SCIREA Journal of Philosophy

ISSN: 2995-7788

http://www.scirea.org/journal/Philosophy

September 7, 2026

Volume 6, Issue 1, February 2026

https://doi.org/10.54647/philosophy720132

Knowledge Reasoning Based on the Classical Modal Syllogism ◇A□O◇O-2 from the Perspective of Information Processing

Yijiang Hao [1,*]

[1] Institute of Philosophy, Chinese Academy of Social Sciences, Beijing, China

*Corresponding author: 2151499207@qq.com

Abstract

Based on the classical modal syllogism ◇A□O◇O-2, this paper formalizes the four types of syllogisms and then respectively deduces other 48 valid classical modal syllogisms, 30 non-trivial valid generalized modal syllogisms, 15 non-trivial valid generalized syllogisms, and 24 valid classical syllogisms. The rules, definitions, and facts involved in the deductive process fall within the scope of first-order logic and generalized quantifier theory. By making full use of reduction operations, this paper explores the reducible relationships between/among the above four types of syllogisms. Subsequently, this paper discusses how to judge the validity of discourse reasoning nested by these four types of syllogisms. In this sense, this paper accomplishes a cross-type study of syllogisms, offering a unified method to derive valid syllogisms across different types. Following the proposed method in this paper, one can derive more valid syllogisms of various types from the syllogism ◇A□O◇O-2. Similarly, this method can also be applied to other valid classical modal syllogisms, enabling further exploration of deriving different syllogisms. Due to the use of deductive reasoning throughout the entire process of mining other valid syllogisms from one valid syllogism, the

conclusions of this paper are logically consistent. Thus, the research method utilized in this study demonstrates considerable universality in the realm of syllogistic research, and it also provides inspiration for natural language information processing and knowledge mining in artificial intelligence.

Keywords: classical modal syllogisms; knowledge representation; knowledge mining; generalized syllogisms

1 Introduction

Syllogistic reasoning, common in natural language and social life, plays an important role in logic. Syllogisms can be categorized into several distinct types, including classical syllogisms [1], classical modal syllogisms [2, 3], generalized syllogisms [4], and generalized modal syllogisms [5], among others. The following study only involves these four syllogisms.

The study of classical modal syllogisms dates back to Aristotle's era. Works such as those by [6-11] are representative in this field. While these studies have primarily focused on syllogistic validity, some of their findings are inconsistent and subject to criticism. In contrast, the reducible relationships between/among different types of syllogisms have been a less explored topic, and this paper attempts to do some work on this topic.

This paper specifically explores how to derive the four types of valid syllogisms based on the validity of the classical modal syllogism ◇A□O◇O-2. To be specific, this study first proves that the syllogism ◇A□O◇O-2 is valid. Based on this, other classical modal syllogisms, generalized modal syllogisms, generalized syllogisms, and classical syllogisms are respectively derived from the syllogism ◇A□O◇O-2 by applying related facts, definitions and inference rules, thus maintaining consistency in the results.

2 Knowledge Representation about Classical Modal Syllogisms

In what follows, let $Q \in$ Square{*all*}={*all*, *not all*, *some*, *no*}, with $Q\neg$ and $\neg Q$ representing its inner and outer negation, respectively. And let b, r and w be lexical variables, K be their domain, and B, R, and W denote the set composed of b, r and w, respectively. The symbol '$=_{def}$' signifies definitional equivalence. Let π, ω, χ and α be well-formed formulas (shortened to wff). '$\vdash\alpha$' denotes that the wff α is provable. The logical operators in the paper are the fundamental symbols in modal logic [12] and set theory [13], including $\neg$ (negation), $\wedge$ (conjunction), $\rightarrow$ (conditionality), $\leftrightarrow$ (biconditionality), $\Box$ (necessity) and $\Diamond$ (possibility).

There are four propositional forms in classical syllogisms: Proposition *A* ('all *b*s are *w*s'), *O* ('not all *b*s are *w*s'), *I* ('some *b*s are *w*s'), and *E* ('no *b*s are *w*s'), which correspond to the formalization *all(b, w)*, *not all(b, w)*, *some(b, w)*, and *no(b, w)*, respectively. Following standard practice [14], classical syllogisms possess four figures.

A classical modal syllogism is constructed by augmenting a classical syllogism with necessity ($\Box$) and/or possibility ($\Diamond$) operators. In a non-trivial classical modal syllogism, there are at least one and at most three non-overlapping modalities. Apart from the aforementioned four fundamental propositions, there are eight other proposition types in non-trivial classical modal syllogisms, namely, $\Box$*all(b, w)*, $\Box$*not all(b, w)*, $\Box$*some(b, w)*, $\Box$*no(b, w)*, $\Diamond$*all(b, w)*, $\Diamond$*not all(b, w)*, $\Diamond$*some(b, w)*, and $\Diamond$*no(b, w)*. And they are denoted by Proposition $\Box$*A*, $\Box$*O*, $\Box$*I*, $\Box$*E*, $\Diamond$*A*, $\Diamond$*O*, $\Diamond$*I* and $\Diamond$*E*, respectively. This can be illustrated with the syllogism $\Diamond$A$\Box$O$\Diamond$O-2, which takes the form $\Diamond$*all(w, r)*$\wedge\Box$*not all(b, r)*$\rightarrow\Diamond$*not all(b, w)*. And this syllogism can be exemplified as follows:

Major premise: All birds possibly like to eat insects.

<u>Minor premise: Not all animals necessarily like to eat insects.</u>

Conclusion: Not all animals are possibly birds.

Let *w*, *r*, and *b* be the variables representing birds, animals that like to eat insects, and animals, respectively. Then, the above instance can be formalized as $\Diamond$*all(w, r)*$\wedge\Box$*not all(b, r)*$\rightarrow\Diamond$*not all(b, w)*, shortened to $\Diamond$A$\Box$O$\Diamond$O-2. Other cases follow the same pattern.

3 Classical Modal Syllogistic System

The following components constitute this formal system: initial symbols, formation rules, basic axioms, relevant definitions, relevant facts, and inference rules. In the following, $Q\in$Square{*all*}.

3.1 Initial Symbols

[1] lexical variables: *b*, *r*, *w*

[2] quantifier: *all*

[3] operators: $\neg$, $\rightarrow$

[4] modality: $\Box$

[5] brackets: (,)

3.2 Formation Rules

[1] If Q is a quantifier, b and w are lexical variables, then $Q(b, w)$ is a wff.

[2] If α is a wff, then so are $\neg\alpha$ and $\Box\alpha$.

[3] If σ and α are wffs, then so is $\sigma\rightarrow\alpha$.

[4] Only the formulas that follow the above rules are wffs.

3.3 Basic Axioms

A1: If α is a valid wff in first-order logic [15], then $\vdash\alpha$.

A2: $\vdash\Diamond \mathit{all}(w, r)\wedge\Box \mathit{not\ all}(b, r)\rightarrow\Diamond \mathit{not\ all}(b, w)$ (i.e. the syllogism $\Diamond$A$\Box$O$\Diamond$O-2).

3.4 Relevant Definitions

D1: $(\chi\wedge\alpha)=_{\text{def}}\neg(\chi\rightarrow\neg\alpha)$;

D2: $(\chi\leftrightarrow\alpha)=_{\text{def}}(\chi\rightarrow\alpha)\wedge(\alpha\rightarrow\chi)$;

D3: $Q\neg(b, w)=_{\text{def}}Q(b, K-w)$;

D4: $(\neg Q)(b, w)=_{\text{def}}$It is not the case that $Q(b, w)$;

D5: $\Diamond\phi=_{\text{def}}\neg\Box\neg\phi$;

D6: $\mathit{all}(b, w)$ is true iff $B\subseteq W$ is true;

D7: $\mathit{not\ all}(b, w)$ is true iff $B\not\subseteq W$ is true;

D8: $\mathit{some}(b, w)$ is true iff $B\cap W\neq\varnothing$ is true;

D9: $\mathit{no}(b, w)$ is true iff $B\cap W=\varnothing$ is true;

D10: $\mathit{most}(b, w)$ is true iff $|B\cap W|>0.5|B|$ is true;

D11: *fewer than half of the(b, w)* is true iff $|B\cap W|<0.5|B|$ is true;

D12: $Q(b, w)$ is true iff $Q(b, w)$ is true in any real world;

D13: $\Box Q(b, w)$ is true iff $Q(b, w)$ is true in any possible world;

D14: $\Diamond Q(b, w)$ is true iff $Q(b, w)$ is true in at least one possible world.

3.5 Relevant Facts

Fact 1 (inner negation):

[1.1] $\vdash all(b, w) \leftrightarrow no\neg(b, w)$;

[1.2] $\vdash not\ all(b, w) \leftrightarrow some\neg(b, w)$;

[1.3] $\vdash some(b, w) \leftrightarrow not\ all\neg(b, w)$;

[1.4] $\vdash no(b, w) \leftrightarrow all\neg(b, w)$;

[1.5] $\vdash fewer\ than\ half\ of\ the(b, w) \leftrightarrow most\neg(b, w)$;

[1.6] $\vdash most(b, w) \leftrightarrow fewer\ than\ half\ of\ the\neg(b, w)$;

[1.7] $\vdash at\ least\ half\ of\ the(b, w) \leftrightarrow at\ most\ half\ of\ the\neg(b, w)$;

[1.8] $\vdash at\ most\ half\ of\ the(b, w) \leftrightarrow at\ least\ half\ of\ the\neg(b, w)$.

Fact 2 (outer negation):

[2.1] $\vdash \neg not\ all(b, w) \leftrightarrow all(b, w)$;

[2.2] $\vdash \neg all(b, w) \leftrightarrow not\ all(b, w)$;

[2.3] $\vdash \neg no(b, w) \leftrightarrow some(b, w)$;

[2.4] $\vdash \neg some(b, w) \leftrightarrow no(b, w)$;

[2.5] $\vdash \neg fewer\ than\ half\ of\ the(b, w) \leftrightarrow at\ least\ half\ of\ the(b, w)$;

[2.6] $\vdash \neg most(b, w) \leftrightarrow at\ most\ half\ of\ the(b, w)$;

[2.7] $\vdash \neg at\ least\ half\ of\ the(b, w) \leftrightarrow fewer\ than\ half\ of\ the(b, w)$;

[2.8] $\vdash \neg at\ most\ half\ of\ the(b, w) \leftrightarrow most(b, w)$.

Fact 3 (dual):

[3.1] $\vdash \neg\Box Q(b, w) \leftrightarrow \Diamond\neg Q(b, w)$;

[3.2] $\vdash \neg\Diamond Q(b, w) \leftrightarrow \Box\neg Q(b, w)$.

Fact 4 (Subordination):

[4.1] $\vdash all(b, w) \rightarrow some(b, w)$;

[4.2] $\vdash$*no(b, w)*$\rightarrow$*not all(b, w)*;

[4.3] $\vdash$*all(b, w)*$\rightarrow$*most(b, w)*;

[4.4] $\vdash$*most(b, w)*$\rightarrow$*some(b, w)*;

[4.5] $\vdash$*all(b, w)*$\rightarrow$*at least half of the(b, w)*;

[4.6] $\vdash$*fewer than half of the(b, w)*$\rightarrow$*not all(b, w)*;

[4.7] $\vdash$*at most half of the(b, w)*$\rightarrow$*not all(b, w)*;

[4.8] $\vdash$*at least half of the(b, w)*$\rightarrow$*some(b, w)*;

[4.9] $\vdash\Box Q(b, w)\rightarrow Q(b, w)$;

[4.10] $\vdash\Box Q(b, w)\rightarrow\Diamond Q(b, w)$;

[4.11] $\vdash Q(b, w)\rightarrow\Diamond Q(b, w)$.

Fact 5 (Symmetry):

[5.1] $\vdash$*some(b, w)*$\leftrightarrow$*some(w, b)*;

[5.2] $\vdash$*no(b, w)*$\leftrightarrow$*no(w, b)*.

3.6 Inference Rules

Rule 1 (antecedent strengthening): From $\vdash(\chi\wedge\alpha\rightarrow\pi)$ and $\vdash(\omega\rightarrow\chi)$ infer $\vdash(\omega\wedge\alpha\rightarrow\pi)$.

Rule 2 (subsequent weakening): From $\vdash(\chi\wedge\alpha\rightarrow\pi)$ and $\vdash(\pi\rightarrow\omega)$ infer $\vdash(\chi\wedge\alpha\rightarrow\omega)$.

Rule 3 (anti-syllogism): From $\vdash(\chi\wedge\alpha\rightarrow\pi)$ infer $\vdash(\neg\pi\wedge\chi\rightarrow\neg\alpha)$.

Rule 4 (anti-syllogism): From $\vdash(\chi\wedge\alpha\rightarrow\pi)$ infer $\vdash(\neg\pi\wedge\alpha\rightarrow\neg\chi)$.

Because the foregoing facts are elementary in first-order logic and generalized quantifier theory [16], their proofs are omitted.

4 Knowledge Mining Based on the Classical Modal Syllogism ◇A□O◇O-2

In the subsequent discussion, the validity of the syllogism ◇A□O◇O-2 is proved in Theorem 1. '[1] ◇A□O◇O-2→A□O◇O-2' in Theorem 2 signifies that the validity of the right syllogism can be proved based on that of the left, demonstrating a reducible relationship between them. Other cases follow the same pattern. From Theorem 2 to Theorem 5, the 48 valid classical modal syllogisms, the 30 non-trivial valid generalized modal syllogisms, the 15 non-trivial valid generalized syllogisms, and the 24 valid classical syllogisms are derived from the classical modal syllogism ◇A□O◇O-2, respectively.

Theorem 1 (◇A□O◇O-2): The classical modal syllogism ◇*all(w, r)*∧□*not all(b, r)*→◇*not all(b, w)* is valid.

Proof: Assume that ◇*all(w, r)* and □*not all(b, r)* are true, then $W\subseteq R$ is true in at least one possible world in the light of Definition D6 and D14, and $B\nsubseteq R$ is true in any possible world in line with Definition D7 and D13. Since any real world is a possible world, $B\nsubseteq W$ is true in at least one possible world. Accordingly, ◇*not all(b, w)* is true with the help of Definition D7 and D14. Thus, the validity of the syllogism ◇*all(w, r)*∧□*not all(b, r)*→◇*not all(b, w)* is proved.

Theorem 2: At least the following 48 valid classical modal syllogisms can be deduced from the valid syllogism ◇A□O◇O-2:

[1] ⊢◇A□O◇O-2→A□O◇O-2

[2] ⊢◇A□O◇O-2→□A□O◇O-2

[3] ⊢◇A□O◇O-2→◇A□E◇O-2

[4] ⊢◇A□O◇O-2→◇A□E◇O-2→A□E◇O-2

[5] ⊢◇A□O◇O-2→◇A□E◇O-2→□A□E◇O-2

[6] ⊢◇A□O◇O-2→◇A□E◇O-2→◇A□E◇O-4

[7] ⊢◇A□O◇O-2→◇A□E◇O-2→A□E◇O-2→A□E◇O-4

[8] ⊢◇A□O◇O-2→◇A□E◇O-2→□A□E◇O-2→□A□E◇O-4

[9] ⊢◇A□O◇O-2→◇E□I◇O-2

[10] ⊢◇A□O◇O-2→◇E□I◇O-2→E□I◇O-2

[11] ⊢◇A□O◇O-2→◇E□I◇O-2→□E□I◇O-2

[12] ⊢◇A□O◇O-2→◇E□I◇O-2→◇E□I◇O-1

[13] ⊢◇A□O◇O-2→◇E□I◇O-2→E□I◇O-2→E□I◇O-1

[14] ⊢◇A□O◇O-2→◇E□I◇O-2→□E□I◇O-2→□E□I◇O-1

[15] ⊢◇A□O◇O-2→◇E□I◇O-2→◇E□I◇O-4

[16] ⊢◇A□O◇O-2→◇E□I◇O-2→E□I◇O-2→E□I◇O-4

[17] ⊢◇A□O◇O-2→◇E□I◇O-2→□E□I◇O-2→□E□I◇O-4

[18] ⊢◇A□O◇O-2→◇E□I◇O-2→◇E□I◇O-1→◇E□I◇O-3

[19] ⊢◇A□O◇O-2→◇E□I◇O-2→E□I◇O-2→E□I◇O-1→E□I◇O-3

[20] ⊢◇A□O◇O-2→◇E□I◇O-2→□E□I◇O-2→□E□I◇O-1→□E□I◇O-3

[21] ⊢◇A□O◇O-2→◇E□I◇O-2→◇E□A◇O-2

[22] ⊢◇A□O◇O-2→◇E□I◇O-2→E□I◇O-2→E□A◇O-2

[23] ⊢◇A□O◇O-2→◇E□I◇O-2→□E□I◇O-2→□E□A◇O-2

[24] ⊢◇A□O◇O-2→◇E□I◇O-2→◇E□I◇O-1→◇E□A◇O-1

[25] ⊢◇A□O◇O-2→◇E□I◇O-2→E□I◇O-2→E□I◇O-1→E□A◇O-1

[26] ⊢◇A□O◇O-2→◇E□I◇O-2→□E□I◇O-2→□E□I◇O-1→□E□A◇O-1

[27] ⊢◇A□O◇O-2→◇E□I◇O-2→◇E□I◇O-4→◇E□A◇O-4

[28] ⊢◇A□O◇O-2→◇E□I◇O-2→E□I◇O-2→E□I◇O-4→E□A◇O-4

[29] ⊢◇A□O◇O-2→◇E□I◇O-2→□E□I◇O-2→□E□I◇O-4→□E□A◇O-4

[30] ⊢◇A□O◇O-2→◇E□I◇O-2→◇E□I◇O-1→◇E□I◇O-3→◇E□A◇O-3

[31] ⊢◇A□O◇O-2→◇E□I◇O-2→E□I◇O-2→E□I◇O-1→E□I◇O-3→E□A◇O-3

[32] ⊢◇A□O◇O-2→◇E□I◇O-2→□E□I◇O-2→□E□I◇O-1→□E□I◇O-3
→□E□A◇O-3

[33] ⊢◇A□O◇O-2→□O□A□O-3

[34] ⊢◇A□O◇O-2→□O□A□O-3→□O□AO-3

[35] ⊢◇A□O◇O-2→□O□A□O-3→□O□A◇O-3

[36] ⊢◇A□O◇O-2→□O□A□O-3→□E□A□O-3

[37] ⊢◇A□O◇O-2→□O□A□O-3→□O□AO-3→□E□AO-3

[38] ⊢◇A□O◇O-2→□O□A□O-3→□E□A□O-3→□E□A□O-4

[39] ⊢◇A□O◇O-2→□O□A□O-3→□O□AO-3→□E□AO-3→□E□AO-4

[40] ⊢◇A□O◇O-2→◇A□A◇A-1

[41] ⊢◇A□O◇O-2→◇A□A◇A-1→A□A◇A-1

[42] ⊢◇A□O◇O-2→◇A□A◇A-1→□A□A◇A-1

[43] ⊢◇A□O◇O-2→◇A□A◇A-1→◇A□A◇I-1

[44] ⊢◇A□O◇O-2→◇A□A◇A-1→A□A◇A-1→A□A◇I-1

[45] ⊢◇A□O◇O-2→◇A□A◇A-1→□A□A◇A-1→□A□A◇I-1

[46] ⊢◇A□O◇O-2→◇A□A◇A-1→◇A□A◇I-1→□A◇A◇I-4

[47] ⊢◇A□O◇O-2→◇A□A◇A-1→A□A◇A-1→A□A◇I-1→□AA◇I-4

[48] ⊢◇A□O◇O-2→◇A□A◇A-1→□A□A◇A-1→□A□A◇I-1→□A□A◇I-4

Proof:

(1) $\vdash\Diamond all(w, r)\wedge\Box not\ all(b, r)\rightarrow\Diamond not\ all(b, w)$ (i.e. $\Diamond$A$\Box$O$\Diamond$O-2, Axiom A2)

(2) $\vdash all(w, r)\wedge\Box not\ all(b, r)\rightarrow\Diamond not\ all(b, w)$ (i.e. A$\Box$O$\Diamond$O-2, by (1), Fact [4.11] and Rule 1)

(3) $\vdash\Box all(w, r)\wedge\Box not\ all(b, r)\rightarrow\Diamond not\ all(b, w)$(i.e. $\Box$A$\Box$O$\Diamond$O-2, by (1), Fact [4.10] and Rule 1)

(4) $\vdash\Diamond all(w, r)\wedge\Box no(b, r)\rightarrow\Diamond not\ all(b, w)$ (i.e. $\Diamond$A$\Box$E$\Diamond$O-2, by (1), Fact [4.2] and Rule 1)

(5) $\vdash all(w, r)\wedge\Box no(b, r)\rightarrow\Diamond not\ all(b, w)$ (i.e. A$\Box$E$\Diamond$O-2, by (4), Fact [4.11] and Rule 1)

(6) $\vdash\Box all(w, r)\wedge\Box no(b, r)\rightarrow\Diamond not\ all(b, w)$ (i.e. $\Box$A$\Box$E$\Diamond$O-2, by (4), Fact [4.10] and Rule 1)

(7) $\vdash\Diamond all(w, r)\wedge\Box no(r, b)\rightarrow\Diamond not\ all(b, w)$ (i.e. $\Diamond$A$\Box$E$\Diamond$O-4, by (4), Fact [5.2] and Rule 1)

(8) $\vdash all(w, r)\wedge\Box no(r, b)\rightarrow\Diamond not\ all(b, w)$ (i.e. A$\Box$E$\Diamond$O-4, by (5), Fact [5.2] and Rule 1)

(9) $\vdash\Box all(w, r)\wedge\Box no(r, b)\rightarrow\Diamond not\ all(b, w)$ (i.e. $\Box$A$\Box$E$\Diamond$O-4, by (6), Fact [5.2] and Rule 1)

(10) $\vdash\Diamond no\neg(w, r)\wedge\Box some\neg(b, r)\rightarrow\Diamond not\ all(b, w)$ (by (1), Fact [1.1] and Fact [1.4])

(11) $\vdash\Diamond no(w, K\neg r)\wedge\Box some(b, K\neg r)\rightarrow\Diamond not\ all(b, w)$

(i.e. $\Diamond$E$\Box$I$\Diamond$O-2, by (10) and Definition D3)

(12) $\vdash no(w, K\neg r)\wedge\Box some(b, K\neg r)\rightarrow\Diamond not\ all(b, w)$

(i.e. E$\Box$I$\Diamond$O-2, by (11), Fact [4.11] and Rule 1)

(13) $\vdash\Box no(w, K\neg r)\wedge\Box some(b, K\neg r)\rightarrow\Diamond not\ all(b, w)$

(i.e. $\Box$E$\Box$I$\Diamond$O-2, by (11), Fact [4.10] and Rule 1)

(14) $\vdash\Diamond no(K\neg r, w)\wedge\Box some(b, K\neg r)\rightarrow\Diamond not\ all(b, w)$ (i.e. $\Diamond$E$\Box$I$\Diamond$O-1, by (11) and Fact [5.2])

(15) $\vdash no(K\neg r, w)\wedge\Box some(b, K\neg r)\rightarrow\Diamond not\ all(b, w)$ (i.e. E$\Box$I$\Diamond$O-1, by (12) and Fact [5.2])

(16) $\vdash\Box no(K\neg r, w)\wedge\Box some(b, K\neg r)\rightarrow\Diamond not\ all(b, w)$ (i.e. $\Box$E$\Box$I$\Diamond$O-1, by (13) and Fact [5.2])

(17) $\vdash\Diamond no(w, K\neg r)\wedge\Box some(K\neg r, b)\rightarrow\Diamond not\ all(b, w)$ (i.e. $\Diamond$E$\Box$I$\Diamond$O-4, by (11) and Fact [5.1])

(18) $\vdash no(w, K\neg r)\wedge\Box some(K\neg r, b)\rightarrow\Diamond not\ all(b, w)$ (i.e. E$\Box$I$\Diamond$O-4, by (12) and Fact [5.1])

(19) $\vdash\Box no(w, K\neg r)\wedge\Box some(K\neg r, b)\rightarrow\Diamond not\ all(b, w)$ (i.e. $\Box$E$\Box$I$\Diamond$O-4, by (13) and Fact [5.1])

(20) $\vdash\Diamond no(K\neg r, w)\wedge\Box some(K\neg r, b)\rightarrow\Diamond not\ all(b, w)$ (i.e. $\Diamond$E$\Box$I$\Diamond$O-3, by (14) and Fact [5.1])

(21) $\vdash no(K\neg r, w)\wedge\Box some(K\neg r, b)\rightarrow\Diamond not\ all(b, w)$ (i.e. E$\Box$I$\Diamond$O-3, by (15) and Fact [5.1])

(22) $\vdash\Box no(K\neg r, w)\wedge\Box some(K\neg r, b)\rightarrow\Diamond not\ all(b, w)$ (i.e. $\Box$E$\Box$I$\Diamond$O-3, by (16) and Fact [5.1])

(23) $\vdash\Diamond no(w, K\neg r)\wedge\Box all(b, K\neg r)\rightarrow\Diamond not\ all(b, w)$

(i.e. $\Diamond$E$\Box$A$\Diamond$O-2, by (11), Fact [4.1] and Rule 1)

(24) $\vdash no(w, K\neg r)\wedge\Box all(b, K\neg r)\rightarrow\Diamond not\ all(b, w)$ (i.e. E$\Box$A$\Diamond$O-2, by (12), Fact [4.1] and Rule 1)

(25) $\vdash\Box no(w, K\neg r)\wedge\Box all(b, K\neg r)\rightarrow\Diamond not\ all(b, w)$

(i.e. $\Box$E$\Box$A$\Diamond$O-2, by (13), Fact [4.1] and Rule 1)

(26) $\vdash\Diamond no(K\neg r, w)\wedge\Box all(b, K\neg r)\rightarrow\Diamond not\ all(b, w)$

(i.e. $\Diamond$E$\Box$A$\Diamond$O-1, by (14), Fact [4.1] and Rule 1)

(27) $\vdash no(K{\neg}r, w) \wedge \Box all(b, K{\neg}r) \rightarrow \Diamond not\ all(b, w)$ (i.e. E□A◇O-1, by (15), Fact [4.1] and Rule 1)

(28) $\vdash \Box no(K{\neg}r, w) \wedge \Box all(b, K{\neg}r) \rightarrow \Diamond not\ all(b, w)$

(i.e. □E□A◇O-1, by (16), Fact [4.1] and Rule 1)

(29) $\vdash \Diamond no(w, K{\neg}r) \wedge \Box all(K{\neg}r, b) \rightarrow \Diamond not\ all(b, w)$

(i.e. ◇E□A◇O-4, by (17), Fact [4.1] and Rule 1)

(30) $\vdash no(w, K{\neg}r) \wedge \Box all(K{\neg}r, b) \rightarrow \Diamond not\ all(b, w)$ (i.e. E□A◇O-4, by (18), Fact [4.1] and Rule 1)

(31) $\vdash \Box no(w, K{\neg}r) \wedge \Box all(K{\neg}r, b) \rightarrow \Diamond not\ all(b, w)$

(i.e. □E□A◇O-4, by (19), Fact [4.1] and Rule 1)

(32) $\vdash \Diamond no(K{\neg}r, w) \wedge \Box all(K{\neg}r, b) \rightarrow \Diamond not\ all(b, w)$

(i.e. ◇E□A◇O-3, by (20), Fact [4.1] and Rule 1)

(33) $\vdash no(K{\neg}r, w) \wedge \Box all(K{\neg}r, b) \rightarrow \Diamond not\ all(b, w)$ (i.e. E□A◇O-3, by (21), Fact [4.1] and Rule 1)

(34) $\vdash \Box no(K{\neg}r, w) \wedge \Box all(K{\neg}r, b) \rightarrow \Diamond not\ all(b, w)$

(i.e. □E□A◇O-3, by (22), Fact [4.1] and Rule 1)

(35) $\vdash \neg\Diamond not\ all(b, w) \wedge \Box not\ all(b, r) \rightarrow \neg\Diamond all(w, r)$ (by (1) and Rule 4)

(36) $\vdash \Box\neg not\ all(b, w) \wedge \Box not\ all(b, r) \rightarrow \Box\neg all(w, r)$ (by (35) and Fact [3.2])

(37) $\vdash \Box all(b, w) \wedge \Box not\ all(b, r) \rightarrow \Box not\ all(w, r)$

(i.e. □O□A□O-3, by (36), Fact [2.1] and Fact [2.2])

(38) $\vdash \Box all(b, w) \wedge \Box not\ all(b, r) \rightarrow not\ all(w, r)$ (i.e. □O□AO-3, by (37), Fact [4.9] and Rule 2)

(39) $\vdash \Box all(b, w) \wedge \Box not\ all(b, r) \rightarrow \Diamond not\ all(w, r)$

(i.e. □O□A◇O-3, by (37), Fact [4.10] and Rule 2)

(40) $\vdash \Box all(b, w) \wedge \Box no(b, r) \rightarrow \Box not\ all(w, r)$ (i.e. □E□A□O-3, by (37), Fact [4.2] and Rule 1)

(41) $\vdash \Box all(b, w) \wedge \Box no(b, r) \rightarrow not\ all(w, r)$ (i.e. □E□AO-3, by (38), Fact [4.2] and Rule 1)

(42) $\vdash \Box all(b, w) \wedge \Box no(r, b) \rightarrow \Box not\ all(w, r)$ (i.e. □E□A□O-4, by (40) and Fact [5.2])

(43) $\vdash \Box all(b, w) \wedge \Box no(r, b) \rightarrow not\ all(w, r)$ (i.e. □E□AO-4, by (41) and Fact [5.2])

(44) $\vdash \neg\Diamond not\ all(b, w) \wedge \Diamond all(w, r) \rightarrow \neg\Box not\ all(b, r)$ (by (1) and Rule 3)

(45) $\vdash \Box\neg not\ all(b, w) \wedge \Diamond all(w, r) \rightarrow \Diamond\neg not\ all(b, r)$ (by (44), Fact [3.1] and Fact [3.2])

(46) $\vdash \Box all(b, w) \wedge \Diamond all(w, r) \rightarrow \Diamond all(b, r)$ (i.e. ◇A□A◇A-1, by (45) and Fact [2.1])

(47) $\vdash \Box all(b, w) \wedge all(w, r) \rightarrow \Diamond all(b, r)$ (i.e. A□A◇A-1, by (46), Fact [4.11] and Rule 1)

(48) $\vdash \Box all(b, w) \wedge \Box all(w, r) \rightarrow \Diamond all(b, r)$ (i.e. □A□A◇A-1, by (46), Fact [4.10] and Rule 1)

(49) $\vdash \Box all(b, w) \wedge \Diamond all(w, r) \rightarrow \Diamond some(b, r)$ (i.e. ◇A□A◇I-1, by (46) and Fact [4.1] and Rule 2)

(50) $\vdash \Box all(b, w) \wedge all(w, r) \rightarrow \Diamond some(b, r)$ (i.e. A□A◇I-1, by (47), Fact [4.1] and Rule 2)

(51) $\vdash \Box all(b, w) \wedge \Box all(w, r) \rightarrow \Diamond some(b, r)$ (i.e. □A□A◇I-1, by (48), Fact [4.1] and Rule 2)

(52) ⊢□*all(b, w)*∧◇*all(w, r)*→◇*some(r, b)* (i.e. □A◇A◇I-4, by (49) and Fact [5.1])

(53) ⊢□*all(b, w)*∧*all(w, r)*→◇*some(r, b)* (i.e. □AA◇I-4, by (50) and Fact [5.1])

(54) ⊢□*all(b, w)*∧□*all(w, r)*→◇*some(r, b)* (i.e. □A□A◇I-4, by (51) and Fact [5.1])

So far, the above 48 classical modal syllogisms have been deduced from the syllogism ◇A□O◇O-2. Indeed, there are 138 valid classical modal syllogisms can be inferred from ◇A□O◇O-2 in total. To maintain structural conciseness of this paper, other valid syllogisms and their proofs are omitted here. Through further deduction from the above derived syllogisms, one can attempt to derive more valid classical modal syllogisms.

Theorem 2 shows that other classical modal syllogisms can be derived from the classical modal syllogism ◇A□O◇O-2 by deductive reasoning. It naturally follows to inquire whether the other three types of syllogisms can also be deduced from the syllogism ◇A□O◇O-2. The following sections will focus on investigating this possibility.

Theorem 3 discusses how to deduce non-trivial valid generalized modal syllogisms from the classical modal syllogism ◇A□O◇O-2. The method comprises the following steps: (i) The generalized modal syllogism ◇A□F◇O-2 is deduced from the syllogism ◇A□O◇O-2 by applying the antecedent strengthening rule. (ii) Other valid generalized modal syllogisms can be inferred from the derived syllogism ◇A□F◇O-2, following the similar reduction operations as presented in Theorem 2.

Theorem 3: At least the following 30 non-trivial valid generalized modal syllogisms can be deduced from the valid classical modal syllogism ◇A□O◇O-2:

[1] ⊢◇A□O◇O-2→◇A□F◇O-2

[2] ⊢◇A□O◇O-2→◇A□F◇O-2→A□F◇O-2

[3] ⊢◇A□O◇O-2→◇A□F◇O-2→□A□F◇O-2

[4] ⊢◇A□O◇O-2→◇A□H◇O-2

[5] ⊢◇A□O◇O-2→◇A□H◇O-2→A□H◇O-2

[6] ⊢◇A□O◇O-2→◇A□H◇O-2→□A□H◇O-2

[7] ⊢◇A□O◇O-2→◇A□F◇O-2→◇E□M◇O-2

[8] ⊢◇A□O◇O-2→◇A□F◇O-2→◇E□M◇O-2→E□M◇O-2

[9] ⊢◇A□O◇O-2→◇A□F◇O-2→◇E□M◇O-2→□E□M◇O-2

[10] ⊢◇A□O◇O-2→◇A□F◇O-2→◇E□M◇O-2→◇E□M◇O-1

[11] ⊢◇A□O◇O-2→◇A□F◇O-2→◇E□M◇O-2→E□M◇O-2→E□M◇O-1

[12] ⊢◇A□O◇O-2→◇A□F◇O-2→◇E□M◇O-2→□E□M◇O-2→□E□M◇O-1

[13] ⊢◇A□O◇O-2→◇A□F◇O-2→◇E□S◇O-2

[14] ⊢◇A□O◇O-2→◇A□F◇O-2→◇E□S◇O-2→E□S◇O-2

[15] ⊢◇A□O◇O-2→◇A□F◇O-2→◇E□S◇O-2→□E□S◇O-2

[16] ⊢◇A□O◇O-2→◇A□F◇O-2→◇E□S◇O-2→◇E□S◇O-1

[17] ⊢◇A□O◇O-2→◇A□F◇O-2→◇E□S◇O-2→E□S◇O-2→E□S◇O-1

[18] ⊢◇A□O◇O-2→◇A□F◇O-2→◇E□S◇O-2→□E□S◇O-2→□E□S◇O-1

[19] ⊢◇A□O◇O-2→◇A□F◇O-2→◇E□M◇O-2→◇E□M◇O-1→◇A□M◇I-1

[20] ⊢◇A□O◇O-2→◇A□F◇O-2→◇E□M◇O-2→◇E□M◇O-1→◇A□M◇I-1

→A□M◇I-1

[21] ⊢◇A□O◇O-2→◇A□F◇O-2→◇E□M◇O-2→◇E□M◇O-1→◇A□M◇I-1

→□A□M◇I-1

[22] ⊢◇A□O◇O-2→◇A□F◇O-2→◇E□M◇O-2→◇E□M◇O-1→◇A□M◇I-1

→□M◇A◇I-4

[23] ⊢◇A□O◇O-2→◇A□F◇O-2→◇E□M◇O-2→◇E□M◇O-1→◇A□M◇I-1

→A□M◇I-1→□MA◇I-4

[24] ⊢◇A□O◇O-2→◇A□F◇O-2→◇E□M◇O-2→◇E□M◇O-1→◇A□M◇I-1

→□A□M◇I-1→□M□A◇I-4

[25] ⊢◇A□O◇O-2→◇A□F◇O-2→◇E□S◇O-2→◇E□S◇O-1→◇A□S◇I-1

[26] ⊢◇A□O◇O-2→◇A□F◇O-2→◇E□S◇O-2→◇E□S◇O-1→◇A□S◇I-1

→A□S◇I-1

[27] ⊢◇A□O◇O-2→◇A□F◇O-2→◇E□S◇O-2→◇E□S◇O-1→◇A□S◇I-1

→□A□S◇I-1

[28] $\vdash$◇A□O◇O-2→◇A□F◇O-2→◇E□S◇O-2→◇E□S◇O-1→◇A□S◇I-1 →□S◇A◇I-4

[29] $\vdash$◇A□O◇O-2→◇A□F◇O-2→◇E□S◇O-2→◇E□S◇O-1→◇A□S◇I-1 →A□S◇I-1→□SA◇I-4

[30] $\vdash$◇A□O◇O-2→◇A□F◇O-2→◇E□S◇O-2→◇E□S◇O-1→◇A□S◇I-1 →□A□S◇I-1→□S□A◇I-4

Proof:

(1) $\vdash$◇*all(w, r)*∧□*not all(b, r)*→◇*not all(b, w)* (i.e. ◇A□O◇O-2, Axiom A2)

(2) $\vdash$◇*all(w, r)*∧□*fewer than half of the(b, r)*→◇*not all(b, w)*

(i.e. ◇A□F◇O-2, by (1), Fact [4.6] and Rule 1)

(3) $\vdash$*all(w, r)*∧□*fewer than half of the(b, r)*→◇*not all(b, w)*

(i.e. A□F◇O-2, by (2), Fact [4.11] and Rule 1)

(4) $\vdash$□*all(w, r)*∧□*fewer than half of the(b, r)*→◇*not all(b, w)*

(i.e. □A□F◇O-2, by (2), Fact [4.10] and Rule 1)

(5) $\vdash$◇*all(w, r)*∧□*at most half of the(b, r)*→◇*not all(b, w)*

(i.e. ◇A□H◇O-2, by (1), Fact [4.7] and Rule 1)

(6) $\vdash$*all(w, r)*∧□*at most half of the(b, r)*→◇*not all(b, w)*

(i.e. A□H◇O-2, by (5), Fact [4.11] and Rule 1)

(7) $\vdash$□*all(w, r)*∧□*at most half of the(b, r)*→◇*not all(b, w)*

(i.e. □A□H◇O-2, by (5), Fact [4.10] and Rule 1)

(8) $\vdash$◇*no¬(w, r)*∧□*most¬(b, r)*→◇*not all(b, w)* (by (2), Fact [1.1] and Fact [1.5])

(9) $\vdash$◇*no(w, K¬r)*∧□*most(b, K¬r)*→◇*not all(b, w)*(i.e. ◇E□M◇O-2, by (8) and Definition D3)

(10) $\vdash$*no(w, K¬r)*∧□*most(b, K¬r)*→◇*not all(b, w)*

(i.e. E□M◇O-2, by (9), Fact [4.11] and Rule 1)

(11) $\vdash\Box no(w, K\neg r)\wedge\Box most(b, K\neg r)\rightarrow\Diamond not\ all(b, w)$

(i.e. □E□M◇O-2, by (9), Fact [4.10] and Rule 1)

(12) $\vdash\Diamond no(K\neg r, w)\wedge\Box most(b, K\neg r)\rightarrow\Diamond not\ all(b, w)$ (i.e. ◇E□M◇O-1, by (9) and Fact [5.2])

(13) $\vdash no(K\neg r, w)\wedge\Box most(b, K\neg r)\rightarrow\Diamond not\ all(b, w)$

(i.e. E□M◇O-1, by (12), Fact [4.11] and Rule 1)

(14) $\vdash\Box no(K\neg r, w)\wedge\Box most(b, K\neg r)\rightarrow\Diamond not\ all(b, w)$

(i.e. □E□M◇O-1, by (12), Fact [4.10] and Rule 1)

(15) $\vdash\Diamond no\neg(w, r)\wedge\Box at\ least\ half\ of\ the\neg(b, r)\rightarrow\Diamond not\ all(b, w)$ (by (5), Fact [1.1] and Fact [1.8])

(16) $\vdash\Diamond no(w, K\neg r)\wedge\Box at\ least\ half\ of\ the(b, K\neg r)\rightarrow\Diamond not\ all(b, w)$

(i.e. ◇E□S◇O-2, by (15) and Definition D3)

(17) $\vdash no(w, K\neg r)\wedge\Box at\ least\ half\ of\ the(b, K\neg r)\rightarrow\Diamond not\ all(b, w)$

(i.e. E□S◇O-2, by (16), Fact [4.11] and Rule 1)

(18) $\vdash\Box no(w, K\neg r)\wedge\Box at\ least\ half\ of\ the(b, K\neg r)\rightarrow\Diamond not\ all(b, w)$

(i.e. □E□S◇O-2, by (16), Fact [4.10] and Rule 1)

(19) $\vdash\Diamond no(K\neg r, w)\wedge\Box at\ least\ half\ of\ the(b, K\neg r)\rightarrow\Diamond not\ all(b, w)$

(i.e. ◇E□S◇O-1, by (16) and Fact [5.2])

(20) $\vdash no(K\neg r, w)\wedge\Box at\ least\ half\ of\ the(b, K\neg r)\rightarrow\Diamond not\ all(b, w)$

(i.e. E□S◇O-1, by (17) and Fact [5.2])

(21) $\vdash\Box no(K\neg r, w)\wedge\Box at\ least\ half\ of\ the(b, K\neg r)\rightarrow\Diamond not\ all(b, w)$

(i.e. □E□S◇O-1, by (18) and Fact [5.2])

(22) $\vdash\Diamond all\neg(K\neg r, w)\wedge\Box most(b, K\neg r)\rightarrow\Diamond some\neg(b, w)$ (by (12), Fact [1.4] and Fact [1.2])

(23) $\vdash\Diamond all(K\neg r, K\neg w)\wedge\Box most(b, K\neg r)\rightarrow\Diamond some(b, K\neg w)$

(i.e. $\Diamond A\Box M\Diamond I$-1, by (22) and Definition D3)

(24) $\vdash all(K{-}r, K{-}w)\wedge\Box most(b, K{-}r)\rightarrow\Diamond some(b, K{-}w)$

(i.e. $A\Box M\Diamond I$-1, by (23), Fact [4.11] and Rule 1)

(25) $\vdash\Box all(K{-}r, K{-}w)\wedge\Box most(b, K{-}r)\rightarrow\Diamond some(b, K{-}w)$

(i.e. $\Box A\Box M\Diamond I$-1, by (23), Fact [4.10] and Rule 1)

(26) $\vdash\Diamond all(K{-}r, K{-}w)\wedge\Box most(b, K{-}r)\rightarrow\Diamond some(K{-}w, b)$

(i.e. $\Box M\Diamond A\Diamond I$-4, by (23) and Fact [5.1])

(27) $\vdash all(K{-}r, K{-}w)\wedge\Box most(b, K{-}r)\rightarrow\Diamond some(K{-}w, b)$ (i.e. $\Box MA\Diamond I$-4, by (24) and Fact [5.1])

(28) $\vdash\Box all(K{-}r, K{-}w)\wedge\Box most(b, K{-}r)\rightarrow\Diamond some(K{-}w, b)$

(i.e. $\Box M\Box A\Diamond I$-4, by (25) and Fact [5.1])

(29) $\vdash\Diamond all\neg(K{-}r, w)\wedge\Box at\ least\ half\ of\ the(b, K{-}r)\rightarrow\Diamond some\neg(b, w)$

(by (19), Fact [1.4] and Fact [1.2])

(30) $\vdash\Diamond all(K{-}r, K{-}w)\wedge\Box at\ least\ half\ of\ the(b, K{-}r)\rightarrow\Diamond some(b, K{-}w)$

(i.e. $\Diamond A\Box S\Diamond I$-1, by (29) and Definition D3)

(31) $\vdash all(K{-}r, K{-}w)\wedge\Box at\ least\ half\ of\ the(b, K{-}r)\rightarrow\Diamond some(b, K{-}w)$

(i.e. $A\Box S\Diamond I$-1, by (30), Fact [4.11] and Rule 1)

(32) $\vdash\Box all(K{-}r, K{-}w)\wedge\Box at\ least\ half\ of\ the(b, K{-}r)\rightarrow\Diamond some(b, K{-}w)$

(i.e. $\Box A\Box S\Diamond I$-1, by (30), Fact [4.10] and Rule 1)

(33) $\vdash\Diamond all(K{-}r, K{-}w)\wedge\Box at\ least\ half\ of\ the(b, K{-}r)\rightarrow\Diamond some(K{-}w, b)$

(i.e. $\Box S\Diamond A\Diamond I$-4, by (30) and Fact [5.1])

(34) $\vdash all(K{-}r, K{-}w)\wedge\Box at\ least\ half\ of\ the(b, K{-}r)\rightarrow\Diamond some(K{-}w, b)$

(i.e. $\Box SA\Diamond I$-4, by (31) and Fact [5.1])

(35) $\vdash\Box all(K{-}r, K{-}w)\wedge\Box at\ least\ half\ of\ the(b, K{-}r)\rightarrow\Diamond some(K{-}w, b)$

(i.e. $\Box S\Box A\Diamond I$-4, by (32) and Fact [5.1])

Next, this paper proceeds to study the derivation of non-trivial valid generalized syllogisms from the valid classical modal syllogism ◇A□O◇O-2. As demonstrated by [17], the obtained syllogism AOO-2 is also valid after removing all modalities from the syllogism ◇A□O◇O-2, and the validity of the syllogism AOO-2 can be proven according to Definition 3.4. After that, the generalized syllogism AFO-2 is further inferred from the syllogism AOO-2 by the antecedent strengthening rule. Continuing to conduct deductive reasoning similar to Theorem 2 and Theorem 3, more generalized syllogisms can be deduced from the derived syllogism AFO-2.

Theorem 4: At least the following 15 non-trivial valid generalized syllogisms can be derived from the valid classical modal syllogism ◇A□O◇O-2:

[1] ⊢◇A□O◇O-2→AOO-2→AFO-2

[2] ⊢◇A□O◇O-2→AOO-2→AFO-2→EMO-2

[3] ⊢◇A□O◇O-2→AOO-2→AFO-2→EMO-2→EMO-1

[4] ⊢◇A□O◇O-2→AOO-2→AFO-2→EMO-2→EMO-1→AMI-1

[5] ⊢◇A□O◇O-2→AOO-2→AFO-2→EMO-2→EMO-1→AMI-1→MAI-4

[6] ⊢◇A□O◇O-2→AOO-2→AFO-2→FAO-3

[7] ⊢◇A□O◇O-2→AOO-2→AFO-2→AAS-1

[8] ⊢◇A□O◇O-2→AOO-2→AFO-2→FAO-3→MAI-3

[9] ⊢◇A□O◇O-2→AOO-2→AFO-2→FAO-3→MAI-3→AMI-3

[10] ⊢◇A□O◇O-2→AOO-2→AFO-2→AAS-1→EAH-1

[11] ⊢◇A□O◇O-2→AOO-2→AFO-2→AAS-1→EAH-1→EAH-2

[12] ⊢◇A□O◇O-2→AOO-2→AFO-2→FAO-3→MAI-3→AMI-3→EMO-3

[13] ⊢◇A□O◇O-2→AOO-2→AFO-2→FAO-3→MAI-3→AMI-3→EMO-3→EMO-4

[14] ⊢◇A□O◇O-2→AOO-2→AFO-2→AAS-1→EAH-1→EAH-2→AEH-2

[15] ⊢◇A□O◇O-2→AOO-2→AFO-2→AAS-1→EAH-1→EAH-2→AEH-2→AEH-4

Proof:

(1) ⊢◇*all(w, r)*∧□*not all(b, r)*→◇*not all(b, w)* (i.e. ◇A□O◇O-2, Axiom A2)

(2) ⊢*all(w, r)*∧*not all(b, r)*→*not all(b, w)* (i.e. AOO-2, by (1) and Xu(2023)[16])

(3) ⊢*all(w, r)*∧*fewer than half of the(b, r)*→*not all(b, w)* (i.e. AFO-2, by (2), Fact [4.6] and Rule 1)

(4) $\vdash no\neg(w, r) \wedge most\neg(b, r) \rightarrow not\ all(b, w)$ (by (3), Fact [1.1] and Fact [1.5])

(5) $\vdash no(w, K{-}r) \wedge most(b, K{-}r) \rightarrow not\ all(b, w)$ (i.e. EMO-2, by (4) and Definition D3)

(6) $\vdash no(K{-}r, w) \wedge most(b, K{-}r) \rightarrow not\ all(b, w)$ (i.e. EMO-1, by (5) and Fact [5.2])

(7) $\vdash all\neg(K{-}r, w) \wedge most(b, K{-}r) \rightarrow some\neg(b, w)$ (by (6), Fact [1.4] and Fact [1.2])

(8) $\vdash all(K{-}r, K{-}w) \wedge most(b, K{-}r) \rightarrow some(b, K{-}w)$ (i.e. AMI-1, by (7) and Definition D3)

(9) $\vdash all(K{-}r, K{-}w) \wedge most(b, K{-}r) \rightarrow some(K{-}w, b)$ (i.e. MAI-4, by (8) and Fact [5.1])

(10) $\vdash \neg not\ all(b, w) \wedge fewer\ than\ half\ of\ the(b, r) \rightarrow \neg all(w, r)$ (by (3) and Rule 4)

(11) $\vdash all(b, w) \wedge fewer\ than\ half\ of\ the(b, r) \rightarrow not\ all(w, r)$

(i.e. FAO-3, by (10), Fact [2.1] and Fact [2.2])

(12) $\vdash \neg not\ all(b, w) \wedge all(w, r) \rightarrow \neg fewer\ than\ half\ of\ the(b, r)$ (by (3) and Rule 3)

(13) $\vdash all(b, w) \wedge all(w, r) \rightarrow at\ least\ half\ of\ the(b, r)$ (i.e. AAS-1, by (12), Fact [2.1] and Fact [2.5])

(14) $\vdash all(b, w) \wedge most\neg(b, r) \rightarrow some\neg(w, r)$ (by (11), Fact [1.5] and [1.2])

(15) $\vdash all(b, w) \wedge most(b, K{-}r) \rightarrow some(w, K{-}r)$ (i.e. MAI-3, by (14) and Definition D3)

(16) $\vdash all(b, w) \wedge most(b, K{-}r) \rightarrow some(K{-}r, w)$ (i.e. AMI-3, by (15) and Fact [5.1])

(17) $\vdash all(b, w) \wedge no\neg(w, r) \rightarrow at\ most\ half\ of\ the\neg(b, r)$ (by (13), Fact [1.1] and Fact [1.7])

(18) $\vdash all(b, w) \wedge no(w, K{-}r) \rightarrow at\ most\ half\ of\ the(b, K{-}r)$ (i.e. EAH-1, by (17) and Definition D3)

(19) $\vdash all(b, w) \wedge no(K{-}r, w) \rightarrow at\ most\ half\ of\ the(b, K{-}r)$ (i.e. EAH-2, by (18) and Fact [5.2])

(20) $\vdash no\neg(b, w) \wedge most(b, K{-}r) \rightarrow not\ all\neg(K{-}r, w)$ (by (16), Fact [1.1] and Fact [1.3])

(21) $\vdash no(b, K{-}w) \wedge most(b, K{-}r) \rightarrow not\ all(K{-}r, K{-}w)$ (i.e. EMO-3, by (20) and Definition D3)

(22) $\vdash no(K{-}w, b) \wedge most(b, K{-}r) \rightarrow not\ all(K{-}r, K{-}w)$ (i.e. EMO-4, by (21) and Fact [5.2])

(23) $\vdash no\neg(b, w) \wedge all\neg(K{-}r, w) \rightarrow at\ most\ half\ of\ the(b, K{-}r)$ (by (19), Fact [1.1] and Fact [1.4])

(24) $\vdash no(b, K{-}w) \wedge all(K{-}r, K{-}w) \rightarrow at\ most\ half\ of\ the(b, K{-}r)$

(i.e. AEH-2, by (23) and Definition D3)

(25) $\vdash no(K{-}w, b) \wedge all(K{-}r, K{-}w) \rightarrow at\ most\ half\ of\ the(b, K{-}r)$

(i.e. AEH-4, by (24) and Fact [5.2])

The final part illustrates the method for deriving classical syllogisms from the classical modal syllogism $\Diamond A \Box O \Diamond O$-2. The procedure involves: (i) obtaining the classical syllogism

AOO-2 by following the method in Theorem 4; (ii) deducing the other 23 valid classical syllogisms from the syllogism AOO-2 by applying deductive steps.

Theorem 5: At least the following 24 classical syllogisms can be derived from the valid classical modal syllogism ◇A□O◇O-2:

[1] ⊢◇A□O◇O-2→AOO-2

[2] ⊢◇A□O◇O-2→AOO-2→AEO-2

[3] ⊢◇A□O◇O-2→AOO-2→AEO-2→AEO-4

[4] ⊢◇A□O◇O-2→AOO-2→EIO-2

[5] ⊢◇A□O◇O-2→AOO-2→EIO-2→EIO-1

[6] ⊢◇A□O◇O-2→AOO-2→EIO-2→EIO-4

[7] ⊢◇A□O◇O-2→AOO-2→EIO-2→EIO-1→EIO-3

[8] ⊢◇A□O◇O-2→AOO-2→EIO-2→EAO-2

[9] ⊢◇A□O◇O-2→AOO-2→EIO-2→EIO-1→EAO-1

[10] ⊢◇A□O◇O-2→AOO-2→EIO-2→EIO-4→EAO-4

[11] ⊢◇A□O◇O-2→AOO-2→EIO-2→EIO-1→EIO-3→EAO-3

[12] ⊢◇A□O◇O-2→AOO-2→EIO-2→EIO-1→AII-1

[13] ⊢◇A□O◇O-2→AOO-2→EIO-2→EIO-1→AII-1→AII-3

[14] ⊢◇A□O◇O-2→AOO-2→EIO-2→EIO-1→AII-1→IAI-4

[15] ⊢◇A□O◇O-2→AOO-2→EIO-2→EIO-1→AII-1→AII-3→IAI-3

[16] ⊢◇A□O◇O-2→AOO-2→EIO-2→EIO-1→AII-1→AAI-1

[17] ⊢◇A□O◇O-2→AOO-2→EIO-2→EIO-1→AII-1→AII-3→AAI-3

[18] ⊢◇A□O◇O-2→AOO-2→EIO-2→EIO-1→AII-1→IAI-4→AAI-4

[19] ⊢◇A□O◇O-2→AOO-2→EIO-2→EIO-1→AII-1→AII-3→IAI-3→OAO-3

[20] ⊢◇A□O◇O-2→AOO-2→AAA-1

[21] ⊢◇A□O◇O-2→AOO-2→EIO-2→EIO-1→AII-1→AEE-2

[22] ⊢◇A□O◇O-2→AOO-2→EIO-2→EIO-1→AII-1→AEE-2→AEE-4

[23] ⊢◇A□O◇O-2→AOO-2→EIO-2→EIO-1→AII-1→AEE-2→EAE-2

[24] ⊢◇A□O◇O-2→AOO-2→EIO-2→EIO-1→AII-1→AEE-2→AEE-4→EAE-1

Proof:

(1) $\vdash\Diamond all(w, r)\wedge\Box not\ all(b, r)\rightarrow\Diamond not\ all(b, w)$ (i.e. $\Diamond$A$\Box$O$\Diamond$O-2, Axiom A2)

(2) $\vdash all(w, r)\wedge not\ all(b, r)\rightarrow not\ all(b, w)$ (i.e. AOO-2, by (1) and Xu(2023)[16])

(3) $\vdash all(w, r)\wedge no(b, r)\rightarrow not\ all(b, w)$ (i.e. AEO-2, by (2), Fact [4.2] and Rule 1)

(4) $\vdash all(w, r)\wedge no(r, b)\rightarrow not\ all(b, w)$ (i.e. AEO-4, by (3) and Fact [5.2])

(5) $\vdash no\neg(w, r)\wedge some\neg(b, r)\rightarrow not\ all(b, w)$ (by (2), Fact [1.1] and Fact [1.2])

(6) $\vdash no(w, K\neg r)\wedge some(b, K\neg r)\rightarrow not\ all(b, w)$ (i.e. EIO-2, by (5) and Definition D3)

(7) $\vdash no(K\neg r, w)\wedge some(b, K\neg r)\rightarrow not\ all(b, w)$ (i.e. EIO-1, by (6) and Fact [5.2])

(8) $\vdash no(w, K\neg r)\wedge some(K\neg r, b)\rightarrow not\ all(b, w)$ (i.e. EIO-4, by (6) and Fact [5.1])

(9) $\vdash no(K\neg r, w)\wedge some(K\neg r, b)\rightarrow not\ all(b, w)$ (i.e. EIO-3, by (7) and Fact [5.1])

(10) $\vdash no(w, K\neg r)\wedge all(b, K\neg r)\rightarrow not\ all(b, w)$ (i.e. EAO-2, by (6), Fact [4.1] and Rule 1)

(11) $\vdash no(K\neg r, w)\wedge all(b, K\neg r)\rightarrow not\ all(b, w)$ (i.e. EAO-1, by (7), Fact [4.1] and Rule 1)

(12) $\vdash no(w, K\neg r)\wedge all(K\neg r, b)\rightarrow not\ all(b, w)$ (i.e. EAO-4, by (8), Fact [4.1] and Rule 1)

(13) $\vdash no(K\neg r, w)\wedge all(K\neg r, b)\rightarrow not\ all(b, w)$ (i.e. EAO-3, by (9), Fact [4.1] and Rule 1)

(14) $\vdash all\neg(K\neg r, w)\wedge some(b, K\neg r)\rightarrow some\neg(b, w)$ (by (7), Fact [1.4] and Fact [1.2])

(15) $\vdash all(K\neg r, K\neg w)\wedge some(b, K\neg r)\rightarrow some(b, K\neg w)$ (i.e. AII-1, by (14) and Definition D3)

(16) $\vdash all((K\neg r, K\neg w)\wedge some(K\neg r, b)\rightarrow some(b, K\neg w)$ (i.e. AII-3, by (15) and Fact [5.1])

(17) $\vdash all(K\neg r, K\neg w)\wedge some(b, K\neg r)\rightarrow some(K\neg w, b)$ (i.e. IAI-4, by (15) and Fact [5.1])

(18) $\vdash all(K\neg r, K\neg w)\wedge some(K\neg r, b)\rightarrow some(K\neg w, b)$ (i.e. IAI-3, by (16) and Fact [5.1])

(19) $\vdash all(K\neg r, K\neg w)\wedge all(b, K\neg r)\rightarrow some(b, K\neg w)$ (i.e. AAI-1, by (15), Fact [4.1] and Rule 1)

(20) $\vdash all(K\neg r, K\neg w)\wedge all(K\neg r, b)\rightarrow some(b, K\neg w)$ (i.e. AAI-3, by (16), Fact [4.1] and Rule 1)

(21) $\vdash all(K\neg r, K\neg w)\wedge all(b, K\neg r)\rightarrow some(K\neg w, b)$ (i.e. AAI-4, by (17), Fact [4.1] and Rule 1)

(22) $\vdash all(K\neg r, K\neg w)\wedge not\ all\neg(K\neg r, b)\rightarrow not\ all\neg(K\neg w, b)$ (by (18) and Fact [1.3])

(23) $\vdash all(K\neg r, K\neg w)\wedge not\ all(K\neg r, K\neg b)\rightarrow not\ all(K\neg w, K\neg b)$

(i.e. OAO-3, by (22) and Definition D3)

(24) $\vdash\neg not\ all(b, w)\wedge all(w, r)\rightarrow\neg not\ all(b, r)$ (by (1) and Rule 3)

(25) $\vdash all(b, w)\wedge all(w, r)\rightarrow all(b, r)$ (i.e. AAA-1, by (24) and Fact [2.1])

(26) $\vdash\neg some(b, K\neg w)\wedge all(K\neg r, K\neg w)\rightarrow\neg some(b, K\neg r)$ (by (15) and Rule 3)

(27) $\vdash no(b, K\neg w) \wedge all(K\neg r, K\neg w) \rightarrow no(b, K\neg r)$ (i.e. AEE-2, by (26) and Fact [2.4])

(28) $\vdash no(K\neg w, b) \wedge all(K\neg r, K\neg w) \rightarrow no(b, K\neg r)$ (i.e. AEE-4, by (27) and Fact [5.2])

(29) $\vdash no(b, K\neg w) \wedge all(K\neg r, K\neg w) \rightarrow no(K\neg r, b)$ (i.e. EAE-2, by (27) and Fact [5.2])

(30) $\vdash no(K\neg w, b) \wedge all(K\neg r, K\neg w) \rightarrow no(K\neg r, b)$ (i.e. EAE-1, by (28) and Fact [5.2])

As demonstrated above, this section discusses how to derive other classical modal syllogisms, generalized modal syllogisms, generalized syllogisms, and classical syllogisms from the valid classical modal syllogism ◇A□O◇O-2 by using the method of knowledge mining. There are (48+30+15+24=) 117 syllogisms in total derived through related definitions, deductive rules, and facts. This part seeks to inspire further research into the reducible relationships between/among various syllogisms with different figures and forms. What is presented in this section is merely the possibility of knowledge mining for the specific valid syllogism ◇A□O◇O-2, and that of other valid syllogisms remains open for exploration.

5 Conclusion and Future Work

This paper firstly employs knowledge of modal logic, set theory, and generalized quantifier theory to prove the validity of the classical modal syllogism ◇A□O◇O-2 in the context of knowledge representation. Subsequently, by making the best of relevant definitions, facts, and inference rules, this paper formally derives 117 valid different types of syllogisms from this foundational proof. The deductive method yields consistent results throughout. The conclusions drawn in this paper demonstrate the reducible relationships between/among various syllogisms with different figures and forms. Specifically, Theorem 2 uncovers the reducible relationships between/ among classical modal syllogisms with different figures and forms, while Theorem 3-Theorem 5 demonstrate the reducible relationships between/among the classical modal syllogism ◇A□O◇O-2 and the other three types of syllogisms (i.e., generalized modal syllogisms, generalized syllogisms, and classical syllogisms).

This paper is merely an attempt at cross-syllogism research. Further studies are still needed in this area. The proposed method is expected to facilitate new research paradigms to promote cross-syllogism research. It is hoped that this paper provides enlightenment for natural language information processing and knowledge mining in artificial intelligence.

Acknowledgement

This work was supported by the National Social Science Fund of China under Grant No.24FZXB068.

Reference

[1] Moss L S. (2008). Completeness Theorems for Syllogistic Fragments. Hamm F, Kepser S (Eds.) *Log. Linguist. Struc*t, 143-173.

[2] Johnson F. (1989). Models for modal syllogisms. *Notre Dame Journal of Formal Logic*, (30): 271-284.

[3] Hao H W. (2025). Deductibility about the Classical Modal Syllogism □AEE-4. *SCIREA Journal of Philosophy*, 5(1): 18-26.

[4] Endrullis J and Moss L S. (2015). Syllogistic logic with 'most'. V. de Paiva et al. (eds.), *Logic, Language, Information, and Computation*, 124-139.

[5] Fritz P. (2013). Modal ontology and generalized quantifiers. *Journal of Philosophical Logic*, 42(4): 643-678.

[6] Łukasiewicz J. (1957). *Aristotle's Syllogistic: From the Standpoint of Modern Formal Logic (2nd Edition)*. Oxford: Clerndon Press, 1957.

[7] McCall S. (1963). *Studies in Logic and the Foundations of Mathematics, Aristotle's Modal Syllogisms*. Amsterdam: North-Holland Publishing Company.

[8] Thomason S K. (1997). Relational Modal for the Modal Syllogistic". *Journal of Philosophical Logic*, (26): 129-141.

[9] Johnson F. (2004). Aristotle's modal syllogism. *Handbook of the History of Logic*, (1): 247 -338.

[10] Malink M. (2013). *Aristotle's Modal Syllogistic*. Cambridge, MA: Harvard University Press.

[11] Zhang, X J. (2020). Screening out All Valid Classical Modal Syllogisms. *Applied and Computational Mathematic*s, 8(6): 95-104.

[12] Chellas F. (1980). *Modal Logic: an Introduction*. Cambridge: Cambridge University Press.

[13] Halmos P R. (1974). *Naive Set Theo*ry. New York: Springer-Verlag.

[14] Chen B. (2020). *Introduction to Logic (4th Edition)*. Beijing: China Renmin University of Press. (Chinese)

[15] Hamilton A G. (1978). *Logic for Mathematicians*. Cambridge: Cambridge University Press.

[16] Peters S and Westerståhl D. (2006). *Quantifiers in Language and Logic*. Oxford: Claredon Press.

[17] Xu J. (2023). *Progressive Research from Aristotelian Syllogisms to Generalized Modal Syllogisms for Natural Language Information Processing*. Doctoral Dissertation, Anhui University. (Chinese)