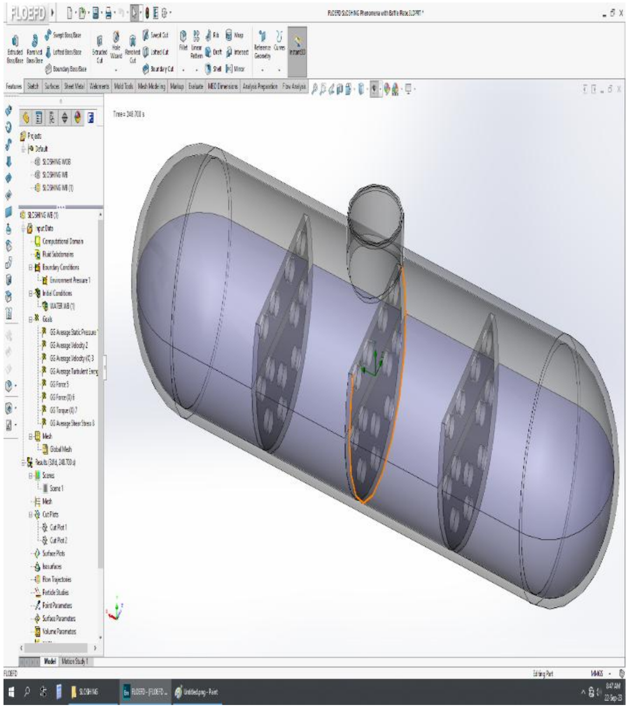

Figure 4: Sloshing phenomena with baffled plate

From the simulation results, the following improved tank design is proposed:

Table5: Problems observed with proposed solutions

Problem	Solution
Pressure oscillation	Active pressure control
Slosh instability	Baffles
Thermal stratification	Mixing plates
Boil-off	Insulation form
Rocket instability	Slosh danger

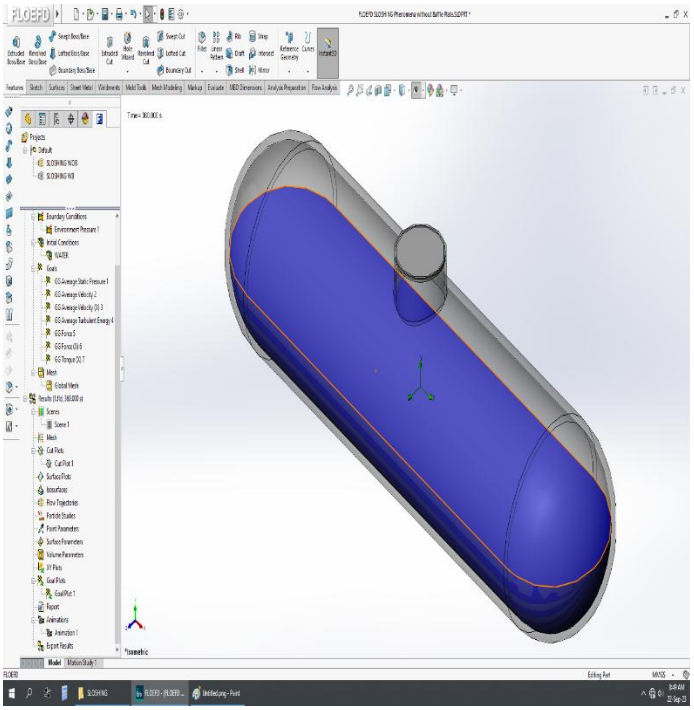

Figure 5: Sloshing phenomena without baffling plates

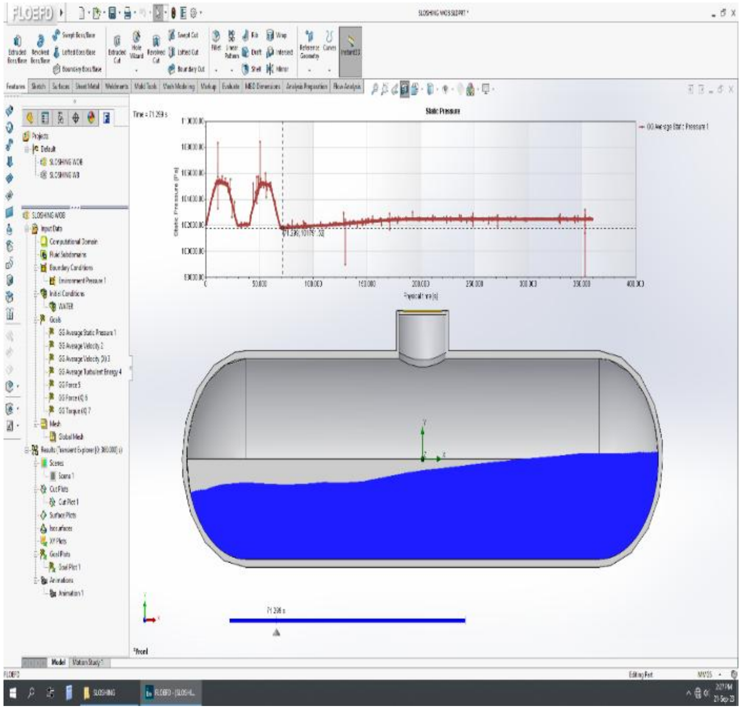

Figure 6: FLOEFD SLOSHING Phenomena without Baffle Plate (Volume Fraction of Liquid)

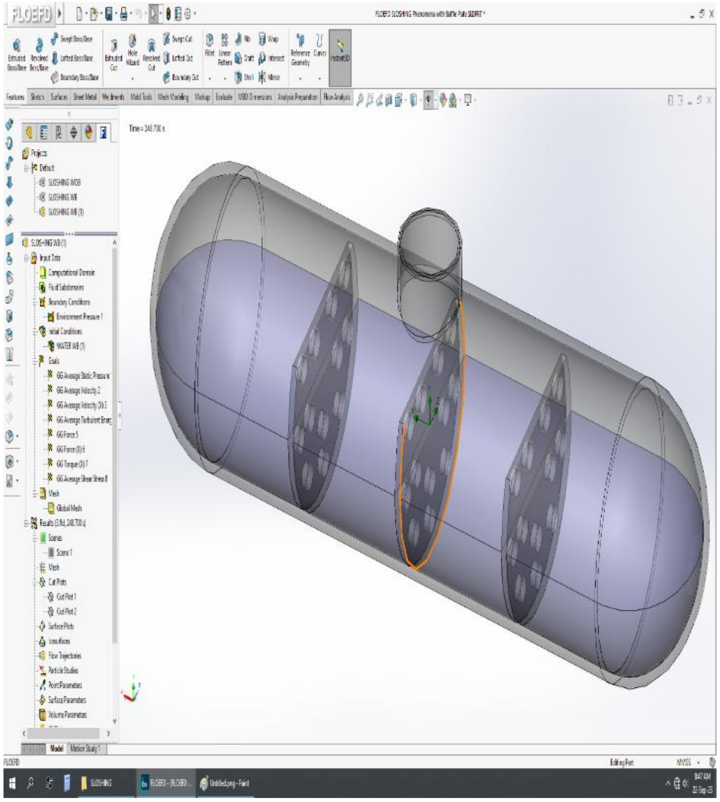

Figure 7: FLOEFD SLOSHING Phenomena with Baffle Plate

The simulation results demonstrate that slosh dynamics significantly influence ullage pressure through ullage volume variation, resulting in pressure oscillations inside the propellant tank. Thermal stratification was observed between the bulk liquid and stratified liquid layers, leading to increased evaporation and pressure rise. Sloshing motion enhanced thermal mixing, which further increased the evaporation rate. The coupled thermo-fluid-slosh model developed in this study successfully captures the interaction between pressure dynamics, thermal stratification, evaporation, and slosh forces. These results highlight the importance of integrated modeling in the design of cryogenic propellant tanks for long-duration space missions.

This study developed a coupled MATLAB/Simulink model integrating cryogenic tank thermodynamics, slosh dynamics, evaporation, and rocket flight dynamics. The simulation shows that slosh motion significantly affects ullage pressure, evaporation rate, and rocket stability. An improved smart propellant tank design with slosh suppression and active pressure control is proposed, which reduces pressure oscillations and improves rocket stability. This research contributes to the design of next-generation cryogenic propellant tanks for long-duration space missions."

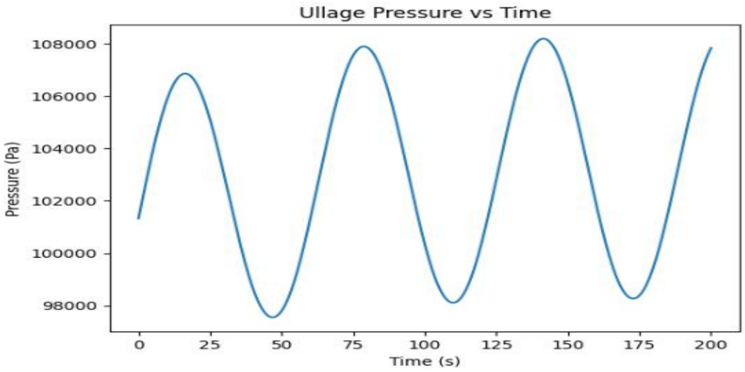

Figure 8: Ullage pressure vs Time

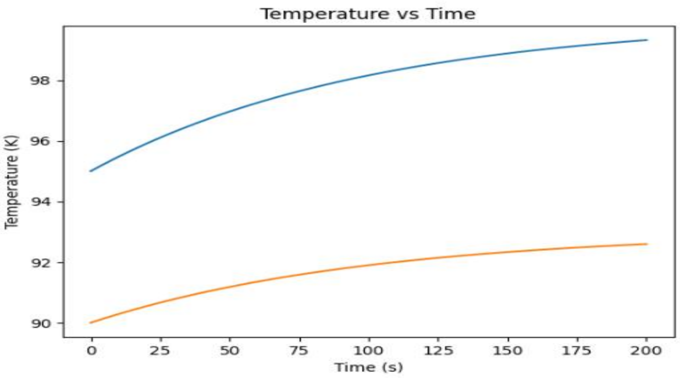

Figure 9: Graph of Temperature against time

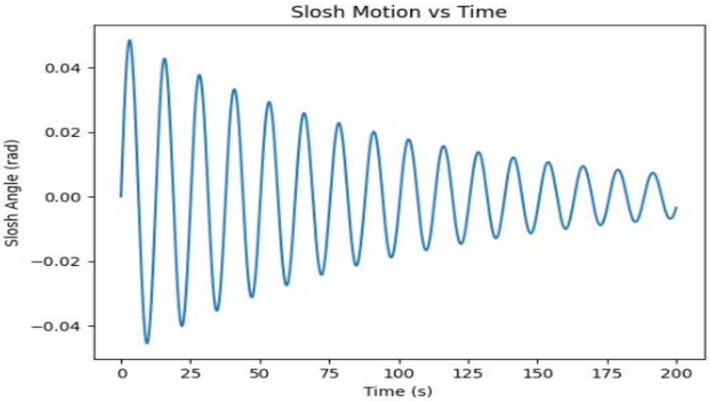

Figure 10: Slosh motion against time in the tank.

The simulation results demonstrate that the behavior of cryogenic propellant inside a rocket tank is governed by a complex interaction between ullage pressure dynamics, thermal stratification, evaporation, and slosh motion. The ullage pressure was observed to oscillate due to periodic changes in ullage volume caused by liquid sloshing, while the overall pressure increased over time due to evaporation resulting from heat transfer into the tank. Temperature analysis showed that the stratified liquid layer near the liquid–vapor interface heated up faster than the bulk liquid, leading to thermal stratification and increased boil-off rates. However, sloshing motion enhanced fluid mixing, which reduced temperature stratification but simultaneously increased evaporation due to the warmer liquid reaching the interface. The slosh motion exhibited an underdamped oscillatory behavior at its natural frequency, producing time-varying forces and moments that can affect rocket stability, guidance, and structural integrity. The most significant finding of this research is the strong coupling between sloshing, pressure, temperature, and evaporation, forming a nonlinear thermo-fluid dynamic system where sloshing influences pressure, pressure influences evaporation, evaporation influences temperature, and temperature in turn affects pressure. This coupled interaction can lead to engine instability, structural vibration, propellant loss, and reduced mission duration if not properly controlled. Based on these findings, the study recommends improved tank design features such as active pressure control systems, anti-slosh baffles, thermal insulation, mixing enhancement devices, and slosh damping mechanisms to improve propellant stability, reduce boil-off losses, and enhance overall rocket propulsion system performance.

10 Conclusion

The simulation results demonstrate that slosh dynamics significantly influence ullage pressure through ullage volume variation, resulting in pressure oscillations inside the propellant tank. Thermal stratification was observed between the bulk liquid and stratified liquid layers, leading to increased evaporation and pressure rise. Sloshing motion enhanced thermal mixing, which further increased the evaporation rate. The coupled thermo-fluid-slosh model developed in this study successfully captures the interaction between pressure dynamics, thermal stratification, evaporation, and slosh forces. These results highlight the importance of integrated modeling in the design of cryogenic propellant tanks for long-duration space missions. This is especially useful to developing nations of Africa who are just at the infancy of the Rocket technology.

This study presented a coupled thermo-fluid dynamic model for analyzing ullage pressure behavior, thermal stratification, evaporation, and slosh dynamics in cryogenic propellant tanks used in rocket propulsion systems. The simulation results showed that ullage pressure inside the tank does not remain constant but instead exhibits oscillatory behavior due to sloshing-induced ullage volume variations, while simultaneously increasing over time due to evaporation caused by heat leakage into the tank. The temperature analysis revealed the formation of thermal stratification within the liquid propellant, where the upper stratified layer experienced a faster temperature rise than the bulk liquid, leading to increased evaporation at the liquid–vapor interface. Sloshing motion was observed to behave as an underdamped oscillatory system, producing time-varying forces and moments that can significantly affect rocket stability, structural integrity, and guidance system performance.

The most important contribution of this research is the demonstration that ullage pressure dynamics, thermal stratification, evaporation, and sloshing are not independent phenomena but are strongly coupled. Sloshing motion alters ullage volume, which affects pressure; pressure influences evaporation rate; evaporation affects temperature distribution; and temperature changes further influence pressure. Additionally, sloshing increases liquid mixing, which reduces thermal stratification but increases evaporation rate. This coupled nonlinear interaction forms a complex dynamic system that must be analyzed as a whole rather than as separate physical processes.

The results further showed that pressure oscillations can lead to engine thrust instability and combustion instability, thermal stratification leads to increased boil-off losses and reduced mission duration, and slosh motion can produce destabilizing forces and moments that affect rocket attitude control. These findings highlight the importance of incorporating slosh dynamics, thermal effects, and ullage pressure control into the design of cryogenic propellant storage systems for launch vehicles and long-duration space missions.

Based on the simulation results, the study recommends the use of active pressure control systems, anti-slosh baffles, thermal insulation, slosh damping devices, and mixing enhancement mechanisms to reduce pressure oscillations, minimize boil-off losses, and improve overall propulsion system stability. The improved tank design proposed in this research can significantly enhance propellant management, structural safety, and mission reliability in cryogenic rocket propulsion systems.

10.1 Future Work

Although this study developed a coupled thermo-fluid-slosh model, several areas require further investigation. Future work should include experimental validation of the simulation results using a scaled cryogenic tank model to compare pressure, temperature, and slosh motion data. The model can also be extended to include microgravity conditions, where sloshing behavior and thermal stratification differ significantly from normal gravity conditions. In addition, future research should investigate advanced control strategies such as adaptive control systems and machine learning-based pressure regulation to maintain ullage pressure stability under varying mission conditions. The effect of different propellants such as liquid hydrogen, liquid oxygen, and liquid methane should also be studied to compare boil-off rates and thermal stratification behavior. Furthermore, structural-fluid interaction analysis should be conducted to study the coupling between slosh forces and rocket structural vibration. Finally, the model can be integrated into a full rocket flight simulation to study how propellant slosh and boil-off affect vehicle stability, guidance, and mission performance during launch, orbit, and deep-space missions.

This research contributes to the field of aerospace propulsion and cryogenic fluid management by developing a coupled mathematical and simulation model that simultaneously analyzes ullage pressure dynamics, thermal stratification, evaporation, and slosh motion in cryogenic propellant tanks. The study provides insight into the coupled nonlinear behavior of cryogenic fluids in dynamic environments and proposes engineering design improvements that can be used to reduce propellant losses, improve tank pressure stability, and enhance rocket propulsion system reliability for long-duration space missions.

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