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The Reducible Relations between/among Valid Generalized Syllogisms with the Generalized Quantifiers in Square{most}

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Abstract

This paper firstly proves that the syllogism EMO-4 is valid, and secondly indicates the validity of the other 14 syllogisms can be deduced from that of EMO-4 with the help of generalized quantifier theory, set theory, and first-order logic. And more valid generalized syllogisms can be deduced when one continues to infer. It indicates that there are reducible relations between/among valid generalized syllogisms. It is hoped that this research will not only promote the development of modern logic, but also provide assistance for knowledge reasoning in natural language.

Keywords: generalized quantifiers; first-order logic; reducible relation; generalized syllogisms

1. Introduction

There are two types of quantifiers in natural language: Aristotelian quantifiers (that is, *all*, *not all*, *some*, *no*) and generalized quantifiers (Zhang, 2018). The former is a special case of the latter (Wei, 2023). A generalized syllogism contains at least one generalized quantifier (Hao, 2024). This paper only studies non-trivial generalized syllogisms(Moss, 2008; Endrullis and Moss, 2015), which include at least one generalized quantifier beyond Aristotelian quantifiers. There are infinitely many non-trivial generalized quantifiers in natural language (Barwise and Cooper, 1981; Westerståhl, 1989), such as *most*, *both*, *fewer than half of the*, which can form an infinite number of non-trivial generalized syllogisms. Generalized syllogism reasoning is one of the important forms of syllogistic reasoning which has been a widespread and significant form of reasoning in human thinking (Xu and Zhang, 2023).

But there are few works about generalized syllogisms. Different from the previous studies, this paper is devoted to studying the validity and reducibility of generalized syllogisms with the non-trivial generalized quantifier *most* and its outer, inner, and dual negative quantifiers, that is, *at most half of the*, *fewer than half of the*, and *at least half of the*, respectively. The above four quantifiers form a modern square{most}.

2. Preliminary Knowledge

In this paper, *g, k*, and *r* denote lexical variables, and *D* represents their domain. The sets composed of *g, k*, and *r* are denoted by *G*, *K,* and *R*, respectively. Let θ, γ, φ and ϕ be well-formed formulas (abbreviated as wff). '$|G \cap R|$' represents the cardinality for the intersection of the sets *G* and *R* (Halmos, 1974). '$\vdash\theta$'indicates that the formula θ is provable, and '$\gamma =_{def} \phi$'indicates that γ can be defined by ϕ. Others are similar. The connectives in the paper such as, ¬, →, ∧, ↔ are symbols in first-order logic.

Generalized syllogisms in this paper only involves the following eight propositions: *all (g, r), no (g, r), some (g, r), not all (g, r), most (g, r), fewer than half of the (g, r), at most half of the (g, r), at least half of the (g, r)*, which are respectively abbreviated as Propositions *A, E, I, O, M, F, H*, and *S*. In this paper, *Q* just refers to one of the eight quantifiers contained in these eight propositions. The fourth figure generalized syllogism *no(g, k)∧most(k, r)→not all(r, g)* is abbreviated as EMO-4, which is the foundation of syllogism deductive reasoning in this paper. A natural language example of it is as follows:

Major premise: No plastic conducts electricity.

Minor premise: Most conductive things are metals.

Conclusion: Not all metals are plastic.

Let *g* be a lexical variable that stands for plastic in the domain, *k* be a lexical variable that denotes conductive things in the domain, and *r* be a lexical variable that represents metals in the domain. Then this syllogism can be formalized as '$no(g, k) \wedge most(k, r) \rightarrow not\ all(r, g)$', which is abbreviated as EMO-4. Others are similar to this.

3. Generalized Syllogism System Including the Generalized Quantifier '*Most*'

This system contains the following: primitive symbols, formation and deductive rules, and basic axioms, and so on.

3.1 Primitive Symbols

(1) lexical variables: *g*, *k*, *r*

(2) quantifier: *no*

(3) quantifier: *most*

(4) unary connectives: $\neg$, $\rightarrow$

(5) brackets: (,)

3.2 Formation Rules

(1) If *Q* is a quantifier, *g* and *r* are lexical variables, then $Q(g, r)$ is a wff.

(2) If φ and θ are wffs, then so is $\varphi \rightarrow \theta$.

(3) Only the formulas obtained by the rules are wffs.

3.3 Basic Axioms

A1: If ϕ is a valid formula in first-order logic, then $\vdash \phi$.

A2: $\vdash no(g, k) \wedge most(k, r) \rightarrow not\ all(r, g)$ (that is, the syllogism EMO-4).

3.4 Deductive Rules

Rule 1 (subsequent weakening): From $\vdash(\theta\wedge\delta\rightarrow\phi)$ and $\vdash(\phi\rightarrow\psi)$ infer $\vdash(\theta\wedge\delta\rightarrow\psi)$.

Rule 2 (anti-syllogism): From $\vdash(\theta\wedge\delta\rightarrow\phi)$ infer $\vdash(\neg\phi\wedge\theta\rightarrow\neg\delta)$.

Rule 3 (anti-syllogism): From $\vdash(\theta\wedge\delta\rightarrow\phi)$ infer $\vdash(\neg\phi\wedge\delta\rightarrow\neg\theta)$.

3.5 Relevant Definitions

D1: $(\theta\wedge\phi)=_{def}\neg(\theta\rightarrow\neg\phi)$;

D2: $(\theta\leftrightarrow\phi)=_{def}(\theta\rightarrow\phi)\wedge(\phi\rightarrow\theta)$;

D3: $(Q\neg)(g, r)=_{def} Q(g, D-r)$

D4: $(\neg Q)(g, r)=_{def}$ It is not that $Q(g, r)$

D5: *all*(g, r) is true when and only when $G\subseteq R$ is true;

D6: *some*(g, r) is true when and only when $G\cap R\neq\varnothing$ is true;

D7: *no*(g, r) is true when and only when $G\cap R=\varnothing$ is true;

D8: *not all*(g, r) is true when and only when $G\nsubseteq R$ is true;

D9: *most*(g, r) is true when and only when $|G\cap R|>0.5|G|$ is true;

D10: *fewer than half of the*(g, r) is true when and only when $|G\cap R|<0.5|G|$ is true ;

D11: *at most half of the*(g, r) is true when and only when $|G\cap R|\leq0.5|G|$ is true;

D12: *at least half of the*(g, r) is true when and only when $|G\cap R|\geq0.5|G|$ is true.

Fact 1(Inner Negation):

(1.1) $\vdash$*all*(g, r)=*no*$\neg(g, r)$;

(1.2) $\vdash$*no*(g, r)=*all*$\neg(g, r)$;

(1.3) $\vdash$*some*(g, r)=*not all*$\neg(g, r)$;

(1.4) $\vdash$*not all*(g, r)=*some*$\neg(g, r)$;

(1.5) $\vdash$*most*(g, r)=*fewer than half of the*$\neg(g, r)$;

(1.6) $\vdash$*fewer than half of the*(g, r)=*most*$\neg(g, r)$;

(1.7) $\vdash$*at least half of the*(g, r)=*at most half of the*$\neg(g, r)$;

(1.8) ⊢*at most half of the*(*g, r*)=*at least half of the*¬(*g, r*).

Fact 2(Outer Negation)::

(2.1) ⊢¬*all*(*g, r*)=*not all*(*g, r*);

(2.2) ⊢¬*not all*(*g, r*)=*all*(*g, r*);

(2.3) ⊢¬*no*(*g, r*)=*some*(*g, r*);

(2.4) ⊢¬*some*(*g, r*)=*no*(*g, r*);

(2.5) ⊢¬*most*(*g, r*)=*at most half of the*(*g, r*);

(2.6) ⊢¬*at most half of the*(*g, r*)=*most*(*g, r*);

(2.7) ⊢¬*fewer than half of the*(*g, r*)=*at least half of the*(*g, r*);

(2.8) ⊢¬*at least half of the*(*g, r*)=*fewer than half of the*(*g, r*).

Fact 3 (Symmetry):

(3.1) ⊢*some*(*g, r*)↔*some*(*r, g*);

(3.2) ⊢*no*(*g, r*)↔*no*(*r*, *g*).

Fact 4 (Subordination):

(4.1) ⊢*all(g, r)*→*some(g, r)*;

(4.2) ⊢*no(g, r)*→*not all(g, r)*;

(4.3) ⊢*all(g, r)*→*most(g, r)*;

(4.4) ⊢*most(g, r)*→*some(g, r)*;

(4.5) ⊢*at least half of the(g, r)*→*some(g, r)*;

(4.6) ⊢*all(g, r)*→*at least half of the(g, r)*;

(4.7) ⊢*at most half of the(g, r)*→*not all(g, r)*;

(4.8) ⊢*fewer than half of the(g, r)*→*not all(g, r)*.

4. The Validity and Reducibility of the Generalized Syllogism EMO-4

The following Theorem 1 indicates that the syllogism EMO-4 is valid. In the following Theorem 2, EMO-4 $\rightarrow$ EMO-3 means that the validity of the syllogism EMO-3 can be inferred from that of EMO-4. One can say that there are reducible relations between the two syllogisms. In other words, the syllogism EMO-3 has reducibility. The others are similar.

Theorem 1 (EMO-4): The generalized syllogism *no(g, k)∧most(k, r)→not all(r, g)* is valid.

Proof: According to Example 1, EMO-4 is the abbreviation of the fourth figure syllogism *no(g, k)∧most(k, r)→not all(r, g)*. Suppose that *no(g, k)* and *most(k, r)* are true, then $G\cap K=\varnothing$ is true and $|K\cap R|>0.5|K|$ is true in line with Definition D7 and D9, respectively. Now it follows that $R\cap G=\varnothing$ is true. Thus, *not all(r, g)* is true according to Definition D8. This proves that the syllogism *no(g, k)∧most(k, r)→not all(r, g)* is valid.

Theorem 2: There are the following 14 valid generalized syllogisms inferred from the syllogism EMO-4:

(2.1) ⊢EMO-4→EMO-3

(2.2) ⊢EMO-4→AEH-4

(2.3) ⊢EMO-4→AEH-4→AEH-2

(2.4) ⊢EMO-4→MAI-4

(2.5) ⊢EMO-4→MAI-4→AMI-1

(2.6) ⊢EMO-4→EMO-3→AMI-3

(2.7) ⊢EMO-4→EMO-3→AMI-3→MAI-3

(2.8) ⊢EMO-4→AEH-4→AEH-2→EAH-2

(2.9) ⊢EMO-4→AEH-4→AEH-2→EAH-2→EAH-1

(2.10) ⊢EMO-4→MAI-4→AMI-1→EMO-1

(2.11) ⊢EMO-4→MAI-4→AMI-1→EMO-1→EMO-2

(2.12) ⊢EMO-4→EMO-3→AMI-3→MAI-3→FAO-3

(2.13) ⊢EMO-4→AEH-4→AEH-2→EAH-2→EAH-1→AAS-1

(2.14) ⊢EMO-4→MAI-4→AMI-1→EMO-1→EMO-2→AFO-2

Proof

[1] ⊢*no(g, k)∧most(k, r)→not all(r, g)* (i.e.EMO-4, basic axiom)

[2] ⊢*no(k, g)∧most(k, r)→not all(r, g)* (i.e.EMO-3, by[1] and Fact 3)

[3] ⊢*¬not all(r, g)∧no(g, k)→¬most(k, r)* (by[1] and Rule 2)

[4] $\vdash$*all(r, g)∧no(g, k)→at most half of the(k, r)* (i.e.AEH-4, by[3] and Fact 2)

[5] $\vdash$*all(r, g)∧no(k, g)→at most half of the(k, r)* (i.e.AEH-2, by[4] and Fact 3)

[6] $\vdash$*¬not all(r, g)∧most(k, r)→¬no(g, k)* (by[1] and Rule 3)

[7] $\vdash$*all(r, g)∧most(k, r)→*some*(g, k)* (i.e.MAI-4, by[6] and Fact 2)

[8] $\vdash$*all(r, g)∧most(k, r)→*some*(k, g)* (i.e.AMI-1, by[7] and Fact 3)

[9] $\vdash$*all¬(k, g)∧most(k, r)→some¬(r, g)* (by[2] and Fact 1)

[10] $\vdash$*all(k, D−g)∧most(k, r)→some(r, D−g)* (i.e.AMI-3, by[9] and D3)

[11] $\vdash$*all(k, D−g)∧most(k, r)→some(D−g, r)* (i.e.MAI-3, by[10] and Fact 3)

[12] $\vdash$*no¬(r, g)∧all¬(k, g)→at most half of the(k, r)* (by[5] and Fact 1)

[13] $\vdash$*no(r, D−g)∧all(k, D−g)→at most half of the(k, r)* (i.e.EAH-2, by[12] and D3)

[14] $\vdash$*no(D−g, r)∧all(k, D−g)→at most half of the(k, r)* (i.e.EAH-1, by[13] and Fact 3)

[15] $\vdash$*no¬(r, g)∧most(k, r)→not all¬(k, g)* (by[8] and Fact 1)

[16] $\vdash$*no(r, D−g)∧most(k, r)→not all(k, D−g)* (i.e.EMO-1, by[15] and D3)

[17] $\vdash$*no(D−g, r)∧most(k, r)→not all(k, D−g)* (i.e.EMO-2, by[16] and Fact 3)

[18] $\vdash$*all(k, D−g)∧fewer than half of the¬(k, r)→not all¬(D−g, r)* (by[11] and Fact 1)

[19] $\vdash$*all(k, D−g)∧fewer than half of the(k, D−r)→not all(D−g, D−r)* (i.e.FAO-3, by[18] and D3)

[20] $\vdash$*all¬(D−g, r)∧all(k, D−g)→at least half of the¬(k, r)* (by[14] and Fact 1)

[21] $\vdash$*all(D−g, D−r)∧all(k, D−g)→at least half of the(k, D−r)* (i.e.AAS-1, by[20] and D3)

[22] $\vdash$*all¬(D−g, r)∧fewer than half of the¬(k, r)→not all(k, D−g)* (by[17] and Fact 1)

[23] $\vdash$*all(D−g, D−r)∧fewer than half of the(k, D−r)→not all(k, D−g)* (i.e.AFO-2, by[22] and D3)

Now, the other 14 valid generalized syllogisms have been deduced from the validity of EMO-4.

4. Conclusion

To sum up, Theorem 1 firstly proves that the syllogism EMO-4 is valid. Theorem 2 indicates the validity of the other 14 syllogisms can be deduced from that of EMO-4 with the help of generalized quantifier theory, set theory, and first-order logic. It shows that there are reducible relations between/among valid generalized syllogisms of different figures and forms.

This paper only studies the validity and reducibility of the generalized syllogisms that involve the generalized quantifiers in modern Square{*most*}. This work provides a concise and universal deductive method for generalized syllogisms that include other non-trivial generalized quantifiers (such as *at least two-thirds*, *both*, *infinitely many*).

This study has important theoretical value and practical significance for knowledge reasoning in natural language. Therefore, it is crucial to conduct in-depth research on the validity and reducibility of other generalized syllogisms.

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