SCIREA Journal of Philosophy
ISSN: 2995-7788
http://www.scirea.org/journal/Philosophy
June 18, 2025
Volume 5, Issue 1, February 2025
https://doi.org/10.54647/philosophy720126

# Deductibility about the Classical Modal Syllogism □AEE-4

**Haowei Shi**
Institute of Logic and Information Technology, Sichuan Normal University, Chengdu, China
**Email:** 597387580@qq.com

## Abstract

This paper firstly formalizes the classical modal syllogism □AEE-4, and then proves its validity, and finally derives the other 21 valid classical modal syllogisms from the validity of the syllogism □AEE-4. These knowledge deduction processes not only illustrate the deductibility of □AEE-4, but also demonstrate the materialist view that there are universal connectiones between things. This innovative study is based on logical deduction, therefore its conclusions have logical consistency. And the study will contribute to knowledge mining in big data.

**Keywords:** Classical modal syllogisms; Validity; Knowledge deduction; Knowledge mining; Deductibility

## 1. Introduction

Syllogism reasoning is one of the common forms of reasoning in natural language and occupies an important position in human thinking [1]. There are various forms of syllogisms [2-5], such as classical syllogisms [6-7], classical modal syllogisms [8], generalized syllogisms [9], generalized modal syllogisms [10], and so on. Classic syllogisms only contain the following four classic quantifiers: *some*, *all*, *no*, *not all*. Generalized syllogisms include generalized quantifiers. A classical (/generalized) modal syllogism is obtained by adding at least one and at most three of non-overlapping necessary or possible modal operators (i.e. □ or ◇ ) to a classical(/generalized) syllogism [11].

Although there are many achievements on various syllogisms, the related studies are still not perfect, and there are even inconsistencies. This paper focuses on the deducability of the classical modal syllogism □AEE-4 by means of logical deduction, aiming to provide consistent conclusions.

## 2. Formalization for Classical Modal Syllogisms

In this paper, let *Q* be any of the four classical quantifiers (namely, *some*, *all*, *no*, and *not all*), ¬*Q* and *Q*¬ be respectively its outer and inner negation. And let *u*, *v* and *w* be variables, and *D* be their domain. The set composed of *u*, *v* and *w* is respectively *U, V*, and *W*. Let α, θ, μ, and ξ be well-formed formulas (shorted as wff). '$\alpha =_{def} \xi$' states that α is defined by ξ, and '⊢ ξ' that ξ is provable. The operators, such as ¬, ∧, →, ↔, □, and ◇ are the symbols in modal logic [12].

This paper only studies the following 4 propositions: 'all *u*s are *v*s', 'some *u*s are *v*s', 'no *u*s are *v*s' and 'not all *u*s are *v*s', which are denoted as *all(u, v)*, *some(u, v), no(u, v)*, and *not all(u, v),* respectively, and they are called Proposition *A*, *I*, *E*, and *O,* respectively. Then, for example, the syllogism □AEE-4 is an abbreviation for '□*all*(*u, v*)∧*no*(*v, w*)→*no*(*w, u*)'. Its instance is as follows:

Major premise: All chickens are necessarily oviparous animals.

Minor premise: No oviparous animals is a dog.

Conclusion: No dog is a chicken.

If *u*, *v*, and *w* represent variables for chickens, oviparous animals, and dogs, respectively. then the instance of syllogism can be denoted as $\Box all(u, v) \wedge no(v, w) \rightarrow no(w, u)$, and abbreviated as □AEE-4.

## 3. Classical Modal Syllogism Formal System

This system consists of the following parts.

### 3.1 Initial Symbols

[1] variables: *u*, *v*, *w*

[2] quantifier: *all*

[3] operators: ¬, ∧, □

[4] brackets: (, )

### 3.3 Relevant Definitions

D1: $(\mu \rightarrow \xi) =_{def} \neg(\mu \wedge \neg\xi)$.

D 2: $(\mu \quad \xi) =_{def} (\neg(\mu \wedge \neg\xi)) \wedge (\neg(\xi \wedge \neg\mu))$.

D3: $Q\neg(u, v) =_{def} Q(u, D{-}v)$.

D4: $(\neg Q)(u, v) =_{def}$ It is not case that $Q(u, v)$.

D5: $\Diamond\phi =_{def} \neg\Box\neg\phi$.

D6: *all(u, v)* is true iff so is $U \subseteq V$ in any real world.

D7: *some(u, v)* is true iff so is $U \cap V \neq \varnothing$ in any real world.

D8: *no(u, v)* is true iff so is $U \cap V = \varnothing$ in any real world.

D9: *not all(u, v)* is true iff so is $U \nsubseteq V$ in any real world.

D10: $\Box all(u, v)$ is true iff so is $U \subseteq V$ in any possible world.

### 3.2 Formation Rules

[1] If $Q$ is a quantifier, $u$ and $v$ are variables, then $Q(u, v)$ is a wff.

[2] If $\xi$ and $\alpha$ are wffs, then so are $\neg\xi$, $\xi\wedge\alpha$, and $\Box\xi$.

[3] Merely the formulas formed by the above rules are wffs.

### 3.3 Basic Axioms

A1: if $\xi$ is a valid formula in classical logic, then $\vdash \xi$.

A2: $\vdash \Box all(u, v)\wedge no(v, w)\rightarrow no(u, v)$ (i.e. the syllogism □AEE-4).

### 3.5 Relevant Facts

**Fact 1** (inner negation):

(1.1) $\vdash$ *all(u, v)* *no¬(u, v)*; (1.2) $\vdash$ *no(u, v)* *all¬(u, v)*;

(1.3) $\vdash$ *some(u, v)* *not all¬(u, v)*; (1.4) $\vdash$ *not all(u, v)* *some¬(u, v)*.

**Fact 2** (outer negation):

(2.1) $\vdash$ *¬not all(u, v)* *all(u, v)*; (2.2) $\vdash$ *¬all(u, v)* *not all(u, v)*;

(2.3) $\vdash$ *¬no(u, v)* *some(u, v)*; (2.4) $\vdash$ *¬some(u, v)* *no(u, v)*.

**Fact 3 (**dual**)**:

(3.1) $\vdash \neg\Box Q(u, v)$ $\Diamond\neg Q(u, v)$; (3.2) $\vdash \neg\Diamond Q(u, v)$ $\Box\neg Q(u, v)$.

**Fact 4**: $\vdash \Box Q(u, v)\rightarrow Q(u, v)$ .

**Fact 5**: $\vdash \Box Q(u, v)\rightarrow\Diamond Q(u, v)$.

**Fact 6**: $\vdash Q(u, v)\rightarrow\Diamond Q(u, v)$.

**Fact 7**: (7.1) $\vdash$ *all(u, v)→some(u, v)*; (7.2) $\vdash$ *no(u, v)→not all(u, v)*.

**Fact 8** (symmetry of *some* and *no*):

(8.1) $\vdash$ *some(u, v)↔some(v, u)*; (8.2) $\vdash$ *no(u, v)↔no(v, u)*.

Fact 8 intuitively holds. The other facts are the basic facts in modal logic.

### 3.6 Inference Rules

Rule 1: If ⊢ $(\alpha\wedge\theta\rightarrow\mu)$ and ⊢ $(\xi\rightarrow\alpha)$, then ⊢ $(\xi\wedge\theta\rightarrow\mu)$.

Rule 2: If ⊢ $(\alpha\wedge\theta\rightarrow\mu)$ and ⊢ $(\mu\rightarrow\xi)$ infer ⊢ $(\alpha\wedge\theta\rightarrow\xi)$.

Rule 3(anti-syllogism): From ⊢ $(\alpha\wedge\theta\rightarrow\mu)$ infer ⊢ $(\neg\mu\wedge\alpha\rightarrow\neg\theta)$.

The above inference rules are basic rules in first-order logic [13].

## 4. Knowledge Deduction Based on the Classical Modal Syllogism □AEE-4

The following Theorem 1 states that the syllogism □AEE-4 is valid. Theorem 2 proves that the validity of the other classical modal syllogisms can be deduced from □AEE-4. In other words, there are reducible relationships between these 22 syllogisms.

**Theorem 1** (□AEE-4): The classical modal syllogism ⊢ $\Box all(u, v)\wedge no(v, w)\rightarrow no(w, u)$ is valid.

Proof: Suppose that $\Box all(u, v)$ and $no(v, w)$ are true, then $U\subseteq V$ is true in any possible world and $V\cap W=\varnothing$ is true in any real world accorrding to Definition D10 and D8, respectively. In fact, all real worlds are possible worlds, it can be concluded that $U\subseteq V$ is true in any real world. That means that $U\subseteq V$ and $V\cap W=\varnothing$ are true in any real world. Hence $U\cap W=\varnothing$ is true in any real world. It follows that $no(w, u)$ is true in line with Definition D8, just as expected.

**Theorem 2**: The following 21 valid classical modal syllogisms can be inferred from the syllogism □AEE-4.

(2.1) ⊢ □AEE-4→□AEE-2

(2.2) ⊢ □AEE-4→E□AE-1

(2.3) ⊢ □AEE-4→□AEE-2→E□AE-2

(2.4) ⊢ □AEE-4→□AEO-4

(2.5) ⊢ □AEE-4→□AEE-2→□AEO-2

(2.6) ⊢ □AEE-4→E□AE-1→E□AO-1

(2.7) ⊢ □AEE-4→□AEE-2→E□AE-2→E□AO-2

(2.7) ⊢ □AEE-4→□A□EE-4

(2.8) ⊢ □AEE-4→E□AE-1→□E□AE-1

(2.9) ⊢ □AEE-4→EI◇O-4

(2.10) ⊢ □AEE-4→EI◇O-4→EI◇O-2

(2.11) ⊢ □AEE-4→EI◇O-4→EI◇O-3

(2.12) ⊢ □AEE-4→EI◇O-4→EI◇O-2→EI◇O-1

(2.13) ⊢ □AEE-4→EI◇O-4→□EI◇O-4

(2.14) ⊢ □AEE-4→EI◇O-4→EI◇O-3→EA◇O-3

(2.15) ⊢ □AEE-4→I□AI-4

(2.16) ⊢ □AEE-4→I□AI-4→I□AI-3

(2.17) ⊢ □AEE-4→I□AI-4→□AII-1

(2.18) ⊢ □AEE-4→I□AI-4→I□AI-3→□AII-3

(2.15) ⊢ □AEE-4→I□AI-4→A□AI-4

(2.16) ⊢ □AEE-4→I□AI-4→I□AI-3→A□AI-3

(2.17) ⊢ □AEE-4→I□AI-4→□AII-1→□AAI-1

(2.18) ⊢ □AEE-4→I□AI-4→I□AI-3→□AII-3→□AAI-3

(2.19) ⊢ □AEE-4→I□AI-4→I□AI-3→O□AO-3

(2.20) ⊢ □AEE-4→I□AI-4→I□AI-3→O□AO-3→A□AA-1

(2.21) ⊢ □AEE-4→I□AI-4→I□AI-3→O□AO-3→A□AA-1→A□A◇A-1

Proof:

[1] ⊢ □*all*(*u*, *v*)∧*no*(*v*, *w*)→*no*(*w*, *u*) (i.e. □AEE-4, basic axiom A2)

[2] ⊢ □*all*(*u*, *v*)∧*no*(*w*, *v*)→*no*(*w*, *u*) (i.e. □AEE-2, by [1] and Fact (8.2))

[3] ⊢ □*all*(*u*, *v*)∧*no*(*v*, *w*)→*no*(*u*, *w*) (i.e. E□AE-1, by [1] and Fact (8.2))

[4] ⊢ □*all*(*u*, *v*)∧*no*(*w*, *v*)→*no*(*u*, *w*) (i.e. E□AE-2, by [2] and Fact (8.2))

[5] ⊢ $\Box all(u, v) \wedge no(v, w) \rightarrow not\ all(w, u)$ (i.e. □AEO-4, by [1], Fact (7.2) and Rule 2)

[6] ⊢ $\Box all(u, v) \wedge no(w, v) \rightarrow not\ all(w, u)$ (i.e. □AEO-2, by [2], Fact (7.2) and Rule 2)

[7] ⊢ $\Box all(u, v) \wedge no(v, w) \rightarrow not\ all(u, w)$ (i.e. E□AO-1, by [3], Fact (7.2) and Rule 2)

[8] ⊢ $\Box all(u, v) \wedge no(w, v) \rightarrow not\ all(u, w)$ (i.e. E□AO-2, by [4], Fact (7.2) and Rule 2)

[9] ⊢ $\Box all(u, v) \wedge \Box no(v, w) \rightarrow no(w, u)$ (i.e. □A□EE-4, by [1], Fact 4 and Rule 1)

[10] ⊢ $\Box all(u, v) \wedge \Box no(v, w) \rightarrow no(u, w)$ (i.e. □E□AE-1, by [3], Fact 4 and Rule 1)

[11] ⊢ $\neg no(w, u) \wedge no(v, w) \rightarrow \neg \Box all(u, v)$ (by [1] and Rule 3)

[12] ⊢ $some(w, u) \wedge no(v, w) \rightarrow \Diamond not\ all(u, v)$ (i.e. EI◇O-4, by [11], Fact (2.2), (2.3) and (3.1))

[13] ⊢ $some(u, w) \wedge no(v, w) \rightarrow \Diamond not\ all(u, v)$ (i.e. EI◇O-2, by [12] and Fact (8.1))

[14] ⊢ $some(w, u) \wedge no(w, v) \rightarrow \Diamond not\ all(u, v)$ (i.e. EI◇O-3, by [12] and Fact (8.2))

[15] ⊢ $some(u, w) \wedge no(w, v) \rightarrow \Diamond not\ all(u, v)$ (i.e. EI◇O-1, by [13] and Fact (8.2))

[16] ⊢ $\Box some(w, u) \wedge no(v, w) \rightarrow \Diamond not\ all(u, v)$ (i.e. □EI◇O-4, by [12], Fact 4 and Rule 1)

[17] ⊢ $all(w, u) \wedge no(w, v) \rightarrow \Diamond not\ all(u, v)$ (i.e. EA◇O-3, by [14], Fact (7.1) and Rule 1)

[18] ⊢ $\neg no(w, u) \wedge \Box all(u, v) \rightarrow \neg no(v, w)$ (by [1] and Rule 3)

[19] ⊢ $some(w, u) \wedge \Box all(u, v) \rightarrow some(v, w)$ (i.e. I□AI-4, by [18] and Fact (2.3))

[20] ⊢ $some(u, w) \wedge \Box all(u, v) \rightarrow some(v, w)$ (i.e. I□AI-3, by [19] and Fact (8.1))

[21] ⊢ $some(w, u) \wedge \Box all(u, v) \rightarrow some(w, v)$ (i.e. □AII-1, by [19] and Fact (8.1))

[22] ⊢ $some(u, w) \wedge \Box all(u, v) \rightarrow some(w, v)$ (i.e. □AII-3, by [20] and Fact (8.1))

[23] ⊢ $all(w, u) \wedge \Box all(u, v) \rightarrow some(v, w)$ (i.e. A□AI-4, by [19], Fact (7.1) and Rule 1)

[24] ⊢ $all(u, w) \wedge \Box all(u, v) \rightarrow some(v, w)$ (i.e. A□AI-3, by [20], Fact (7.1) and Rule 1)

[25] ⊢ $all(w, u) \wedge \Box all(u, v) \rightarrow some(w, v)$ (i.e. □AAI-1, by [21], Fact (7.1) and Rule 1)

[26] ⊢ $all(u, w) \wedge \Box all(u, v) \rightarrow some(w, v)$ (i.e. □AAI-3, by [22], Fact (7.1) and Rule 1)

[27] ⊢ $not\ all\neg(u, w) \wedge \Box all(u, v) \rightarrow not\ all\neg(v, w)$ (by [20] and Fact (1.3))

[28] ⊢ $not\ all(u, D-w) \wedge \Box all(u, v) \rightarrow not\ all(v, D-w)$

(i.e. O□AO-3, by [27], Fact (7.1) and Definition D3)

[29] ⊢ $\neg$*not all*($v$, $D-w$)$\wedge$□*all*($u$, $v$)$\rightarrow\neg$*not all*($u$, $D-w$) (by [28] and Rule 3)

[30] ⊢ *all*($v$, $D-w$)$\wedge$□*all*($u$, $v$)$\rightarrow$*all*($u$, $D-w$) (i.e. A□AA-1, by [29] and Fact (2.1))

[31] ⊢ *all*($v$, $D-w$)$\wedge$□*all*($u$, $v$)$\rightarrow$◇*all*($u$, $D-w$) (i.e. A□A◇A-1, by [30], Fact 6 and Rule 2)

The above processes of knowledge deduction once again manifest the materialist view that there are universal connectiones between things. In fact, there are multiple paths to deducing a valid syllogism.

## 5. Conclusion

This paper firstly formalizes the classical modal syllogism □AEE-4, and then proves its validity, and through 31 steps of logical deduction, finally derives the other 21 valid classical modal syllogisms from the validity of □AEE-4. These knowledge deduction processes not only illustrate the deductibility of the syllogism □AEE-4, but also demonstrate the universal connectiones between things. This innovative research will contribute to knowledge mining in big data.


## Acknowledgement

This work was supported by the National Social Science Foundation of China under Grant No.21BZX100.


## Reference


[1] Yang F F and Zhang X J. (2024). Natural language information processing based on the valid traditional syllogisms EIO-4. *SCIREA Journal of Information Science and Systems Science*, 8(3): 95-102.

[2] Hao L H. (2024a). Knowledge reasoning based on the generalized syllogism *AHH-2*. *SCIREA Journal of Computer*, 9(1): 1-8.

[3] Hao L H. (2024b). Knowledge representation and knowledge reasoning based on the Aristotelian modal syllogism □AE◇E-4. *SCIREA Journal of Information Science and Systems Science*, 9(1): 1-8.

[4] Hao L H. (2024c).The validity of generalized modal syllogisms based on the syllogism E□M◇O-1. *SCIREA Journal of Mathematics*, 9(1): 11-22.

[5] Hao L H. (2024d). Generalized syllogism reasoning with the quantifiers in modern Square{no} and Square{most}. *Applied Science and Innovative Research*, 8(1): 31-38.

[6] Moss L S. (2008). Completeness theorems for syllogistic fragments. In Hamm F and Kepser S. (eds.), *Logics for Linguistic Structures*. Berlin: Mouton de Gruyter, pp. 143-173.

[7] Zhang X J. (2018). Axiomatization of Aristotelian syllogistic logic based on generalized quantifier theory. *Applied and Computational Mathematics*, 7(3): 167-172.

[8] Wei L and Zhang X J. (2023). How to dedrive the other 37 valid modal syllogisms from the syllogism ◇A□I◇I-1. International Journal of Social Science Studies, 11(3): 32-37.

[9] Wang H P and Yuan J J. (2024). The reducibility of the generalized syllogism *MMI-4* with the quantifiers in Square{most} and Square{some}. *SCIREA Journal of Mathematics*, 9(4): 84-92.

[10] Xu J and Zhang X J. (2023). How to obtain valid generalized modal syllogisms from valid generalized syllogisms. *Applied Science and Innovative Research*, 7(2): 45-51.

[11] Yu S Y and Zhang X J. (2023).The validity of generalized modal syllogisms with the generalized quantifiers in Square{most}, SCIREA Journal of Philosophy, 2024, 4(1): 11-22.

[12] Chellas F. (1980). *Modal Logic: an Introduction*. Cambridge: Cambridge University Press.

[13] Hamilton A G (1978). *Logic for Mathematicians*. Cambridge: Cambridge University Press.