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The Reducibility of Generalized Modal Syllogisms Based on □AM◇I-1

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Abstract

There is the reducibility between the generalized modal syllogism □AM◇I-1 and the other 20 valid generalized modal syllogisms. This paper first proves the validity of the generalized modal syllogism based on the truth value definitions of sentences with quantification, set theory and modal logic, then derives the other 20 valid generalized modal syllogisms from the syllogism □AM◇I-1 in line with some facts and inference rules. The reason why these syllogisms are reducible is that: (1) any of the Aristotelian quantifiers can be defined by the other three Aristotelian quantifiers; (2) any of the four generalized quantifiers in this paper (that is, *most*, *at most half of the*, *fewer than half of the* and *at least half of the*) can be defined by the other three generalized quantifiers; (3) the Aristotelian quantifiers *some* and *no* have symmetry; (4) a necessary modality □ and a possible modality ◇ can be mutually defined. And the process of these reductions are ultimately presented in a structured formalization way. Thus, this paper provides a fragmentary research approach for other generalized modal syllogisms including four generalized quantifiers with transformation relations. There are many generalized modal syllogisms in natural language. Therefore, this study has practical

significance and theoretical value for knowledge representation and reasoning in artificial intelligence.

Keywords: Generalized modal syllogism; Reducibility; Generalized quantifier; Validity

1. Introduction

Syllogistic reasoning is one of the important forms of reasoning in natural language and human thinking. There are various kinds of syllogisms in natural language, such as Aristotelian syllogisms (Hui, 2023), Aristotelian modal syllogisms (Johnson, 2004), generalized syllogisms (Murinová & Novák, 2012; Endrullis et al., 2015), generalized modal syllogisms, and so on. This paper shall restrict attention mainly to the reducibility of generalized modal syllogisms.

There are some studies on the reducibility of syllogisms. Łukasiewic (1957) studied the reducibility between the two Aristotelian syllogisms (AAA-1 and AII-3) and the other 22 valid syllogisms. Long (2023) derived the remaining 23 valid syllogisms only from the valid Aristotelian syllogism AEE-4. Xiaojun (2020) discussed reducible relations between/among Aristotelian modal syllogisms. Cheng (2023a) studied how to deduce the other 91 valid Aristotelian modal syllogisms from the Aristotelian modal syllogism □I□A□I-3. Although there are many generalized modal syllogisms in natural language, there is no literature on the reducibility of this kind of syllogisms up till now. This paper focuses on this topic.

2. Preliminaries

In the paper, let R, S and T be the lexical variables in sentences with quantification, and D the domain of lexical variables. The generalized modal syllogisms discussed in this paper contain sentences in the following forms: All Rs are T, No Rs are T, Some Rs are T, Not all Rs are T, *most* Rs are T, *fewer than half of the* Rs are T, *at most half of the* Rs are T, and *at least half of the* Rs are T. The eight sentences can be respectively symbolized as *all*(R, T), *no*(R, T), *some*(R, T), *not all*(R, T), *most*(R, T), *fewer than half of the*(R, T), *at most half of the*(R, T), *at least half of the*(R, T), and abbreviated as the proposition A, E, I, O, M, F, H, and L respectively. $|R|$ stands for the cardinality of the set composed of the variable R, Q for any generalized quantifiers, $\neg Q$ and $Q\neg$ for the outer and inner negative quantifier of Q respectively.

Generalized modal syllogisms refer to the syllogisms that contain modalities (that is, the necessary modality □ and/or the possible modality ◇) and generalized quantifiers.

Example 1:

Major premise: All peach trees are necessarily flowering plants.

Minor premise: Most trees in this farm are peach trees.

Conclusion: Some trees in this farm are possibly flowering plants.

Let R be the set of all trees in the domain, S the set of all peach trees in the domain, and T the set of all flowering plants in the domain. Then the generalized modal syllogism in Example 1 can be formalized as $\Box all(S, T) \wedge most(R, S) \rightarrow \Diamond some(R, T)$, and abbreviated as □AM◇I-1.

According to the generalized quantifier theory (Peters and Westerståhl, 2006), set theory (Halmos, 1974) and possible world semantics (Chellas, 1980), the truth value definitions of the sentences with quantification as follows (Cheng, 2023b):

Definition 1 (truth value definitions):

(1) $all(R, T)$ is true just in case $R \subseteq T$ is true.

(2) $some(R, T)$ is true just in case $R \cap T \neq \varnothing$ is true.

(3) $no(R, T)$ is true just in case $R \cap T = \varnothing$ is true.

(4) $not\ all(R, T)$ is true just in case $R \not\subseteq T$ is true.

(5) $most(R, T)$ is true just in case $|R \cap T| \geq 0.6|R|$ is true.

(6) $\Box all(R, T)$ is true just in case $R \subseteq T$ is true in any possible world.

(7) $\Diamond all(R, T)$ is true just in case $R \subseteq T$ is true in at least one possible world.

(8) $\Box some(R, T)$ is true just in case $R \cap T \neq \varnothing$ is true in any possible world.

(9) $\Diamond some(R, T)$ is true just in case $R \cap T \neq \varnothing$ is true in at least one possible world.

(10) $\Box no(R, T)$ is true just in case $R \cap T = \varnothing$ is true in any possible world.

(11) $\Diamond no(R, T)$ is true just in case $R \cap T = \varnothing$ is true in at least one possible world.

(12) $\Box not\ all(R, T)$ is true just in case $R \not\subseteq T$ is true in any possible world.

(13) ◇*not all*(*R*, *T*) is true just in case $R \nsubseteq T$ is true in at least one possible world.

(14) □*most*(*R*, *T*) is true just in case $|R \cap T| \geq 0.6|R|$ is true in any possible world.

(15) ◇*most*(*R*, *T*) is true just in case $|R \cap T| \geq 0.6|R|$ is true in at least one possible world.

Definition 2 (inner negation): $Q\neg(R, T) =_{\mathrm{def}} Q(R, D-T)$.

Definition 3 (outer negation): $\neg Q(R, T) =_{\mathrm{def}}$ It is not that $Q(R, T)$.

Fact 1 (inner negation for Aristotelian quantifiers)

(1) *all*(*R*, *T*)=*no*¬(*R*, *T*); (2) *no*(*R*, *T*)=*all*¬(*R*, *T*);

(3) *some*(*R*, *T*)=*not all*¬(*R*, *T*); (4) *not all*(*R*, *T*)=*some*¬(*R*, *T*);

(5) *fewer than half of the*(*R*, *T*)=*most*¬(*R*, *T*); (6) *most*(*R*, *T*)=*fewer than half of the*¬(*R*, *T*);

(7) *at most half of the*(*R*, *T*)=*at least half of the* (*R*, *T*);

(8) *at least half of the*(*R*, *T*)=*at most half of the* (*R*, *T*).

Fact 1 can be easily proved by Definition 2 (Cheng, 2022).

Fact 2 (outer negation for Aristotelian quantifiers):

(1) ¬*not all*(*R*, *T*)=*all*(*R*, *T*); (2) ¬*all*(*R*, *T*)=*not all*(*R*, *T*);

(3) ¬*no*(*R*, *T*)=*some*(*R*, *T*); (4) ¬*some*(*R*, *T*)=*no*(*R*, *T*);

(5) ¬*most*(*R*, *T*)=*at most half of the*(*R*, *T*); (6) ¬*at most half of the*(*R*, *T*)=*most*(*R*, *T*);

(7) ¬*fewer than half of the*(*R*, *T*)=*at least half of the*(*R*, *T*);

(8) ¬*at least half of the*(*R*, *T*)=*fewer than half of the*(*R*, *T*).

Fact 2 can be easily proved by Definition 3.

A necessary modality □ and a possible modality ◇ are mutually dual. Let $Q(R, T)$ be a categorical proposition, then $\Diamond Q(R, T) =_{\mathrm{def}} \neg\Box\neg Q(R, T)$ and $\Box Q(R, T) =_{\mathrm{def}} \neg\Diamond\neg Q(R, T)$. Hence, the following Fact 3 can be obtained.

Fact 3: (1) $\neg\Box Q(R, T) = \Diamond\neg Q(R, T)$; (2) $\neg\Diamond Q(R, T) = \Box\neg Q(R, T)$.

The following facts are the basic knowledge in classical modal logic (Chagrov and Zakharyaschev, 1997) or generalized quantifier theory (Peters and Westerståhl, 2006). Thus their proofs are omitted.

Fact 4 (a necessary proposition implies an assertoric proposition): $\vdash\ \Box Q(R, T)\rightarrow Q(R, T)$.

Fact 5 (a necessary proposition implies a possible proposition): $\vdash\ \Box Q(R, T)\rightarrow\Diamond Q(R, T)$.

Fact 6 (an assertoric proposition implies a possible proposition): $\vdash\ Q(R, T)\rightarrow\Diamond Q(R, T)$.

Fact 7 (a universal proposition implies a particular proposition):

(1) $\vdash\ all(R, T)\rightarrow some(R, T)$; (2) $\vdash\ no(R, T)\rightarrow not\ all(R, T)$.

Fact 8 (symmetry of *some* and *no*): (1) $some(R, T)\leftrightarrow some(T, R)$; (2) $no(R, T)\leftrightarrow no(T, R)$.

The basic rules in classical propositional logic are suitable for generalized modal syllogistic. For example, let α, β, γ and δ be proposition variables,

Rule 1 (subsequent weakening): If $\vdash\ (\alpha\wedge\beta\rightarrow\gamma)$ and $\vdash\ (\gamma\rightarrow\delta)$, then $\vdash\ (\alpha\wedge\beta\rightarrow\delta)$.

Rule 2 (anti-syllogism): If $\vdash\ (\alpha\wedge\beta\rightarrow\gamma)$, then $\vdash\ (\neg\gamma\wedge\alpha\rightarrow\neg\beta)$ or $\vdash\ (\neg\gamma\wedge\beta\rightarrow\neg\alpha)$.

3. Reduction between the □AM◇I-1 and the Other 20 Valid Generalized Modal Syllogisms

The following Theorem 1 shows that □AM◇I-1 is valid. And '□AM◇I-1⇒ □EM◇O-1' in the Theorem 2(1) means that the validity of syllogism □EM◇O-1 can be deduced from the validity of syllogism □AM◇I-1. Then one can say that there is reducibility between these two syllogisms. The other cases in the Theorem 2 are similar.

Theorem 1 (□AM◇I-1): The generalized modal syllogism $\Box all(S, T)\wedge most(R, S)\rightarrow\Diamond some(R, T)$ is valid.

Proof: According to Example 1, □AM◇I-1 is the abbreviation of the syllogism $\Box all(S, T)\wedge most(R, S)\rightarrow\Diamond some(R, T)$. Suppose that $\Box all(S, T)$ and $most(R, S)$ are true. Then in virtue of the clause (6) in Definition 1, $\Box all(S, T)$ is true just in case $S\subseteq T$ is true in any possible world. Similarly, in line with the clause (5) in Definition 1, $most(R, S)$ is true just in case $|R\cap S|\geq 0.6|R|$ is true. Thus, it is easily seen that $S\subseteq T$ and $|R\cap S|\geq 0.6|R|$ are true. It follows that $|R\cap T|\geq 0.6|R|$, and it is clear that $R\cap T\neq\varnothing$. Then, on the basis of the clause (2) in Definition 1, $some(R, T)$ is true. Therefore, $\Diamond some(R, T)$ comes out ture in terms of the Fact 6, just as required.

Theorem 2: The following valid generalized modal syllogisms can be derived from □AM◇I-1:

(1) □AM◇I-1⇒□EM◇O-1

(2) □AM◇I-1⇒M□A◇I-4

(3) □AM◇I-1⇒□EM◇O-3

(4) □AM◇I-1⇒□A□EH-2

(5) □AM◇I-1⇒□A□EH-2⇒□A□E◇H-2

(6) □AM◇I-1⇒□EM◇O-1⇒□EM◇O-2

(7) □AM◇I-1⇒□EM◇O-1⇒□E□AH-2

(8) □AM◇I-1⇒□EM◇O-1⇒□E□AH-2⇒□E□A◇H-2

(9) □AM◇I-1⇒□EM◇O-1⇒□AM◇I-3

(10) □AM◇I-1⇒M□A◇I-4⇒□A□EH-4

(11) □AM◇I-1⇒M□A◇I-4⇒□A□EH-4⇒□A□E◇H-4

(12) □AM◇I-1⇒M□A◇I-4⇒□EM◇O-4

(13) □AM◇I-1⇒□EM◇O-1⇒□EM◇O-2⇒M□A◇I-3

(14) □AM◇I-1⇒□EM◇O-1⇒□EM◇O-2⇒□E□AH-1

(15) □AM◇I-1⇒□EM◇O-1⇒□EM◇O-2⇒□E□AH-1⇒□E□A◇H-1

(16) □AM◇I-1⇒□EM◇O-1⇒□EM◇O-2⇒□AF◇O-2

(17) □AM◇I-1⇒□EM◇O-1⇒□EM◇O-2⇒□AF◇O-2⇒□A□AL-1

(18) □AM◇I-1⇒□EM◇O-1⇒□EM◇O-2⇒□AF◇O-2⇒□A□AL-1⇒□A□A◇L-1

(19) □AM◇I-1⇒□EM◇O-1⇒□EM◇O-2⇒□AF◇O-2⇒F□A◇O-3

(20) □AM◇I-1⇒□A□EH-2⇒□A□E◇H-2⇒□A□M◇I-1

Proof:

[1] ⊢ $\Box all(S, T) \wedge most(R, S) \rightarrow \Diamond some(R, T)$ (i. e.□AM◇I-1)

[2] ⊢ $all(S, T) = no\neg(S, T)$ (by Fact 1(1))

[3] ⊢ $some(R, T) = not\ all\neg(R, T)$ (by Fact 1(3))

[4] ⊢ $\Box no\neg(S, T)\wedge most(R, S)\rightarrow\Diamond not\ all\neg(R, T)$ (by [1], [2], [3])

[5] ⊢ $no\neg(S, T)=no(S, D-T)$ (by Definition 2)

[6] ⊢ $not\ all\neg(R, T)=not\ all(R, D-T)$ (by Definition 2)

[7] ⊢ $\Box no(S, D-T)\wedge most(R, S)\rightarrow\Diamond not\ all(R, D-T)$ (i. e. □EM◇O-1, by [4], [5], [6])

[8] ⊢ $some(R, T)\leftrightarrow some(T, R)$ (by Fact 8(1))

[9] ⊢ $\Box all(S, T)\wedge most(R, S)\rightarrow\Diamond some(T, R)$ (i. e. M□A◇I-4, by [1] and [8])

[10] ⊢ $\neg\Diamond some(R, T)\wedge most(R, S)\rightarrow\neg\Box all(S, T)$ (by [1] and Rule 2)

[11] ⊢ $\neg\Diamond some(R, T)=\Box\neg some(R, T)$ (by Fact 3(2))

[12] ⊢ $\neg\Box all(S, T)=\Diamond\neg all(S, T)$ (by Fact 3(1))

[13] ⊢ $\Box\neg some(R, T)\wedge most(R, S)\rightarrow\Diamond\neg all(S, T)$ (by [10], [11], [12])

[14] ⊢ $\neg some(R, T)=no(R, T)$ (by Fact 2(4))

[15] ⊢ $\neg all(S, T)=not\ all(S, T)$ (by Fact 2(2))

[16] ⊢ $\Box no(R, T)\wedge most(R, S)\rightarrow\Diamond not\ all(S, T)$ (i. e. □EM◇O-3, by [13], [14], [15])

[17] ⊢ $\neg\Diamond some(R, T)\wedge\Box all(S, T)\rightarrow\neg most(R, S)$ (by [1] and Rule 2)

[18] ⊢ $\neg most(R, S)=at\ most\ half\ of\ the(R, S)$ (by Fact 2(5), [17])

[19] ⊢ $\Box no(R, T)\wedge\Box all(S, T)\rightarrow at\ most\ half\ of\ the(R, S)$ (i. e. □A□EH-2, by [11], [14], [17], [18])

[20] ⊢ $at\ most\ half\ of\ the(R, S)\rightarrow\Diamond at\ most\ half\ of\ the(R, S)$ (by Fact 6)

[21] ⊢ $\Box no(R, T)\wedge\Box all(S, T)\rightarrow\Diamond at\ most\ half\ of\ the(R, S)$

(i. e. □A□E◇H-2, by [19], [20] and Rule 1)

[22] ⊢ $no(S, D-T)\leftrightarrow no(D-T, S)$ (by Fact 8(2))

[23] ⊢ $\Box no(D-T, S)\wedge most(R, S)\rightarrow\Diamond not\ all(R, D-T)$ (i. e. □EM◇O-2, by [7], [22])

[24] ⊢ $\neg\Diamond not\ all(R, D-T)\wedge\Box no(S, D-T)\rightarrow\neg most(R, S)$ (by [7] and Rule 2)

[25] ⊢ $\Box\neg not\ all(R, D-T)\wedge\Box no(S, D-T)\rightarrow\neg most(R, S)$ (by Fact 3(2))

[26] ⊢ $\Box all(R, D-T)\wedge\Box no(S, D-T)\rightarrow at\ most\ half\ of\ the(R, S)$

(i. e. □E□AH-2,by Fact 2(1), Fact 2(5), [25])

[27] ⊢ $\Box all(R, D-T)\wedge\Box no(S, D-T)\rightarrow\Diamond at\ most\ half\ of\ the(R, S)$

(i. e. □E□A◇H-2, by [20], [26] and Rule 1)

[28] ⊢ $\neg\Diamond not\ all(R, D-T)\wedge most(R, S)\rightarrow\neg\Box no(S, D-T)$ (by [7] and Rule 2)

[29] ⊢ $\Box\neg$*not all*$(R, D-T)\wedge$*most*$(R, S)\rightarrow\Diamond\neg$*no*$(S, D-T)$ (by Fact 3 and [28])

[30] ⊢ $\Box$*all*$(R, D-T)\wedge$*most*$(R, S)\rightarrow\Diamond$*some*$(S, D-T)$

(i. e. □AM◇I-3, by Fact 2(1), Fact 2(3), [29])

[31] ⊢ $\neg\Diamond$*some*$(T, R)\wedge\Box$*all*$(S, T)\rightarrow\neg$*most*(R, S) (by [9] and Rule 2)

[32] ⊢ $\Box\neg$*some*$(T, R)\wedge\Box$*all*$(S, T)\rightarrow\neg$*most*(R, S) (by Fact 3(2) and [31])

[33] ⊢ $\Box$*no*$(T, R)\wedge\Box$*all*$(S, T)\rightarrow$*at most half of the*(R, S)

(i. e. □A□EH-4, by Fact 2(4), Fact 2(5), [32])

[34] ⊢ $\Box$*no*$(T, R)\wedge\Box$*all*$(S, T)\rightarrow\Diamond$*at most half of the*$(R, S)$

(i. e. □A□E◇H-4,by [20], [33] and Rule 1)

[35] ⊢ $\neg\Diamond$*some*$(T, R)\wedge$*most*$(R, S)\rightarrow\neg\Box$*all*$(S, T)$ (by [9] and Rule 2)

[36] ⊢ $\Box\neg$*some*$(T, R)\wedge$*most*$(R, S)\rightarrow\Diamond\neg$*all*$(S, T)$ (by Fact 3 and [35])

[37] ⊢ $\Box$*no*$(T, R)\wedge$*most*$(R, S)\rightarrow\Diamond$*not all*$(S, T)$

(i. e. □EM◇O-4, by Fact 2(4), Fact 2(2), [36])

[38] ⊢ $\neg\Diamond$*not all*$(R, D-T)\wedge$*most*$(R, S)\rightarrow\neg\Box$*no*$(D-T, S)$ (by [23] and Rule 2)

[39] ⊢ $\Box\neg$*not all*$(R, D-T)\wedge$*most*$(R, S)\rightarrow\Diamond\neg$*no*$(D-T, S)$ (by Fact 3 and [38])

[40] ⊢ $\Box$*all*$(R, D-T)\wedge$*most*$(R, S)\rightarrow\Diamond$*some*$(D-T, S)$

(i. e. M□A◇I-3, by Fact 2(1), Fact 2(3), [39])

[41] ⊢ $\neg\Diamond$*not all*$(R, D-T)\wedge\Box$*no*$(D-T, S)\rightarrow\neg$*most*(R, S) (by [23] and Rule 2)

[42] ⊢ $\Box\neg$*not all*$(R, D-T)\wedge\Box$*no*$(D-T, S)\rightarrow\neg$*most*(R, S) (by Fact 3(2) and [41])

[43] ⊢ $\Box$*all*$(R, D-T)\wedge\Box$*no*$(D-T, S)\rightarrow$*at most half of the*(R, S)

(i. e. □E□AH-1, by Fact 2(1), Fact 2(5), [42])

[44] ⊢ $\Box$*all*$(R, D-T)\wedge\Box$*no*$(D-T, S)\rightarrow\Diamond$*at most half of the*$(R, S)$

(i. e. □E□A◇H-1, by [20], [43] and Rule 1)

[45] ⊢ *no*$(D-T, S)=$*all*$\neg(D-T, S)$ (by Fact 1(2))

[46] ⊢ *most*$(R, S)=$*fewer than half of the*$\neg(R, S)$ (by Fact 1(6))

[47] ⊢ $\Box$*all*$\neg(D-T, S)\wedge$*fewer than half of the*$\neg(R, S)\rightarrow\Diamond$*not all*$(R, D-T)$ (by [23], [45], [46])

[48] ⊢ $\Box$*all*$(D-T, D-S)\wedge$*fewer than half of the*$(R, D-S)\rightarrow\Diamond$*not all*$(R, D-T)$

(i. e. □AF◇O-2, by Definition 2 and [47])

[49] ⊢ $\neg\Diamond$*not all*$(R, D-T)\wedge\Box$*all*$(D-T, D-S)\rightarrow\neg$*fewer than half of the*$(R, D-S)$

(by [48] and Rule 2)

[50] ⊢ $\neg$*fewer than half of the*$(R, D-S)$=*at least half of the*$(R, D-S)$ (by Fact 2(7))

[51] ⊢ $\Box\neg$*not all*$(R, D-T)\wedge\Box$*all*$(D-T, D-S)\rightarrow$*at least half of the*$(R, D-S)$

(by Fact 3(2), [49] and [50])

[52] ⊢ $\Box$*all*$(R, D-T)\wedge\Box$*all*$(D-T, D-S)\rightarrow$*at least half of the*$(R, D-S)$

(i. e. □A□AL-1, by [51] and Fact 2(1))

[53] ⊢ *at least half of the*$(R, D-S)\rightarrow\Diamond$*at least half of the*$(R, D-S)$ (by Fact 5)

[54] ⊢ $\Box$*all*$(R, D-T)\wedge\Box$*all*$(D-T, D-S)\rightarrow\Diamond$*at least half of the*$(R, D-S)$

(i. e. □A□A◇L-1, by [52], [53])

[55] ⊢ $\neg\Diamond$*not all*$(R, D-T)\wedge$*fewer than half of the*$(R, D-S)\rightarrow\neg\Box$*all*$(D-T, D-S)$

(by [48] and Rule 2)

[56] ⊢ $\Box\neg$*not all*$(R, D-T)\wedge$*fewer than half of the*$(R, D-S)\rightarrow\Diamond\neg$*all*$(D-T, D-S)$ (by Fact 3, [55])

[57] ⊢ $\Box$*all*$(R, D-T)\wedge$*fewer than half of the*$(R, D-S)\rightarrow\Diamond$*not all*$(D-T, D-S)$

(i. e. F□A◇O-3, by Fact 2(1), Fact 2(2), [56])

[58] ⊢ $\neg\Diamond$*at most half of the*$(R, S)\wedge\Box$*all*$(S, T)\rightarrow\neg\Box$*no*$(R, T)$ (by [21] and Rule 2)

[59] ⊢ $\Box\neg$*at most half of the*$(R, S)\wedge\Box$*all*$(S, T)\rightarrow\Diamond\neg$*no*$(R, T)$ (by Fact 3 and [58])

[60] ⊢ $\Box$*most*$(R, S)\wedge\Box$*all*$(S, T)\rightarrow\Diamond$*some*$(R, T)$ (i. e. □A□M◇I-1, by Fact 2(3), Fact 2(5), [59])

So far, the other 20 generalized modal syllogisms have been deduced from the validity of the syllogism □AM◇I-1 by means of generalized quantifier theory and modern modal logic. This indicates that there is the reducibility between these 21 syllogisms.

4. Conclusion

The research in this paper illustrates that there is the reducibility between the generalized modal syllogism □AM◇I-1 and the other 20 valid generalized modal syllogisms. Specifically, this paper first proves the validity of the generalized modal syllogism □AM◇I-1 based on the truth value definitions of sentences with quantification, set theory and modal logic, and then derives the other 20 valid generalized modal syllogisms from the syllogism □AM◇I-1 in line with some facts and inference rules. And the entire process of proof is ultimately presented in a structured formalization way. The reason why these syllogisms are reducible is that: (1) any of the Aristotelian quantifiers can be defined by the other three Aristotelian quantifiers; (2) any of the four generalized quantifiers mentioned in this paper (that is, *most*, *at most half of the*, *fewer than half of the* and *at least half of the*) can be defined by the other three generalized quantifiers; (3) the Aristotelian quantifiers *some* and *no* have symmetry; (4) a necessary modality □ and a possible modality ◇ can be mutually defined. Just for the above reasons, more valid generalized modal syllogisms can be also derived from □AM◇I-1.

This paper inspires us to simplify the study of other generalized modal syllogisms through a fragmentary research approach. That is to say, by making most of the transformation relations of the four generalized quantifiers (such as *at most* 1/3 *of the*, *most than* 1/3 *of the*, *at least* 2/3 *of the*, *less than* 2/3 *of the*) and that of the necessary modality □ and the possible modality ◇, one can not only show the reducibility of corresponding valid generalized modal syllogisms, but also discuss the soundness and completeness for the fragment of generalized modal syllogistic. There are many generalized modal syllogisms in natural language. Therefore, this study has practical significance and theoretical value for knowledge representation and reasoning in artificial intelligence.

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