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Knowledge Mining Based on the Valid Aristotelian Modal Syllogism □AA◇A-1

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Abstract

This paper firstly presents knowledge representations of Aristotelian modal syllogisms based on the structure of modal categorical propositions; Then, the validity of syllogism □AA◇A-1 was proved by using the truth value definitions of these propositions. And then this syllogism was used for knowledge reasoning with the help of some reduction operations. More specifically, the validity of the other 30 syllogisms can be inferred from that of □AA◇A-1 with the help of possible world semantics and generalized quantifier theory. Owing to the possible modality (◇) and necessary modality (□) can be defined mutually, and any one Aristotelian quantifier can define the other three, so there are reducible relations between/among 31 valid Aristotelian modal syllogisms. Due to the other 30 valid syllogisms obtained by deductive methods, these results are consistent. That is superior to previous works. This formal method is consistent with the ideas of knowledge mining in artificial intelligence.

Keywords: Aristotelian modal syllogisms; reducible relations; knowledge reasoning; knowledge mining

1. Introduction

The fact that syllogistic reasoning has been a widespread and significant form of reasoning in human thinking. There are different kinds of syllogisms in natural language, such as categorical syllogisms [7], [10], [14], Aristotelian modal syllogisms [6], [16], [17], and syllogisms with Boolean operations [4], etc. This paper focuses on Aristotelian modal syllogisms.

McCall ([9]), Geach ([3]), Johnson ([5]) and other scholars have discussed modal syllogisms and made some achievements, but it's far from establishing a formal consistent axiom system for Aristotelian modal syllogistic. Thomason ([13]) and Malink ([8]) have given adequate semantic analysis or reconstruction of the syllogistic, but the commonplace view is that there are a lot of inconsistencies and faults. Thus, this paper attempts to contribute to the consistency of this syllogistic. Specifically, it takes advantage of related knowledge to infer the validity of other Aristotelian modal syllogisms by proving the validity of one Aristotelian modal syllogism (i.e.$\Box AA\Diamond A$-1).

2.Knowledge Representation for Aristotelian Modal Syllogisms

In the following, *G*, *M* and *N* denote lexical variables, and *D* the domain of lexical variables. *Q* stands for any of Aristotelian quantifiers (that is, *all*, *some*, *no* and *not all*), $\neg Q$ and $Q\neg$ for the outer and inner negative quantifier of *Q*, respectively. And the symbol '$=_{def}$ 'means the left is defined by the right. Let β, ϕ, λ, and θ be propositions. '$\vdash\beta$' stands for that β is provable. The others are similar. And $\wedge$, $\neg$, $\rightarrow$, $\leftrightarrow$, $\Diamond$ and $\Box$ are respectively symbols of conjunction, negation, conditionality, biconditionality, possibility, and necessity.

Aristotelian syllogisms contain the following four kinds of propositions: *all(G, N)*, *some(G, N)*, *no(G, N)*, and *not all(G, N)*, meaning that 'all *G*s are *N*s', 'some *G*s are *N*s', 'no *G*s are *N*s' and 'not all *G*s are *N*s', respectively. They are noted as Proposition *A*, *E*, *I* and *O*, respectively. An Aristotelian modal syllogism is obtained by adding modalities ($\Diamond$ or $\Box$) to an Aristotelian syllogism. A modal syllogism contains at least one possible modality ($\Diamond$)or necessary modality ($\Box$) and at most three modalities. Therefore, there are the following 12 propositions: *A*, *E*, *I*, *O*, $\Diamond A$, $\Diamond E$, $\Diamond I$, $\Diamond O$, $\Box A$, $\Box E$, $\Box I$ and $\Box O$.

Example 1:

Major premise: All birds necessarily can fly.

Minor premise: All robins are birds.

Conclusion: All robins possibly can fly.

The figures of Aristotelian modal syllogisms is as usual [1]. Then the above example is the first figure syllogism. Let G be the set of robins, M the set of birds, and N the set of flying animals in the domain. The above syllogism can be symbolized as $\Box$ *all(M, N)* $\wedge$ *all(G, M)*$\rightarrow\Diamond$*all(G, N)*, abbreviated as $\Box AA\Diamond A$-1.

2.1 Relevant Definitions

According to set theory [1] and possible world semantics [2], and generalized quantifier theory [12], the truth value definitions of the above 12 propositions can be given as follows:

Definition 1 (truth value definition):

(1.1) *all(G, N)* is true when and only when $G\subseteq N$ is true in any real world.

(1.2) *some(G, N)* is true when and only when $G\cap N\neq\varnothing$ is true in any real world.

(1.3) *no(G, N)* is true when and only when $G\cap N=\varnothing$ is true in any real world.

(1.4) *not all(G, N)* is true when and only when $G\not\subseteq N$ is true in any real world.

(1.5) $\Box$*all(G, N)* is true when and only when $G\subseteq N$ is true in any possible world.

(1.6) $\Diamond$*all(G, N)* is true when and only when $G\subseteq N$ is true in at least one possible world.

(1.7) $\Box$*some(G, N)* is true when and only when $G\cap N\neq\varnothing$ is true in any possible world.

(1.8) $\Diamond$*some(G, N)* is true when and only when $G\cap N\neq\varnothing$ is true in at least one possible world.

(1.9) $\Box$*no(G, N)* is true when and only when $G\cap N=\varnothing$ is true in any possible world.

(1.10) $\Diamond$*no(G, N)* is true when and only when $G\cap N=\varnothing$ is true in at least one possible world.

(1.11) $\Box$*not all(G, N)* is true when and only when $G\not\subseteq N$ is true in any possible world.

(1.12) $\Diamond$*not all(G, N)* is true when and only when $G\not\subseteq N$ is true in at least one possible world.

Definition 2 (inner negation**)**: $Q\neg(G, N) =_{\text{def}} Q(G, D-N)$.

Definition 3 (outer negation): $\neg Q(G, N) =_{\text{def}}$ It is not that $Q(G, N)$.

2.2 Relevant Facts

Fact 1 (inner negation)

(1) $all(G, N) \leftrightarrow no\neg(G, N)$; (2) $no(G, N) \leftrightarrow all\neg(G, N)$;

(3) $some(G, N) \leftrightarrow not\ all\neg(G, N)$; (4) $not\ all(G, N) \leftrightarrow some\neg(G, N)$.

Fact 2 (outer negation):

(1) $\neg not\ all(G, N) \leftrightarrow all(G, N)$; (2) $\neg all(G, N) \leftrightarrow not\ all(G, N)$;

(3) $\neg no(G, N) \leftrightarrow some(G, N)$; (4) $\neg some(G, N) \leftrightarrow no(G, N)$.

Fact 3 (dual): (1) $\neg\Box Q(G, N) \leftrightarrow \Diamond\neg Q(G, N)$; (2) $\neg\Diamond Q(G, N) \leftrightarrow \Box\neg Q(G, N)$.

Fact 4 (a necessary proposition implies an assertoric one): $\vdash\Box Q(G, N) \rightarrow Q(G, N)$.

Fact 5 (a necessary proposition implies a possible one): $\vdash\Box Q(G, N) \rightarrow \Diamond Q(G, N)$.

Fact 6 (an assertoric proposition implies a possible one): $\vdash Q(G, N) \rightarrow \Diamond Q(G, N)$.

Fact 7 (a universal proposition implies a particular one):

(1) $\vdash all(G, N) \rightarrow some(G, N)$; (2) $\vdash no(G, N) \rightarrow not\ all(G, N)$.

Fact 8 (symmetry of *some* and *no*): (1) $some(G, N) \leftrightarrow some(N, G)$; (2) $no(G, N) \leftrightarrow no(N, G)$.

The above facts are the basic knowledge in generalized quantifier theory or modal logic, their proofs are omitted [11].

2.3 Relevant Inference Rules

Rule 1 (subsequent weakening): If $\vdash(\beta\wedge\phi\rightarrow\lambda)$ and $\vdash(\lambda\rightarrow\theta)$, then $\vdash(\beta\wedge\phi\rightarrow\theta)$.

Rule 2 (anti-syllogism): If $\vdash(\beta\wedge\phi\rightarrow\lambda)$, then $\vdash(\neg\lambda\wedge\beta\rightarrow\neg\phi)$.

Rule 3 (anti-syllogism): If $\vdash(\beta\wedge\phi\rightarrow\lambda)$, then $\vdash(\neg\lambda\wedge\phi\rightarrow\neg\beta)$.

3. Knowledge Reasoning Based on the Syllogism □AA◇A-1

The following Theorem 1 shows that the syllogism *□AA◇A-1* is valid. In the following Theorem 2, *□AA◇A-1→□AA◇I-1* means that the validity of the syllogism *□AA◇I-1* can be inferred from that of *□AA◇A-1*. One can say that there are reducible relations between the two syllogisms. The others are similar.

Theorem 1 ($\Box AA\Diamond A\text{-}1$): $\Box all(M, N)\wedge all(G, M)\rightarrow\Diamond all(G, N)$ is valid.

Proof: According to Example 1, $\Box AA\Diamond A\text{-}1$ is the abbreviation of the first figure syllogism $\Box all(M, N)\wedge all(G, M)\rightarrow\Diamond all(G, N)$. Suppose that $\Box all(M, N)$ and $all(G, M)$ are true, then $M\subseteq N$ is true at any possible world and $G\subseteq M$ is true at any real world in line with Definition (1.5) and (1.1), respectively. Because all real worlds are possible worlds. Now it follows that $G\subseteq N$ is true in at least one possible world. Thus, $\Diamond all(G, N)$ is true according to Definition (1.6). This proves that the syllogism $\Box all(M, N)\wedge all(G, M)\rightarrow\Diamond all(G, N)$ is valid.

Theorem 2: The validity of the following 30 syllogisms can be inferred from that of $\Box AA\Diamond A\text{-}1$:

(2.1) $\vdash\Box AA\Diamond A\text{-}1\rightarrow\Box AA\Diamond I\text{-}1$

(2.2) $\vdash\Box AA\Diamond A\text{-}1\rightarrow\Box AA\Diamond I\text{-}1\rightarrow A\Box A\Diamond I\text{-}4$

(2.3) $\vdash\Box AA\Diamond A\text{-}1\rightarrow\Box EA\Diamond E\text{-}1$

(2.4) $\vdash\Box AA\Diamond A\text{-}1\rightarrow\Box EA\Diamond E\text{-}1\rightarrow\Box EA\Diamond E\text{-}2$

(2.5) $\vdash\Box AA\Diamond A\text{-}1\rightarrow\Box EA\Diamond E\text{-}1\rightarrow A\Box E\Diamond E\text{-}4$

(2.6) $\vdash\Box AA\Diamond A\text{-}1\rightarrow\Box EA\Diamond E\text{-}1\rightarrow\Box EA\Diamond E\text{-}2\rightarrow A\Box E\Diamond E\text{-}2$

(2.7) $\vdash\Box AA\Diamond A\text{-}1\rightarrow\Box EA\Diamond E\text{-}1\rightarrow\Box EA\Diamond O\text{-}1$

(2.8) $\vdash\Box AA\Diamond A\text{-}1\rightarrow\Box EA\Diamond E\text{-}1\rightarrow\Box EA\Diamond O\text{-}1\rightarrow\Box EA\Diamond O\text{-}2$

(2.9) $\vdash\Box AA\Diamond A\text{-}1\rightarrow\Box OA\Diamond O\text{-}3$

(2.10) $\vdash\Box AA\Diamond A\text{-}1\rightarrow\Box A\Box OO\text{-}2$

(2.11) $\vdash\Box AA\Diamond A\text{-}1\rightarrow\Box AA\Diamond I\text{-}1\rightarrow\Box EA\Diamond O\text{-}3$

(2.12) $\vdash\Box AA\Diamond A\text{-}1\rightarrow\Box AA\Diamond I\text{-}1\rightarrow\Box EA\Diamond O\text{-}3\rightarrow\Box EA\Diamond O\text{-}4$

(2.13) $\vdash\Box AA\Diamond A\text{-}1\rightarrow\Box AA\Diamond I\text{-}1\rightarrow\Box A\Box EO\text{-}2$

(2.14) $\vdash\Box AA\Diamond A\text{-}1\rightarrow\Box AA\Diamond I\text{-}1\rightarrow\Box A\Box EO\text{-}2\rightarrow\Box A\Box EO\text{-}4$

(2.15) $\vdash\Box AA\Diamond A\text{-}1\rightarrow\Box EA\Diamond E\text{-}1\rightarrow\Box IA\Diamond I\text{-}3$

(2.16) $\vdash\Box AA\Diamond A\text{-}1\rightarrow\Box EA\Diamond E\text{-}1\rightarrow\Box IA\Diamond I\text{-}3\rightarrow\Box IA\Diamond I\text{-}4$

(2.17) $\vdash\Box AA\Diamond A\text{-}1\rightarrow\Box EA\Diamond E\text{-}1\rightarrow\Box E\Box IO\text{-}2$

(2.18) $\vdash\Box AA\Diamond A\text{-}1\rightarrow\Box EA\Diamond E\text{-}1\rightarrow\Box E\Diamond IO\text{-}2\rightarrow\Box E\Box IO\text{-}4$

(2.19) $\vdash\Box AA\Diamond A\text{-}1\rightarrow\Box EA\Diamond E\text{-}1\rightarrow\Box E\Diamond IO\text{-}2\rightarrow\Box E\Box IO\text{-}1$

(2.20) $\vdash\Box AA\Diamond A\text{-}1\rightarrow\Box EA\Diamond E\text{-}1\rightarrow\Box E\Diamond IO\text{-}2\rightarrow\Box E\Box IO\text{-}4\ \Box E\Box IO\text{-}4\rightarrow\Box E\Box IO\text{-}3$

(2.21) $\vdash\Box AA\Diamond A\text{-}1\rightarrow\Box EA\Diamond E\text{-}1\rightarrow\Box EA\Diamond E\text{-}2\rightarrow A\Box I\Diamond I\text{-}3$

(2.22) $\vdash\Box AA\Diamond A\text{-}1\rightarrow\Box EA\Diamond E\text{-}1\rightarrow\Box EA\Diamond E\text{-}2\rightarrow A\Box I\Diamond I\text{-}3\rightarrow A\Box I\Diamond I\text{-}1$

(2.23) $\vdash\Box AA\Diamond A\text{-}1\rightarrow\Box EA\Diamond E\text{-}1\rightarrow\Box EA\Diamond E\text{-}2\rightarrow A\Box I\Diamond I\text{-}3\rightarrow\Box IA\Diamond I\text{-}3$

(2.24) $\vdash\Box AA\Diamond A\text{-}1\rightarrow\Box EA\Diamond E\text{-}1\rightarrow\Box EA\Diamond E\text{-}2\rightarrow A\Box I\Diamond I\text{-}3\rightarrow A\Box I\Diamond I\text{-}1\rightarrow\Box IA\Diamond I\text{-}4$

(2.25) $\vdash\Box AA\Diamond A\text{-}1\rightarrow\Box EA\Diamond E\text{-}1\rightarrow\Box EA\Diamond O\text{-}1\rightarrow\Box E\Box AO\text{-}2$

(2.26) $\vdash\Box AA\Diamond A\text{-}1\rightarrow\Box EA\Diamond E\text{-}1\rightarrow\Box EA\Diamond O\text{-}1\rightarrow\Box E\Box AO\text{-}2\rightarrow\Box E\Box AO\text{-}1$

(2.27) $\vdash\Box AA\Diamond A\text{-}1\rightarrow\Box EA\Diamond E\text{-}1\rightarrow\Box EA\Diamond O\text{-}1\rightarrow\Box AA\Diamond I\text{-}3$

(2.28) $\vdash\Box AA\Diamond A\text{-}1\rightarrow\Box EA\Diamond E\text{-}1\rightarrow\Box EA\Diamond O\text{-}1\rightarrow\Box AA\Diamond I\text{-}3\rightarrow A\Box A\Diamond I\text{-}3$

(2.29) $\vdash\Box AA\Diamond A\text{-}1\rightarrow\Box EA\Diamond E\text{-}1\rightarrow A\Box E\Diamond E\text{-}4\rightarrow A\Box E\Diamond O\text{-}4$

(2.30) $\vdash\Box AA\Diamond A\text{-}1\rightarrow\Box AA\Diamond I\text{-}1\rightarrow\Box A\Box EO\text{-}2\rightarrow A\Box E\Diamond O\text{-}2$

Proof:

[1] $\vdash\Box all(M, N)\wedge all(G, M)\rightarrow\Diamond all(G, N)$ (i.e. $\Box AA\Diamond A\text{-}1$, Theorem 1)

[2] $\vdash\Box all(M, N)\wedge all(G, M)\rightarrow\Diamond some(G, N)$ (i.e. $\Box AA\Diamond I\text{-}1$, by [1] and Fact 7)

[3] $\vdash\Box all(M, N)\wedge all(G, M)\rightarrow\Diamond some(N, G)$ (i.e. $A\Box A\Diamond I\text{-}4$, by [2] and Fact 8)

[4] $\vdash\Box no\neg(M, N)\wedge all(G, M)\rightarrow\Diamond no\neg(G, N)$ (by [1] and Fact 1)

[5] $\vdash\Box no(M, D-N)\wedge all(G, M)\rightarrow\Diamond no(G, D-N)$ (i.e. $\Box EA\Diamond E\text{-}1$, by [4] and Definition 2)

[6] $\vdash\Box no(D-N, M)\wedge all(G, M)\rightarrow\Diamond no(G, D-N)$ (i.e. $\Box EA\Diamond E\text{-}2$, by [5] and Fact 8)

[7] $\vdash\Box no(M, D-N)\wedge all(G, M)\rightarrow\Diamond no(D-N, G)$ (i.e. $A\Box E\Diamond E\text{-}4$, by [5] and Fact 8)

[8] $\vdash\Box no(D-N, M)\wedge all(G, M)\rightarrow\Diamond no(D-N, G)$ (i.e. $A\Box E\Diamond E\text{-}2$, by [6] and Fact 8)

[9] $\vdash\Box no(M, D-N)\wedge all(G, M)\rightarrow\Diamond not\ all(G, D-N)$ (i.e. $\Box EA\Diamond O\text{-}1$, by [5] and Fact 7)

[10] $\vdash\Box no(D-N, M)\wedge all(G, M)\rightarrow\Diamond not\ all(G, D-N)$ (i.e. $\Box EA\Diamond O\text{-}2$, by [9] and Fact 8)

[11] $\vdash\neg\Diamond all(G, N)\wedge all(G, M)\rightarrow\neg\Box all(M, N)$ (by [1] and Rule 2)

[12] $\vdash\Box\neg all(G, N)\wedge all(G, M)\rightarrow\Diamond\neg all(M, N)$ (by [11] and Fact 3)

[13] $\vdash\Box not\ all(G, N)\wedge all(G, M)\rightarrow\Diamond not\ all(M, N)$ (i.e. $\Box OA\Diamond O$-3, by [12] and Fact 2)

[14] $\vdash\neg\Diamond all(G, N)\wedge\Box all(M, N)\rightarrow\neg all(G, M)$ (by [1] and Rule 3)

[15] $\vdash\Box\neg all(G, N)\wedge\Box all(M, N)\rightarrow\neg all(G, M)$ (by [14] and Fact 3)

[16] $\vdash\Box not\ all(G, N)\wedge\Box all(M, N)\rightarrow not\ all(G, M)$ (i.e. $\Box A\Box OO$-2, by [15] and Fact 2)

[17] $\vdash\neg\Diamond some(G, N)\wedge all(G, M)\rightarrow\neg\Box all(M, N)$ (by [2] and Rule 2)

[18] $\vdash\Box\neg some(G, N)\wedge all(G, M)\rightarrow\Diamond\neg all(M, N)$ (by [17] and Fact 3)

[19] $\vdash\Box no(G, N)\wedge all(G, M)\rightarrow\Diamond not\ all(M, N)$ (i.e. $\Box EA\Diamond O$-3, by [18] and Fact 2)

[20] $\vdash\Box no(N, G)\wedge all(G, M)\rightarrow\Diamond not\ all(M, N)$ (i.e. $\Box EA\Diamond O$-4, by [19] and Fact 8)

[21] $\vdash\Box\neg some(G, N)\wedge\Box all(M, N)\rightarrow\neg all(G, M)$ (by [2] and Rule 3 and Fact 3)

[22] $\vdash\Box no(G, N)\wedge\Box all(M, N)\rightarrow not\ all(G, M)$ (i.e. $\Box A\Box EO$-2, by [21] and Fact 2)

[23] $\vdash\Box no(N, G)\wedge\Box all(M, N)\rightarrow not\ all(G, M)$ (i.e. $\Box A\Box EO$-4, by [22] and Fact 8)

[24] $\vdash\Box\neg no(G, D\text{–}N)\wedge all(G, M)\rightarrow\Diamond\neg no(M, D\text{–}N)$ (by [5] and Rule 2 and Fact 3)

[25] $\vdash\Box \mathrm{some}(G, D\text{–}N)\wedge all(G, M)\rightarrow\Diamond some(M, D\text{–}N)$ (i.e. $\Box IA\Diamond I$-3, by [24] and Fact 2)

[26] $\vdash\Box some(D\text{–}N, G)\wedge all(G, M)\rightarrow\Diamond some(M, D\text{–}N)$ (i.e. $\Box IA\Diamond I$-4, by [25] and Fact 8)

[27] $\vdash\Box\neg no(G, D\text{–}N)\wedge\Box no(M, D\text{–}N)\rightarrow\neg all(G, M)$ (by [5] and Rule 2 and Fact 3)

[28] $\vdash\Box some(G, D\text{–}N)\wedge\Box no(M, D\text{–}N)\rightarrow not\ all(G, M)$ (i.e. $\Box E\Box IO$-2, by [27] and Fact 2)

[29] $\vdash\Box some(D\text{–}N, G)\wedge\Box no(M, D\text{–}N)\rightarrow not\ all(G, M)$ (i.e. $\Box E\Box IO$-4, by [28] and Fact 8)

[30] $\vdash\Box some(G, D\text{–}N)\wedge\Box no(D\text{–}N, M)\rightarrow not\ all(G, M)$ (i.e. $\Box E\Box IO$-1, by [28] and Fact 8)

[31] $\vdash\Box some(D\text{–}N, G)\wedge\Box no(D\text{–}N, M)\rightarrow not\ all(G, M)$ (i.e. $\Box E\Box IO$-3, by [29] and Fact 8)

[32] $\vdash\Box\neg no(G, D\text{–}N)\wedge all(G, M)\rightarrow\Diamond\neg no(D\text{–}N, M)$ (by [6] and Rule 2 and Fact 3)

[33] $\vdash\Box some(G, D\text{–}N)\wedge all(G, M)\rightarrow\Diamond some(D\text{–}N, M)$ (i.e. $A\Box I\Diamond I$-3, by [32] and Fact 2)

[34] $\vdash\Box some(D\text{–}N, G)\wedge all(G, M)\rightarrow\Diamond some(D\text{–}N, M)$ (i.e. $A\Box I\Diamond I$-1, by [33] and Fact 8)

[35] $\vdash\Box some(G, D\text{–}N)\wedge all(G, M)\rightarrow\Diamond some(M, D\text{–}N)$ (i.e. $\Box IA\Diamond I$-3, by [33] and Fact 8)

[36] $\vdash\Box some(D\text{–}N, G)\wedge all(G, M)\rightarrow\Diamond some(M, D\text{–}N)$ (i.e. $\Box IA\Diamond I$-4, by [34] and Fact 8)

[37] $\vdash\Box\neg not\ all(G, D\text{–}N)\wedge\Box no(M, D\text{–}N)\rightarrow\neg all(G, M)$ (by [9] and Rule 2 and Fact 3)

[38] $\vdash\Box all(G, D\text{–}N)\wedge\Box no(M, D\text{–}N)\rightarrow not\ all(G, M)$ (i.e. □*E*□*AO-2,* by [37] and Fact 2)

[39] $\vdash\Box all(G, D\text{–}N)\wedge\Box no(D\text{–}N, M)\rightarrow not\ all(G, M)$ (i.e. □*E*□*AO-1*, by [38] and Fact 8)

[40] $\vdash\Box\neg not\ all(G, D\text{–}N)\wedge all(G, M)\rightarrow\Diamond\neg no(M, D\text{–}N)$ (by [9] and Rule 3 and Fact 3)

[41] $\vdash\Box all(G, D\text{–}N)\wedge all(G, M)\rightarrow\Diamond some(M, D\text{–}N)$ (i.e. □*AA*◇*I-3*, by [40] and Fact 2)

[42] $\vdash\Box all(G, D\text{–}N)\wedge all(G, M)\rightarrow\Diamond some(D\text{–}N, M)$ (i.e. *A*□*A*◇*I-3,* by [41] and Fact 8)

[43] $\vdash\Box no(M, D\text{–}N)\wedge all(G, M)\rightarrow\Diamond not\ all(D\text{–}N, G)$ (i.e. *A*□*E*◇*O-4*, by [7] and Fact 7)

[44] $\vdash\Box no(D\text{–}N, M)\wedge all(G, M)\rightarrow\Diamond not\ all(D\text{–}N, G)$ (i.e. *A*□*E*◇*O-2*, by [8] and Fact 7)

Theorem 2 shows there are reducible relations between valid Aristotelian modal syllogisms of different figures and forms.

5. Conclusion

To sum up, Theorem 1 firstly proves that the syllogism □*AA*◇*A-1* is valid. Theorem 2 shows the validity of the other 30 syllogisms can be inferred from that of □*AA*◇*A-1* with the help of possible world semantics and generalized quantifier theory. Owing to the possible modality (◇) and necessary modality (□) can be defined mutually, and any Aristotelian quantifier also can be defined by the other three, so there are reducible relations between/among 31 valid Aristotelian modal syllogisms.

This paper provides a concise mathematical method for studying other types of modal syllogisms (such as generalized modal syllogisms). The formal processing technology of natural language in artificial intelligence has developed rapidly and has occupied an important position. Therefore, how to make full use of this method to benefit natural language information processing? This question is worth further study.

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