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Knowledge Mining Based on the Aristotelian Modal Syllogism □EAO-3

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Abstract

This paper formalizes Aristotelian modal syllogism within the framework of knowledge mining and subsequently proves the validity of the Aristotelian modal syllogism □EAO-3 through the application of modal logic and generalized quantifier theory. Moreover, by means of the integration of specific rules and facts derived from first-order logic, the definitions of outer and inner negations of Aristotelian quantifiers in generalized quantifier theory, a minimum of 25 additional valid Aristotelian modal syllogisms based on the validity of the syllogism □EAO-3 have been successfully derived from the perspective of artificial intelligence knowledge mining. The proposed method not only exhibits elegance and simplicity, but also demonstrates its potential for universal applicability to diverse syllogistic scenarios. Undoubtedly, this research makes a significant contribution to the knowledge mining in the field of artificial intelligence.

Keywords: Aristotelian modal syllogisms; knowledge mining; validity; generalized quantifier theory

1. Introduction

Syllogistic reasoning is a scientific method of thinking that enables individuals to arrive at accurate conclusions when engaging in mathematical proofs, handling cases, and conducting scientific research. It represents a form of deductive reasoning characterized by correct logical inference ([1]). This paper primarily focuses on the Aristotelian modal syllogism, which is one of the various types of syllogism ([2]). Therefore, unless explicitly specified otherwise, all subsequent discussions in this paper pertain to Aristotelian modal syllogisms.

Because of its importance, syllogism has been studied by many scholars since Aristotle ([3]). For example, Łukasiewicz(1957)[4], McCall(1963)[5], Thomason(1997)[6], Johnson(2004)[7], Malink(2013) [8], and Zhang (2020)[9], and so on. These studies primarily focused on their validity, however, inconsistencies in research results have been subject to criticism. The reducibility between syllogisms has always received limited attention, and this paper aims to make a significant contribution in addressing this research gap. Specifically, this paper focuses on the reduction between the validity of the syllogism □EAO-3 and other syllogisms. To this end, this paper initially proves the validity of □EAO-3, and subsequently derives the validity of other modal syllogisms through pertinent definitions, facts, and reasoning rules, thereby ensuring consistent outcomes.

2. Preliminaries

In the following, let *h*, *r* and *z* be lexical variables, and *D* be their domain. The sets composed of *h*, *r* and *z* are respectively *H, R*, and *Z*. Let *c*, *n*, *m*, and *t* be well-formed formulas (shortened as wff). Let Q be a quantifier, $\neg Q$, and $Q\neg$ be its outer and inner negation, respectively. '$|H\cap Z|$' states the cardinality of the intersection of the set *H* and *Z*. '$\vdash c$' means that the wff *c* is provable, and '$c =_{\text{def}} n$' that *c* can be defined by *n*. The others are similar. The operators (such as ¬, →, ∧, ↔) in this paper are symbols in set theory ([10]).

The Aristotelian syllogisms studied in this paper involve the following 12 propositions: *all*(*h, z*), *not all*(*h, z*), *no*(*h, z*), *some*(*h, z*), □*all*(*h, z*), □*not all*(*h, z*), □*no*(*h, z*), □*some*(*h, z*), ◇*all*(*h, z*), ◇*not all*(*h, z*), ◇*no*(*h, z*) and ◇*some*(*h, z*) which are respectively referred to as: Proposition A, O, E, I, □A, □O, □E, □I, ◇A, ◇O, ◇E, and ◇I ([11]). A non-trivial Aristotelian syllogism includes at least one and at most three in the last eight propositions. An instance of the Aristotelian syllogism □EAO-3 in natural language is as follows:

Major premise: No cats in her house are necessarily birds.

Minor premise: All the cats in her house are white cats.

Conclusion: Not all the white cats are birds.

Let *r* be cats in her house, *h* be white cats, and *z* be birds in the domain. Thus the syllogism can be symbolized as '$\Box no(r, z) \wedge all(r, h) \rightarrow not\ all(h, z)$', which is shortened to □EAO-3. The Others are similar.

This studies involves the following deductive rules, definitions and facts, etc.

Rule 1 (deductive rules):

R1 (antecedent strengthening 1): From $\vdash(c \wedge n \rightarrow m)$ and $\vdash(t \rightarrow c)$ infer $\vdash(t \wedge n \rightarrow m)$.

R2 (antecedent strengthening 2): From $\vdash(c\wedge n\rightarrow m)$ and $\vdash(t\rightarrow n)$ infer $\vdash(c\wedge t\rightarrow r)$.

R3 (subsequent weakening): From $\vdash(c\wedge n\rightarrow m)$ and $\vdash(m\rightarrow t)$ infer $\vdash(c\wedge n\rightarrow t)$.

R4 (anti-syllogism 1): From $\vdash(c\wedge n\rightarrow m)$ infer $\vdash(\neg m\wedge c\rightarrow\neg n)$.

R5 (anti-syllogism 2): From $\vdash(c\wedge n\rightarrow m)$infer $\vdash(\neg m\wedge n\rightarrow\neg c)$.

Definition 1 (negation and truth value)

D1 (outer negation): $(\neg Q)(h, z)=_{\text{def}}$ It is not that $Q(h, z)$;

D2 (inner negation): $(Q\neg)(h, z)=_{\text{def}} Q(h, D\text{–}x)$;

D3 (truth value): *all*(h, z) is true if and only if $H\subseteq Z$ is true in real world.;

D4 (truth value): *not all*(h, z) is true if and only if $H\not\subseteq Z$ is true in real world.

D5 (truth value): $\Box$*no*(h, z) is true if and only if $H\cap Z=\varnothing$ is true in any possible world.

Fact 1 (inner negation):

(1.1) $\vdash$*all*$(h, z)\leftrightarrow$*no*$\neg(h, z)$; (1.2) $\vdash$*no*$(h, z)\leftrightarrow$*all*$\neg(h, z)$;

(1.3) $\vdash$*some*$(h, z)\leftrightarrow$*not all*$\neg(h, z)$; (1.4) $\vdash$*not all*$(h, z)\leftrightarrow$*some*$\neg(h, z)$.

Fact 2 (outer negation):

(2.1) $\vdash\neg$*all*$(h, z)\leftrightarrow$*not all*(h, z); (2.2) $\vdash\neg$*not all*$(h, z)\leftrightarrow$*all*(h, z);

(2.3) $\vdash\neg$*no*$(h, z)\leftrightarrow$*some*(h, z); (2.4) $\vdash\neg$*some*$(h, z)\leftrightarrow$*no*(h, z).

Fact 3 (symmetry):

(3.1) $\vdash$*some*$(h, z)\leftrightarrow$*some*(z, h); (3.2) $\vdash$*no*$(h, z)\leftrightarrow$*no*(z, h).

Fact 4 (subordination) :

(4.1) $\vdash$*all*$(h, z)\rightarrow$*some*(h, z); (4.2) $\vdash$*no*$(h, z)\rightarrow$*not all*(h, z);

(4.3) $\vdash\Box Q(h, z)\rightarrow Q(h, z)$; (4.4) $\vdash\Box Q(h, z)\rightarrow\Diamond Q(h, z)$;

(4.5) $\vdash Q(h, z)\rightarrow\Diamond Q(h, z)$.

Fact 5 (dual): (5.1) $\neg\Box Q(h, z)=\Diamond\neg Q(h, z)$; (5.2) $\neg\Diamond Q(h, z)=\Box\neg Q(h, z)$.

Fact 1-4 are basic knowledge in first-order logic (Hamilton, 1978) and modal logic ([12]).

3. Knowledge Reasoning of the Aristotelian Modal Syllogism □EAO-3

In the following, Theorem 1 proves the validity of the syllogism □EAO-3. '(2.1) ⊢□EAO-3 →□EAO-4' in Theorem 2 means that the validity of the latter can be proved according to that of the former. In other words, there is reducible relationship between them. Other cases are similar.

Theorem 1 (□EAO-3): The Aristotelian modal syllogism □*no*(*r, z*)∧*all*(*r, h*)→*not all*(*h, z*) is valid.

Proof: □EAO-3 is the abbreviation of the fourth figure syllogism □*no*(*r, z*)∧*all*(*r, h*)→*not all*(*h, z*). Suppose that □*no*(*r, z*) and *all*(*r, h*) are true, then $R\cap Z=\varnothing$ is true in any possible world and $R\subseteq H$ is true in real world in line with Definition D5 and D3 respectively. Because real world are possible worlds. It follows that $H\nsubseteq Z$ is true in real world. Hence *not all*(*h, z*) is true in accordance with Definition D4. This proves that the syllogism □*no*(*r, z*)∧*all*(*r, h*)→*not all*(*h, z*) is valid, just as expected.

Theorem 2: There are at least the following 25 valid generalized syllogisms deduced from □EAO-3:

(2.1) ⊢□EAO-3→□EAO-4

(2.2) ⊢□EAO-3→A□EO-2

(2.3) ⊢□EAO-3→A□EO-2→A□EO-4

(2.4) ⊢□EAO-3→AA◇I-1

(2.5) ⊢□EAO-3→AA◇I-1→AA◇I-4

(2.6) ⊢□EAO-3→□AAI-3

(2.7) ⊢□EAO-3→A□EO-2→E□AO-2

(2.8) ⊢□EAO-3→AA◇I-1→EA◇O-1

(2.9) ⊢□EAO-3→AA◇I-1→EA◇O-1→EA◇O-2

(2.10) ⊢□EAO-3→□EA◇O-3

(2.11) ⊢□EAO-3→□EA◇O-3→□EA◇O-4

(2.12) ⊢□EAO-3→A□EO-2→A□E◇O-2

(2.13) ⊢□EAO-3→A□EO-2→A□EO-4→A□E◇O-4

(2.14) ⊢□EAO-3→□EA◇O-3→□A□EO-2

(2.15) ⊢□EAO-3→□EA◇O-3→□A□EO-2→□A□EO-4

(2.16) ⊢□EAO-3→□EA◇O-3→□AA◇I-1

(2.17) ⊢□EAO-3→□EA◇O-3→□AA◇I-1→A□A◇I-4

(2.18) ⊢□EAO-3→□EA◇O-3→□AA◇I-3

(2.19) ⊢□EAO-3→□EA◇O-3→□A□EO-2→□E□AO-2

(2.20) ⊢□EAO-3→□EA◇O-3→□AA◇I-1→□EA◇O-1

(2.21) ⊢□EAO-3→□EA◇O-3→□AA◇I-1→□EA◇O-1→□EA◇O-2

(2.22) ⊢□EAO-3→□EA◇O-3→□A□EO-2→□A□E◇O-2

(2.23) ⊢□EAO-3→□EA◇O-3→□A□EO-2→□A□EO-4→□A□E◇O-4

(2.24) ⊢□EAO-3→□EA◇O-3→□A□EO-2→□E□AO-2→□E□A◇O-2

(2.25) ⊢□EAO-3→□EA◇O-3→□A□EO-2→□E□AO-2→□E□A◇O-2→□E□A◇O-1

Proof:

[1] ⊢□*no*(*r, z*)∧*all*(*r, h*)→*not all(h, z*) (i.e. □EAO-3, Theorem 1)

[2] ⊢□*no*(*z, r*)∧*all*(*r, h*)→*not all*(*h, z*) (i.e.□EAO-4, by [1] and Fact 3)

[3] ⊢¬*not all*(*h, z*)∧□*no*(*r, z*)→¬*all*(*r, h*) (by [1] and R4)

[4] ⊢*all*(*h, z*)∧□*no*(*r, z*)→¬*all*(*r, h*) (by [3] and Fact 2)

[5] ⊢*all*(*h, z*)∧□*no*(*r, z*)→*not all*(*r, h*) (i.e.A□EO-2, by [4] and Fact 2)

[6] ⊢*all*(*h, z*)∧□*no*(*z, r*)→*not all*(*r, h*) (i.e.A□EO-4, by [5] and Fact 3)

[7] ⊢¬*not all*(*h, z*)∧*all*(*r, h*)→¬□*no*(*r, z*) (by [1] and R5)

[8] ⊢*all*(*h, z*)∧*all*(*r, h*)→◇¬*no*(*r, z*) (by [7], Fact 2 and Fact 5)

[9] ⊢*all*(*h, z*)∧*all*(*r, h*)→◇some(*r, z*) (i.e.AA◇I-1, by [8] and Fact 2)

[10] ⊢*all*(*r, h*)∧*all*(*h, z)*→◇some(*z, r*) (i.e.AA◇I-4, by [9] and Fact 3)

[11] ⊢□*all*¬(*r, z*)∧*all*(*r, h*)→*some*¬(*h, z*) (by [1] and Fact 1)

[12] ⊢□*all*(*r, D−z*)∧*all*(*r, h*)→*some*(*h, D−z*) (i.e.□AAI-3, by [11] and D2)

[13] ⊢*no*¬(*h, z*)∧□*all*¬(*r, z*)→*not all*(*r, h*) (by [5] and Fact 1)

[14] ⊢*no*(*h, D−z*)∧□*all*(*r, D−z*)→*not all*(*r, h*) (i.e.E□AO-2, by [13] and D2)

[15] ⊢*no*¬(*h, z*)∧*all*(*r, h*)→◇*not all*¬(*r, z*) (by [9] and Fact 1)

[16] ⊢*no*(*h, D−z*)∧*all*(*r, h*)→◇*not all*(*r, D−z*) (i.e.EA◇O-1, by [15] and D2)

[17] ⊢*no*(*D−z, h*)∧*all*(*r, h*)→◇*not all*(*r, D−z*) (i.e.EA◇O-2, by [16] and Fact 3)

[18] ⊢□*no*(*r, z*)∧*all*(*r, h*)→◇*not all*(*h, z*) (i.e.□EA◇O-3, by [1] and Fact 4)

[19] ⊢□*no*(*z, r*)∧*all*(*r, h*)→◇*not all*(*h, z*) (i.e.□EA◇O-4, by [18] and Fact 3)

[20] ⊢*all*(*h, z*)∧□*no*(*r, z*)→◇*not all*(*r, h*) (i.e.A□E◇O-2, by [5] and Fact 4)

[21] ⊢*all*(*h, z*)∧□*no*(*z, r*)→◇*not all*(*r, h*) (i.e.A□E◇O-4, by [6] and Fact 4)

[22] ⊢¬◇*not all*(*h, z*)∧□*no*(*r, z*)→¬*all*(*r, h*) (by [18] and R4)

[23] ⊢□¬*not all*(*h, z*)∧□*no*(*r, z*)→*not all*(*r, h*) (by [22], Fact 2 and Fact 5)

[24] $\vdash\Box \mathit{all}(h, z)\wedge\Box \mathit{no}(r, z)\rightarrow \mathit{not\ all}(r, h)$ (i.e.□A□EO-2, by [23] and Fact 2)

[25] $\vdash\Box \mathit{all}(h, z)\wedge\Box \mathit{no}(z, r)\rightarrow \mathit{not\ all}(r, h)$ (i.e.□A□EO-4, by [24] and Fact 3)

[26] $\vdash\neg\Diamond \mathit{not\ all}(h, z)\wedge \mathit{all}(r, h)\rightarrow\neg\Box \mathit{no}(r, z)$ (by [18] and R5)

[27] $\vdash\Box\neg \mathit{not\ all}(h, z)\wedge \mathit{all}(r, h)\rightarrow\Diamond\neg \mathit{no}(r, z)$ (by [22] and Fact 5)

[28] $\vdash\Box \mathit{all}(h, z)\wedge \mathit{all}(r, h)\rightarrow\Diamond \mathrm{some}(r, z)$ (i.e.□AA◇I-1, by [27] and Fact 2)

[29] $\vdash \mathit{all}(r, h)\wedge\Box \mathit{all}(h, z)\rightarrow\Diamond \mathrm{some}(z, r)$ (i.e.A□A◇I-4, by [28] and Fact 3)

[30] $\vdash\Box \mathit{all}\neg(r, z)\wedge \mathit{all}(r, h)\rightarrow\Diamond \mathit{some}\neg(h, z)$ (by [18] and Fact 1)

[31] $\vdash\Box \mathit{all}(r, D-z)\wedge \mathit{all}(r, h)\rightarrow\Diamond \mathit{some}(h, D-z)$ (i.e.□AA◇I-3, by [30] and D2)

[32] $\vdash\Box \mathit{no}\neg(h, z)\wedge\Box \mathit{all}\neg(r, z)\rightarrow \mathit{not\ all}(r, h)$ (by [24] and Fact 1)

[33] $\vdash\Box \mathit{no}(h, D-z)\wedge\Box \mathit{all}(r, D-z)\rightarrow \mathit{not\ all}(r, h)$ (i.e.□E□AO-2, by [32] and D2)

[34] $\vdash\Box \mathit{no}\neg(h, z)\wedge \mathit{all}(r, h)\rightarrow\Diamond \mathit{not\ all}\neg(r, z)$ (by [28] and Fact 1)

[35] $\vdash\Box \mathit{no}(h, D-z)\wedge \mathit{all}(r, h)\rightarrow\Diamond \mathit{not\ all}(r, D-z)$ (i.e.□EA◇O-1, by [34] and D2)

[36] $\vdash\Box \mathit{no}(D-z, h)\wedge \mathit{all}(r, h)\rightarrow\Diamond \mathit{not\ all}(r, D-z)$ (i.e.□EA◇O-2, by [35] and Fact 3)

[37] $\vdash\Box \mathit{all}(h, z)\wedge\Box \mathit{no}(r, z)\rightarrow\Diamond \mathit{not\ all}(r, h)$ (i.e.□A□E◇O-2, by [24] and Fact 4)

[38] $\vdash\Box \mathit{all}(h, z)\wedge\Box \mathit{no}(z, r)\rightarrow\Diamond \mathit{not\ all}(r, h)$ (i.e.□A□E◇O-4, by [25] and Fact 4)

[39] $\vdash\Box \mathit{no}(h, D-z)\wedge\Box \mathit{all}(r, D-z)\rightarrow\Diamond \mathit{not\ all}(r, h)$ (i.e.□E□A◇O-2, by [33] and Fact 4)

[40] $\vdash\Box \mathit{no}(D-z, h)\wedge\Box \mathit{all}(r, D-z)\rightarrow\Diamond \mathit{not\ all}(r, h)$ (i.e.□E□A◇O-1, by [39] and Fact 3)

Through the above proof, the validity of other 25 modal syllogisms has been derived from the validity of the syllogism □EAO-3. If one continue with the above proof, then more valid syllogisms can be deduced from the syllogism. This article solely presents a proof methodology and does not pursue further demonstrations.

5. Conclusion and Future Work

Firstly, the validity of the modal syllogism □EAO-3 is proved by means of modal logic, set theory and generalized quantifier theory in this paper. Subsequently, on the basis of validity of this particular syllogism, the remaining 25 valid modal syllogisms are endeavored to be deduced by employing relevant definitions, facts and reasoning rules. The results obtained through deductive reasoning exhibit consistency, rendering this method not only elegant and straightforward but also widely applicable to the investigation of various forms of syllogism.

This paper does not introduce all modal syllogisms that can be deduced from □EAO-3. Predictably, more syllogisms can be obtained by continuing the deduction. One can get at least 25 syllogisms from □EAO-3 alone, can one deduce all the other syllogisms if a few other modal syllogisms are added? This is a question worthy of further investigation.

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