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The Deductibility of the Aristotelian Modal Syllogism E□I◇O-4 from the Perspective of Mathematical Structuralism

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Abstract

This paper firstly provides knowledge representations of Aristotelian modal syllogisms from the perspective of mathematical structuralism, and proves the validity of Aristotelian modal syllogism E□I◇O-4, and then by making full use of relevant definitions, facts, and some inference rules, formally derive other 30 valid modal syllogisms on the basis of one modal syllogism (i.e. E□I◇O-4) as a basic axiom. The reason why modal syllogisms are deducible is that the four Aristotelian quantifiers (i.e. *all*, *no*, *some*, and *not all*) can be mutually defined, and that so can the two modalities (i.e. □ and ◇). Thus, one can establish a minimalist formal axiomatic system for modal syllogistic logic. This formal method is not only beneficial for the study of other types of syllogisms, but also for the development of a more intelligent inference engine for expert systems.

Keywords: Aristotelian modal syllogism; Aristotelian quantifier; symmetry; deducible relation

1. Introduction

It is known that syllogistic reasoning is one of important themes in natural language information processing [1-2]. There are many types of syllogisms, such as categorical syllogisms [3], generalized ones [4], Aristotelian modal ones [5], generalized modal ones [6], etc. This paper mainly studies on Aristotelian modal syllogisms. There were works on the syllogism of the Aristotelian modal syllogisms, such as Łukasiewicz [7], McCall [8], Triker [9], Brennan [10], Malink [11], Xiaojun [12], Long and Xiaojun [13], and so on.

Inspired by previous works, this paper only takes the valid Aristotelian modal syllogism E□I◇O-4 as a basic axiom to deduce other valid modal syllogisms. That is to say that there are deducible relations between/among valid Aristotelian modal syllogisms.

2. Preliminaries

In this paper, *Q* represents any of the Aristotelian quantifiers (i.e. *all*, *some*, *no*, *not all*). The inner negation of *Q* is denoted as $Q\neg$, and the outer negation of *Q* as $\neg Q$. And *f, u* and *g* stand for the lexical variables. The sets composed of *f*, *u* are *g* is respectively *F*, *U*, and *G*. And *D* denotes the domain of lexical variables. β, δ, ϕ and λ are well-formed formulas. '$\vdash\beta$' indicates that β is provable. The others are similar.

Aristotelian syllogisms contain three categorical propositions which have the following four types: 'all *f*s are *g*s', 'no *f*s are *g*s', 'some *f*s are *g*s', and 'not all *f*s are *g*s'. They are respectively called Proposition *A*, *E*, *I* and *O*. From the perspective of mathematical structuralism, these four propositions can be respectively expressed as follows: *all*(*f*, *g*), *no*(*f*, *g*), *some*(*f*, *g*), and *not all*(*f*, *g*).

An Aristotelian modal syllogism can be obtained by adding at least one necessary modality (i.e. $\Box$) or possible one (i.e. $\Diamond$) to an Aristotelian syllogism. For example, the modal syllogism E$\Box$I$\Diamond$O-4 can be obtained by adding one $\Box$ and one $\Diamond$ to the Aristotelian one EIO-4. The modal syllogism E$\Box$I$\Diamond$O-4 denotes 'no *g*s are *u*s, and some *u*s are necessarily *f*s, so not all *f*s are possibly *g*s.', which can be formalized as $no(g, u) \wedge \Box some(u, f) \rightarrow \Diamond not\ all(f, g)$. The others are similar.

An Aristotelian modal syllogism can be explained in the following example:

Major premise: No wolves are grass eaters.

Minor premise: Some grass eaters are necessarily rabbits.

Conclusion: Not all rabbits are possibly wolves.

According to modal logic [14] and generalized quantifier theory [15-16], the following definitions can be obtained:

Definition 1 (truth value):

(1) *all(F, G)* is true when and only when $F \subseteq G$ is true in any real world.

(2) *no(F, G)* is true when and only when $F \cap G = \varnothing$ is true in any real world.

(3) *some(F, G)* is true when and only when $F \cap G \neq \varnothing$ is true in any real world.

(4) *not all(F, G)* is true when and only when $F \nsubseteq G$ is true in any real world.

(5) $\Box$*all(F, G)* is true when and only when $F \subseteq G$ is true in any possible world.

(6) $\Box$*no(F, G)* is true when and only when $F \cap G = \varnothing$ is true in any possible world.

(7) $\Box$*some(F, G)* is true when and only when $F \cap G \neq \varnothing$ is true in any possible world.

(8) $\Box$*not all(F, G)* is true when and only when $F \nsubseteq G$ is true in any possible world.

(9) $\Diamond$*all(F, G)* is true when and only when $F \subseteq G$ is true in at least one possible world.

(10) $\Diamond$*no(F, G)* is true when and only when $F \cap G = \varnothing$ is true in at least one possible world.

(11) $\Diamond$*some(F, G)* is true when and only when $F \cap G \neq \varnothing$ is true in at least one possible world.

(12) $\Diamond$*not all(F, G)* is true when and only when $F \nsubseteq G$ is true in at least one possible world.

Definition 2 (inner negation): $Q\neg(f, g)=_{\text{def}} Q(f, D-g)$.

Definition 3 (outer negation): $\neg Q(f, g)=_{\text{def}}$ It is not that $Q(f, g)$.

On the basis of generalized quantifier theory [15-16], the following four facts can be obtained:

Fact 1 (inner negation):

(1.1) $\vdash all(f, g)\leftrightarrow no\neg(f, g)$; (1.2) $\vdash no(f, g)\leftrightarrow all\neg(f, g)$;

(1.3) $\vdash some(f, g)\leftrightarrow not\ all\neg(f, g)$; (1.4) $\vdash not\ all(f, g)\leftrightarrow some\neg(f, g)$.

Fact 2 (outer negation):

(2.1) $\vdash\neg not\ all(f, g)\leftrightarrow all(f, g)$; (2.2) $\vdash\neg all(f, g)\leftrightarrow not\ all(f, g)$;

(2.3) $\vdash\neg no(f, g)\leftrightarrow some(f, g)$; (2.4) $\vdash\neg some(f, g)\leftrightarrow no(f, g)$.

Fact 3 (symmetry):

(3.1) $\vdash some(f, g)\leftrightarrow some(g, f)$;

(3.2) $\vdash no(f, g)\leftrightarrow no(g, f)$.

Fact 4 (assertoric subalternations):

(4.1) $\vdash all(f, g)\rightarrow some(f, g)$; (4.2) $\vdash no(f, g)\rightarrow not\ all(f, g)$.

In the light of modal logic [14], the following facts hold:

Fact 5 (dual):

(5.1) $\vdash\neg\Box Q(f, g)=\Diamond\neg Q(f, g)$; (5.2) $\vdash\neg\Diamond Q(f, g)=\Box\neg Q(f, g)$.

Fact 6: $\vdash\Box Q(f, g)\rightarrow Q(f, g)$.

Fact 7: $\vdash\Box Q(f, g)\rightarrow\Diamond Q(f, g)$.

The following deductive rules in propositional logic [17] are also applicable in Aristotelian modal syllogistic.

Rule 1: If $\vdash(\beta\wedge\delta\rightarrow\phi)$ and $\vdash(\phi\rightarrow\lambda)$, then $\vdash(\beta\wedge\delta\rightarrow\lambda)$.

Rule 2: If $\vdash(\beta\wedge\delta\rightarrow\phi)$, then $\vdash(\neg\phi\wedge\beta\rightarrow\neg\delta)$ or $\vdash(\neg\phi\wedge\delta\rightarrow\neg\beta)$.

3. The Reduction from the Modal Syllogism E□I◇O-4 to Other Modal Syllogisms

The validity of the modal syllogism E□I◇O-4 is proved in the following Theorem 1. '(2.1) E□I◇O-4→E□I◇O-3' in Theorem 2 indicates that the validity of E□I◇O-3 can be deduced from the validity of E□I◇O-4. In other words, there is deductibility between these two modal syllogisms. The others are similar.

Theorem 1(E□I◇O-4): The syllogism *no(g, u)*∧□*some(u, f)*→◇*not all(f, g)* is valid.

Proof: Assuming that *no(g, u)* and □*some(u, f)* are true, it follows that $G \cap U = \varnothing$ is true in any real world and $U \cap F \neq \varnothing$ is true in any possible world in terms of Definition (2) and (7), respectively. A real world is a possible world. Then $F \nsubseteq G$ is true in at least one possible world. This can be proven by reductio ad absurdum. Assume that $F \nsubseteq G$ is not true in at least one possible world . That is, $F \subseteq G$ is true in at least one possible world, and $G \cap U = \varnothing$ has been proven to be true. Thus, it follows that $F \cap U = \varnothing$ is true, which contradicts $U \cap F \neq \varnothing$. So, $F \subseteq G$ is not true in at least one possible world. That means $F \nsubseteq G$ is true in at least one possible world. Then in accordance with Definition (7), ◇*not all(f, g)* is true, just as require.

Theorem 2: The following 30 valid modal syllogisms can be derived from the syllogism *EIO-4*:

(2.1) E□I◇O-4→E□I◇O-3

(2.2) E□I◇O-4→E□I◇O-2

(2.3) E□I◇O-4→E□I◇O-3→E□I◇O-1

(2.4) E□I◇O-4→□AE◇E-4

(2.5) E□I◇O-4→□AE◇E-4→□AE◇E-2

(2.6) E□I◇O-4→□AE◇E-4→E□A◇E-1

(2.7) E□I◇O-4→□AE◇E-4→E□A◇E-1→E□A◇E-2

(2.8) E□I◇O-4→□AE◇E-4→□AE◇O-4

(2.9) E□I◇O-4→□AE◇E-4→□AE◇E-2→□AE◇O-2

(2.10) E□I◇O-4→□AE◇E-4→E□A◇E-1→E□A◇O-1

(2.11) E□I◇O-4→□AE◇E-4→E□A◇E-1→E□A◇E-2→E□A◇O-2

(2.12) E□I◇O-4→E□I◇O-3→A□I◇I-3

(2.13) E□I◇O-4→E□I◇O-3→A□I◇I-3→A□I◇I-1

(2.14) E□I◇O-4→E□I◇O-3→A□I◇I-3→□IA◇I-3

(2.15) E□I◇O-4→E□I◇O-3→A□I◇I-3→□IA◇I-3→□IA◇I-4

(2.16) E□I◇O-4→E□I◇O-2→A□O◇O-2

(2.17) E□I◇O-4→□AE◇E-4→E□A◇E-1→A□A◇A-1

(2.18) E□I◇O-4→□AE◇E-4→E□A◇E-1→A□A◇A-1→A□A◇I-1

(2.19) E□I◇O-4→□AE◇E-4→E□A◇E-1→A□A◇A-1→A□A◇I-1→□AA◇I-4

(2.20) E□I◇O-4→□AE◇E-4→E□A◇E-1→A□A◇A-1→□O□AO-3

(2.21) E□I◇O-4→□AE◇E-4→E□A◇E-1→A□A◇A-1→A□A◇I-1→□E□AO-3

(2.22) E□I◇O-4→□AE◇E-4→E□A◇E-1→A□A◇A-1→A□A◇I-1→□E□AO-3→
□E□AO-4

(2.23) E□I◇O-4→□AE◇E-4→E□A◇E-1→E□A◇O-1→□A□AI-3

(2.24) E□I◇O-4→□I□AI-4

(2.25) E□I◇O-4→□E□AE-4→□I□AI-3

(2.26) E□I◇O-4→□E□AE-4→□A□II-1

(2.27) E□I◇O-4→□E□AE-4→□A□EE-1→□A□II-3

(2.28) E□I◇O-4→□AE◇E-4→E□A◇E-1→A□A◇A-1→A□O◇O-2

(2.29) E□I◇O-4→□AE◇E-4→E□A◇E-1→A□A◇A-1→A□A◇I-1→A□E◇O-2

(2.30) E□I◇O-4→□AE◇E-4→E□A◇E-1→A□A◇A-1→A□A◇I-1→A□E◇O-2→
A□E◇O-4

Proof:

[1] $\vdash no(g, u)\wedge\Box some(u, f)\rightarrow\Diamond not\ all(f, g)$ (i.e. E□I◇O-4, basic axiom)

[2] $\vdash no(u, g)\wedge\Box some(u, f)\rightarrow\Diamond not\ all(f, g)$ (i.e. E□I◇O-3, by [1] and Fact (3.2))

[3] $\vdash no(g, u)\wedge\Box some(f, u)\rightarrow\Diamond not\ all(f, g)$ (i.e. E□I◇O-2, by [1] and Fact (3.1))

[4] $\vdash no(u, g)\wedge\Box some(f, u)\rightarrow\Diamond not\ all(f, g)$ (i.e. E□I◇O-1, by [2] and Fact (3.1))

[5] $\vdash\neg\Diamond not\ all(f, g)\wedge no(g, u)\rightarrow\neg\Box some(u, f)$ (by [1] and Rule 2)

[6] $\vdash\Box\neg not\ all(f, g)\wedge no(g, u)\rightarrow\Diamond\neg some(u, f)$ (by [5] and Fact 5)

[7] $\vdash\Box all(f, g)\wedge no(g, u)\rightarrow\Diamond no(u, f)$ (i.e. □AE◇E-4, by [6], Fact (2.1) and (2.4))

[8] $\vdash\Box all(f, g)\wedge no(u, g)\rightarrow\Diamond no(u, f)$ (i.e. □AE◇E-2, by [7] and Fact (3.2))

[9] $\vdash\Box all(f, g)\wedge no(g, u)\rightarrow\Diamond no(f, u)$ (i.e. E□A◇E-1, by [7] and Fact (3.2))

[10]$\vdash\Box all(f, g)\wedge no(u, g)\rightarrow\Diamond no(f, u)$ (i.e. E□A◇E-2, by [9] and Fact (3.2))

[11] $\vdash\Diamond no(u, f)\rightarrow\Diamond not\ all(u, f)$ (by Fact (4.2))

[12] $\vdash\Box all(f, g)\wedge no(g, u)\rightarrow\Diamond not\ all(u, f)$ (i.e. □AE◇O-4, by [7], [11]and Rule 1)

[13] $\vdash\Box all(f, g)\wedge no(u, g)\rightarrow\Diamond not\ all(u, f)$ (i.e. □AE◇O-2, by [8], [11]and Rule 1)

[14] $\vdash\Diamond no(f, u)\rightarrow\Diamond not\ all(f, u)$ (by Fact (4.2))

[15] $\vdash\Box all(f, g)\wedge no(g, u)\rightarrow\Diamond not\ all(f, u)$ (i.e. E□A◇O-1, by [9], [14]and Rule 1)

[16] $\vdash\Box all(f, g)\wedge no(u, g)\rightarrow\Diamond not\ all(f, u)$ (i.e. E□A◇O-2, by [10], [14]and Rule 1)

[17] $\vdash all\neg(u, g)\wedge\Box some(u, f)\rightarrow\Diamond some\neg(f, g)$ (by [2], Fact (1.2) and (1.4))

[18] $\vdash all(u, D-g)\wedge\Box some(u, f)\rightarrow\Diamond some(f, D-g)$ (i.e. A□I◇I-3, by [17] and Definition 2)

[19] $\vdash all(u, D-g)\wedge\Box some(f, u)\rightarrow\Diamond some(f, D-g)$ (i.e. A□I◇I-1, by [18] and Fact (3.1))

[20] $\vdash all(u, D-g)\wedge\Box some(u, f)\rightarrow\Diamond some(D-g, f)$ (i.e. □IA◇I-3, by [18] and Fact (3.1))

[21] $\vdash all(u, D-g)\wedge\Box some(f, u)\rightarrow\Diamond some(D-g, f)$ (i.e. □IA◇I-4, by [20] and Fact (3.1))

[22] $\vdash all\neg(g, u)\wedge\Box not\ all\neg(f, u)\rightarrow\Diamond not\ all(f, g)$ (by [3], Fact (1.2) and (1.3))

[23] $\vdash all(g, D-u)\wedge\Box not\ all(f, D-u)\rightarrow\Diamond not\ all(f, g)$ (i.e. A□O◇O-2, by [22] and Definition 2)

[24] $\vdash\Box all(f, g)\wedge all\neg(g, u)\rightarrow\Diamond all\neg(f, u)$ (by [9] and Fact (1.2))

[25] $\vdash\Box all(f, g)\wedge all(g, D\neg u)\rightarrow\Diamond all(f, D\neg u)$ (i.e. A□A◇A-1, by [24] and Definition 2)

[26] $\vdash\Diamond all(f, D\neg u)\rightarrow\Diamond some(f, D\neg u)$ (by Fact (4.1))

[27] $\vdash\Box all(f, g)\wedge all(g, D\neg u)\rightarrow\Diamond some(f, D\neg u)$ (i.e. A□A◇I-1, by [25], [26]and Rule 1)

[28] $\vdash\Box all(f, g)\wedge all(g, D\neg u)\rightarrow\Diamond some(D\neg u, f)$ (i.e. □AA◇I-4, by [27] and Fact (3.1))

[29] $\vdash\neg\Diamond all(f, D\neg u)\wedge\Box all(f, g)\rightarrow\neg all(g, D\neg u)$ (by [25] and Rule 2)

[30] $\vdash\Box\neg all(f, D\neg u)\wedge\Box all(f, g)\rightarrow\neg all(g, D\neg u)$ (by [29] and Fact (5.2))

[31] $\vdash\Box not\ all(f, D\neg u)\wedge\Box all(f, g)\rightarrow not\ all(g, D\neg u)$ (i.e. □O□AO-3, by [30]and Fact (2.2))

[32] $\vdash\neg\Diamond some(f, D\neg u)\wedge\Box all(f, g)\rightarrow\neg all(g, D\neg u)$ (by [27] and Rule 2)

[33] $\vdash\Box\neg some(f, D\neg u)\wedge\Box all(f, g)\rightarrow\neg all(g, D\neg u)$ (by [32] and Fact (5.2))

[34] $\vdash\Box no(f, D\neg u)\wedge\Box all(f, g)\rightarrow not\ all(g, D\neg u)$ (i.e. □E□AO-3, by [33], Fact (2.2) and (2.4))

[35] $\vdash\Box no(D\neg u, s)\wedge\Box all(f, g)\rightarrow not\ all(g, D\neg u)$ (i.e. □E□AO-4, by [34] and Fact (3.2))

[36] $\vdash\neg\Diamond not\ all(f, u)\wedge\Box all(f, g)\rightarrow\neg no(g, u)$ (by [15] and Rule 2)

[37] $\vdash\Box\neg not\ all(f, u)\wedge\Box all(f, g)\rightarrow\neg no(g, u)$ (by [36] and Fact (5.2))

[38] $\vdash\Box all(f, u)\wedge\Box all(f, g)\rightarrow some(g, u)$ (i.e. □A□AI-3, by [37] and Fact (2.1) and (2.3))

[39] $\vdash\neg\Diamond not\ all(f, g)\wedge\Box some(u, f)\rightarrow\neg no(g, u)$ (by [1] and Rule 2)

[40] $\vdash\Box\neg not\ all(f, g)\wedge\Box some(u, f)\rightarrow\neg no(g, u)$ (by [39] and Fact (5.2))

[41] $\vdash\Box all(f, g)\wedge\Box some(u, f)\rightarrow some(g, u)$ (i.e. □I□AI-4, by [40], Fact (2.1) and (2.3))

[42] $\vdash\Box all(f, g)\wedge\Box some(f, u)\rightarrow some(g, u)$ (i.e. □I□AI-3, by [41] and Fact (3.1))

[43] $\vdash\Box all(f, g)\wedge\Box some(u, f)\rightarrow some(u, g)$ (i.e. □A□II-1, by [41] and Fact (3.1))

[44] $\vdash\Box all(f, g)\wedge\Box some(f, u)\rightarrow some(u, g)$ (i.e. □A□II-3, by [43] and Fact (3.1))

[45] $\vdash\neg\Diamond all(f, D\neg u)\wedge all(g, D\neg u)\rightarrow\neg\Box all(f, g)$ (by [25] and Rule 2)

[46] $\vdash\Box\neg all(f, D\neg u)\wedge all(g, D\neg u)\rightarrow\Diamond\neg all(f, g)$ (by [45] and Fact 5)

[47] $\vdash\Box not\ all(f, D\neg u)\wedge all(g, D\neg u)\rightarrow\Diamond not\ all(f, g)$ (i.e. A□O◇O-2, by [46] and Fact (2.2))

[48] $\vdash\neg\Diamond some(f, D-u)\wedge all(g, D-u)\rightarrow\neg\Box all(f, g)$ (by [27] and Rule 2)

[49] $\vdash\Box\neg some(f, D-u)\wedge all(g, D-u)\rightarrow\Diamond\neg all(f, g)$ (by [48] and Fact 5)

[50] $\vdash\Box no(f, D-u)\wedge all(g, D-u)\rightarrow\Diamond not\ all(f, g)$ (i.e. A□E◇O-2, by [49], Fact (2.2) and (2.4))

[51] $\vdash\Box no(D-u, f)\wedge all(g, D-u)\rightarrow\Diamond not\ all(f, g)$ (i.e. A□E◇O-4, by [50], Fact (3.2))

So far, the validity of the above 30 Aristotelian modal syllogisms have been inferred from that of the one E□I◇O-4 taken as a basic axiom.

4. Conclusion and Future Work

This paper firstly provides knowledge representations of Aristotelian modal syllogisms from the perspective of mathematical structuralism, and proves the validity of the Aristotelian modal syllogism E□I◇O-4, and then derives the other 30 valid modal syllogisms in line with set theory, generalized quantifier theory and modal logic. Then a minimalist formal axiomatic system can be established for Aristotelian modal syllogistic. The deducible relations between/among modal syllogisms are revealed in the process of deduction. The reason why modal syllogisms can be deducible is that the four Aristotelian quantifiers (i.e. *all*, *no*, *some*, and *not all*) can be mutually defined, and that so can the two modalities(i.e. □ and ◇).

In fact, this formal method not only provides a mathematical model for the study of Aristotelian modal syllogisms, but also inspiration for the study of other types of syllogisms(e.g., generalized syllogisms and generalized modal syllogisms), and also for the deeper development of machine reasoning in artificial intelligence. More questions about the deducible relations between/among various syllogisms need further research.

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