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How to Derive Other Valid Generalized Syllogisms from the Generalized Syllogism MAI-4

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Abstract

This paper first formalizes categorical propositions in syllogisms, then proves the validity of the generalized syllogism MAI-4, and then deduces other 15 valid generalized syllogisms from the syllogism MAI-4. In other words, there are reducible relationships between/among the 16 syllogisms. This study highlights the dialectical materialist idea that things are universally interconnected. This study not only provides inspiration for studying generalized syllogisms with other generalized quantifiers but also provides a research perspective for knowledge representation and knowledge reasoning.

Keywords: generalized quantifiers; generalized syllogisms; validity; reducible relationship

1. Introduction

There are a large number of generalized quantifiers in natural language (Westerståhl, 2008), such as '*fewer than half of*', '*at most half of*', '*most*', and '*at least half of*'. Aristotelian quantifiers (that is, *no*, *some*, *not all*, *all)* are trivial generalized quantifiers (Hao, 2024). A syllogism that includes at least one generalized quantifier is a generalized syllogism (Wang and Yuan, 2004). Generalized syllogism reasoning is a common form of reasoning in natural language and scientific language (Wu, 2024), which characterizes the semantic and inferential properties of generalized quantifiers (Peters and Westerståhl, 2006). This paper only focuses on non-trivial generalized syllogisms involving the above eight quantifiers.

2. Preliminaries

Let *g*, *w*, and *z* be lexical variables, the set formed of *g*, *w*, and *z* be *G*, *W*, and *Z*, respectively, and *D* be the domain of a lexical variable. Let *Q* be a quantifier, and $\neg Q$, and $Q\neg$ be its inner and outer negative quantifiers, respectively. Let λ, ϕ, κ, and ξ be well-formed formulas (abbreviated as wff), and '$\vdash\xi$' denotes that ξ can be proved and '$\kappa =_{def} \xi$' that κ is defined by ξ.

The generalized syllogisms discussed in this paper only contains the following eight propositions: *all(g, z)*, *no(g, z)*, *some(g, z)*, *not all(g, z)*, *at most half of the(g, z)*, *most(g, z)*, *at least half of the(g, z)*, and *fewer than half of the(g, z)*, which are respectively abbreviated as *A*, *I*, *E*, *O*, *H*, *M*, *S*, and *F*. A non-trivial generalized syllogism includes at least one of the last four propositions. The proposition *all(g, z)* means that all *gs* are *z*s, and most*(g, z)* means that most *gs* are *z*s. The other propositions are similar.

The generalized syllogism '$most(g, z) \wedge all(z, w) \rightarrow some(w, g)$', which can be shortened as MAI-4. One of its instances in natural language is as follows:

Major premise: Most of the pets in this pet store are kittens.

Minor premise: All kittens are omnivorous animals.

Conclusion: Some omnivorous animals are pets in this pet store.

Let *g*, *z*, and *w* be variables which are representing a pet, a kitten, and omnivorous animals in the domain, respectively. The above syllogism is symbolized as '$most(g, z) \wedge all(z, w) \rightarrow some(w, g)$', which is called the syllogism MAI-4. The others can be similarly symbolized.

3. Relevant Knowledge

In order to discuss the validity and reducibility of a generalized syllogism, the following definitions and reasoning rules need to be provided.

3.1 Primitive Symbols

(1) lexical variables: *g*, *w*, *z*;

(2) quantifiers: *most*, *all*;

(3) operators: $\neg$, $\rightarrow$;

(4) brackets: (,).

3.2 Formation Rules

(1) If *Q* is a quantifier, and *g*, *z* are lexical variables, then *Q(g, z)* is a wff;

(2) If κ is a wff, then $\neg\kappa$ is also a wff;

(3) If κ and ξ are wffs, then $\kappa\rightarrow\xi$ is also a wff;

(4) Only the formulas constructed by (1) to (3) are wffs.

3.3 Basic Axioms

A1: If κ is a wff, then $\vdash\kappa$ is provable;

A2: $\vdash$ *most(g, z)*$\wedge$*all(z, w)*$\rightarrow$*some(w, g)* (i.e. the syllogism MAI-4).

3.4 Deductive Rules

Rule 1 (antecendent strengthening 1): From $\vdash(\lambda\wedge\phi\rightarrow\kappa)$ and $\vdash(\xi\rightarrow\lambda)$ infer $\vdash(\xi\wedge\phi\rightarrow\kappa)$;

Rule 2 (antecendent strengthening 2): From $\vdash(\lambda\wedge\phi\rightarrow\kappa)$ and $\vdash(\xi\rightarrow\phi)$ infer $\vdash(\lambda\wedge\xi\rightarrow\kappa)$;

Rule 3 (subsequent weakening): From $\vdash(\lambda\wedge\phi\rightarrow\kappa)$ and $\vdash(\kappa\rightarrow\xi)$ infer $\vdash(\lambda\wedge\phi\rightarrow\xi)$;

Rule 4 (anti-syllogism 1): From $\vdash(\lambda\wedge\phi\rightarrow\kappa)$ infer $\vdash(\neg\kappa\wedge\phi\rightarrow\neg\lambda)$;

Rule 5 (anti-syllogism 2): From $\vdash(\lambda\wedge\phi\rightarrow\kappa)$ infer $\vdash(\lambda\wedge\neg\kappa\rightarrow\neg\phi)$.

3.5 Relevant Definitions

D1 (conjunction): $(\lambda\wedge\phi)=_{def}\neg(\lambda\rightarrow\neg\phi)$;

D2 (disjunction): $(\lambda\vee\phi)=_{def}(\neg\lambda\rightarrow\phi)$;

D3 (bicondition): $(\lambda\leftrightarrow\phi)=_{def}(\lambda\rightarrow\phi)\wedge(\lambda\rightarrow\phi)$;

D4 (inner negation): $(Q\neg)(g, z)=_{\text{def}} Q(g, D-z)$;

D5 (outer negation): $(\neg Q)(g, z)=_{\text{def}}$ It is not that $Q(g, z)$;

D6 (true value of *all*): $all(g, z)=_{\text{def}} G\subseteq Z$;

D7 (true value of *some*): $some(g, z)=_{\text{def}} G\cap Z\neq\varnothing$;

D8 (true value of *most*): $most(g, z)$ is true iff $|G\cap Z|>0.5|G|$ is true.

3.6 Relevant Facts

Fact 1 (inner negation):

(1.1) $\vdash no\neg(g, z)\leftrightarrow all(g, D-z)$;

(1.2) $\vdash all\neg(g, z)\leftrightarrow no(g, D-z)$;

(1.3) $\vdash not\ all\neg(g, z)\leftrightarrow some(g, D-z)$;

(1.4) $\vdash some\neg(g, z)\leftrightarrow not\ all(g, D-z)$;

(1.5) $\vdash$*fewer than half of the*$\neg(g, z)\leftrightarrow most(g, D-z)$;

(1.6) $\vdash most\neg(g, z)\leftrightarrow$*fewer than half of the*$(g, D-z)$;

(1.7) $\vdash$*at most half of the*$\neg(g, z)\leftrightarrow$*at least half of the*$(g, D-z)$;

(1.8) $\vdash$*at least half of the*$\neg(g, z)\leftrightarrow$*at most half of the*$(g, D-z)$.

Fact 2 (outer negation):

(2.1) $\vdash\neg all(g, z)\leftrightarrow not\ all(g, z)$;

(2.2) $\vdash\neg not\ all(g, z)\leftrightarrow all(g, z)$;

(2.3) $\vdash\neg no(g, z)\leftrightarrow some(g, z)$;

(2.4) $\vdash\neg some(g, z)\leftrightarrow no(g, z)$;

(2.5) $\vdash\neg most(g, z)\leftrightarrow$*at most half of the*$(g, z)$;

(2.6) $\vdash\neg$*at most half of the*$(g, z)\leftrightarrow most(g, z)$;

(2.7) $\vdash\neg$*fewer than half of the*$(g, z)\leftrightarrow$*at least half of the*(g, z);

(2.8) $\vdash\neg$*at least half of the*$(g, z)\leftrightarrow$*fewer than half of the*(g, z).

Fact 3 (symmetry):

(3.1) $\vdash$*some(g, z)*$\leftrightarrow$*some(z, g)*;

(3.2) $\vdash$*no(g, z)*$\leftrightarrow$*no(z, g)*.

Fact 4 (subordination):

(4.1) $\vdash$*all(g, z)*$\rightarrow$most*(g, z)*;

(4.2) $\vdash$*all(g, z)*$\rightarrow$*at least half of the(g, z)*;

(4.3) $\vdash$*all(g, z)*$\rightarrow$*some(g, z)*;

(4.4) $\vdash$*most(g, z)*$\rightarrow$*at least half of the(g, z)*;

(4.5) $\vdash$*most(g, z)*$\rightarrow$*some(g, z)*;

(4.6) $\vdash$*at least half of the(g, z)*$\rightarrow$*some(g, z)*;

(4.7) $\vdash$*no(g, z)*$\rightarrow$*fewer than half of the(g, z)*;

(4.8) $\vdash$*no(g, z)*$\rightarrow$*at most half of the(g, z)*;

(4.9) $\vdash$*no(g, z)*$\rightarrow$*not all(g, z)*;

(4.10) $\vdash$*fewer than half of the(g, z)*$\rightarrow$*at least half of the(g, z)*;

(4.11) $\vdash$*fewer than half of the(g, z)*$\rightarrow$*not all(g, z)*;

(4.12) $\vdash$*at most half of the(g, z)*$\rightarrow$*not all(g, z)*.

The above definitions, rules, and facts are cornerstone in propositional logic (Hamilton, 1978) and generalized quantifier theory (Cao and Li, 2024).

4. From MAI-4 to others Valid Generalized Syllogisms

The validity of syllogism MAI-4 is firstly proved in the following Theorem 1. Theorem 2 discusses the reducible relationships between/among different generalized syllogisms. Specifically, Theorem 2 illustrates how to derive other valid generalized syllogisms from the syllogism MAI-4.

Theorem 1 (MAI-4): The generalized syllogism *most(g, z)*$\wedge$*all(z, w)*$\rightarrow$*some(w, g)* is valid.

Proof: Suppose that *most(g, z)* and *all(z, w)* are true. Then $|G\cap Z|>0.5|G|$ and $Z\subseteq W$ are true according to Definitions D8 and D6, respectively. Then it is obvious that $|W \cap G| \neq \varnothing$. Therefore, *some(w, g)* is true according to Definition D7, as required.

Theorem 2: The following 15 valid generalized syllogisms can be deduced from the syllogism MAI-4:

(2.1) ⊢ MAI-4→MAI-1
(2.2) ⊢ MAI-4→AEH-4
(2.3) ⊢ MAI-4→EMO-4
(2.4) ⊢MAI-4→MAI-1→EMO-1
(2.5) ⊢MAI-4→MAI-1→EMO-3
(2.6) ⊢MAI-4→MAI-1→AEH-2
(2.7) ⊢MAI-4→MAI-1→EMO-1→EMO-2
(2.8) ⊢MAI-4→MAI-1→EMO-1→AMI-3
(2.9) ⊢MAI-4→MAI-1→EMO-1→EAH-2
(2.10) ⊢MAI-4→MAI-1→EMO-1→EMO-2→AFO-2
(2.11) ⊢MAI-4→MAI-1→EMO-1→EMO-2→MAI-3
(2.12) ⊢MAI-4→MAI-1→EMO-1→EMO-2→EAH-1
(2.13) ⊢MAI-4→MAI-1→EMO-1→EMO-2→AFO-2→FAO-3
(2.14) ⊢MAI-4→MAI-1→EMO-1→EMO-2→AFO-2→AAS-1
(2.15) ⊢MAI-4→MAI-1→EMO-1→EAH-2

Proof:

[1] ⊢*most(g, z)∧all(z, w)→some(w, g)* (i.e.MAI-4, Basic Axiom A2)
[2] ⊢*most(g, z)∧all(z, w)→some(g, w)* (i.e.AMI-1,by [1] and Fact (3.1))
[3] ⊢*¬some(w, g)∧all(z, w)→¬most(g, z)* (by [1] and Rule 4)
[4] ⊢*all(z, w)∧no(w, g)→at most half of the(g, z)* (i.e.AEH-4, by [3], Fact (2.4) and (2.5))
[5] ⊢*most(g, z)∧¬some(w, g)→¬all(z, w)* (by [1] and Rule 5)
[6] ⊢no*(w, g)∧most(g, z)→not all(z, w)* (i.e.EMO-4, by [5], Fact (2.1) and (2.4))
[7] ⊢no*(z, D−w)∧most(g, z)→not all(g, D−w)* (i.e.EMO-1, by [2] and Fact (1.1) and Fact (1.3))
[8] ⊢*most(g, z)∧¬some(g, w)→¬all(z, w)* (by [2] and Rule 4)
[9] ⊢*most(g, z)∧no(g, w) →not all(z, w)* (i.e.EMO-3, by [9], Fact (2.1) and (2.4))
[10] ⊢*¬some(g, w)∧all(z, w)→¬most(g, z)* (by [2] and Rule 5)
[11] ⊢*no(g, w)∧all(z, w)→at most half of the(g, z)* (i.e.AEH-2, by [10], Fact (2.4) and (2.5))
[7] ⊢*no(D−w, z)∧most(g, z)→not all(g, D−w)* (i.e.EMO-2, by [7] and Fact (3.2))
[13] ⊢*¬not all(g, D−w)∧most(g, z)→¬no(z, D−w)* (by [7] and Rule 4)
[14] ⊢*all(g, D−w)∧most(g, z)→not all(z, D−w)* (i.e.AMI-3, by [13], Fact (2.2) and (2.3))
[15] ⊢*no(z, D−w)∧¬not all(g, D−)→¬most(g, z)* (by [7] and Rule 5)
[16] ⊢*no(z, D−w)∧all(g, D−w)→at most half of the(g, z)*
(i.e.EAH-2, by [15], Fact (2.2) and (2.5))

[17] $\vdash$all$(g, D{-}z) \wedge$ *fewer than half of*$(w, D{-}z) \rightarrow$ *not all*(w, g)

(i.e.AFO-2, by [12], Fact (1.1) and (1.6))

[18] $\vdash \neg$*not all*$(w, g) \wedge$ *most*$(w, z) \rightarrow \neg$no(g, z) (by [12] and Rule 4)

[19] $\vdash$*most*$(w, z) \wedge$ *all*$(w, g) \rightarrow$ *some*(g, z) (i.e.MAI-3, by [18], Fact (2.1) and (2.2))

[20] $\vdash$*no*$(g, z) \wedge \neg$*not all*$(w, g) \rightarrow \neg$*most*(w, z) (by [12] and Rule 5)

[21] $\vdash$*no*$(g, z) \wedge$ *all*$(w, g) \rightarrow$ *at most half of the*(w, z)

(i.e.EAH-1, by [20], Fact (2.1) and (2.5))

[22] $\vdash$*fewer than half of*$(w, D{-}z) \wedge \neg$*not all*$(w, g) \rightarrow \neg$*all*$(g, D{-}z)$ (by [17] and Rule 4)

[23] $\vdash$*fewer than half of*$(w, D{-}z) \wedge$ *all*$(w, g) \rightarrow$ *not all*$(g, D{-}z)$

(i.e.FAO-3, by [22], Fact (2.1) and (2.2))

[24] $\vdash$*all*$(g, D{-}z) \wedge \neg$*not all*$(w, g) \rightarrow \neg$*fewer than half of*$(w, D{-}z)$ (by [17] and Rule 5)

[25] $\vdash$ *all*$(g, D{-}z) \wedge$ *all*$(w, g) \rightarrow$ *at least half of*$(g, D{-}z)$

(i.e.AAS-1, by [24], Fact (2.2) and(2.7))

[26] $\vdash$*no*$(z, g) \wedge$ *all*$(w, g) \rightarrow$ *at most half of the*(w, z) (i.e.EAH-2, by [21] and Fact (3.2))

[27] $\vdash$*fewer than half of*$(w, D{-}z) \wedge$ *all*$(w, g) \rightarrow$ *not all*$(g, D{-}z)$

(i.e.FAO-3, by [22], Fact (2.1) and(2.2))

More valid generalized syllogisms can be derived from the syllogism MAI-4 if one similarly continue to derive.

5. Conclusion and Future Works

This paper first formalizes categorical propositions in syllogisms, then proves the validity of the generalized syllogism MAI-4, and then deduces other 15 valid generalized syllogisms from the syllogism MAI-4. In other words, there are reducible relationships between/among the 16 syllogisms. This study highlights the dialectical materialist idea that things are universally interconnected.

In fact, there are an infinite number of generalized quantifiers in natural language (Peters and Westerståhl, 2006), and this study not only provides inspiration for studying generalized syllogisms with other generalized quantifiers, such as *at least one-third of*, *both*, *many*, *finite*, and *infinite*, but also provides a research perspective for knowledge representation and knowledge reasoning.

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