SCIREA Journal of Economics

http://www.scirea.org/journal/Economics

July 31, 2022

Volume 7, Issue 4, August 2022

https://doi.org/10.54647/economics79325

IS INVESTING IN SAFETY DETRIMENTAL TO THE FINANCIAL HEALTH OF THE FIRM ?

Nicolas PILUSO

Université Toulouse Paul Sabatier, France

Abstract

The article attempts to analyze the effects of an investment in safety of the production process on the dynamics of the distributed dividends. The aim is to answer the question of whether a risky industry has an incentive to invest in safety independently of the decrease in the probability of accidents generated by the safety measures. Although there is an obvious tension between safety and profitability, it appears that the firm may, in certain cases that we identify, have an interest in investing insofar as capital accumulation and dividend distribution are not durably affected by safety.

Keywords: investment. safety. risk. profit.

Introduction

Investing in safety for a company is costly, but generates, in the more or less long term, numerous benefits, the first of which is obviously the reduction in the probability of accidents. The choice of whether or not to invest in safety, and the level of this investment, are the subject of numerous trade-offs.

In practice, firms reduce the randomness of their economic results through self-protection expenditures (in the sense of Ehrlich and Becker, 1972), self-insurance expenditures and finally insurance.

Prevention expenditures are intended to reduce the probability of an accident, but also to reduce the damage when the risk occurs (self-insurance). Safety investments cover all aspects of prevention (self-protection and self-insurance). They can, for example, aim to make the production technology less dangerous, but also to protect employees and local residents in the event of a risk occurring (Piluso and Rau, 2016). The major industrial risk discussed here is defined as the possibility of an accidental event occurring on an industrial site and resulting in immediate serious consequences for employees, neighboring populations, property or the environment (ministry of ecology, sustainable development and planning , 2007).

Industrialists adopt a risk-based approach and make their decisions on the basis of an efficiency criterion (Chaskiel, 2008, Fiore, 2006): is it not too costly to adopt more secure production techniques in view of the risk reduction they allow? The standard approach to this question is microeconomic. It provides a case-by-case study that consists of evaluating the sum of the benefits and the sum of the costs associated with an investment in security. This method, called "cost-benefit analysis", is advocated by many environmental risk economists.

According to N. Treich (2008), *"CBA (cost-benefit analysis, underlined by us) aims to identify a level of prevention that is effective for civil society. If it is better explained and defined at a public level, it will be more easily reflected and accepted by industry. We see CBA as a decision-making tool that could sometimes be used upstream or as a complement to more social/ethical and engineering approaches. A stronger use of CBA should allow the development of a more objective expertise*.

The guiding principle is simple: a decision is made when the benefits prove to be greater than the costs. The Gollier report (2011) emphasizes the indispensability of the method:

"In a world of limited resources - some much more than others - it is essential to select and prioritize projects that consistently reflect their temporal effects. Not to do so is to accept that part of the community's resources are devoted to investments that could have been of greater use elsewhere, or that we are rashly committing ourselves to actions that have harmful long-term consequences. Furthermore, [industry] must ensure that the expenditures are worthwhile and that the expected benefits of the project are worth the expenditures and costs.... It is the very essence of economic calculation to shed light on the socio-economic efficiency of the

scarce resources mobilized in collective choices.

In this perspective, the microeconomic calculation of the industrialist is to choose the probability of failure (in other words, the level of safety) that minimizes the total cost. He has to make a trade-off between increasing the immediate production costs, through an increase in safety investments, on the one hand, and decreasing the probability of accident on the other hand, which leads to an increase in the expected profit. The optimal probability of failure is that which equals the marginal cost of the safety investment and the expected marginal revenue (Barro, 2006, Farmer, 1977).

In a 2016 paper, Piluso and Rau analyze the determinants of optimal safety investment in a dialogue between industry and local residents[1] .

This optimal security investment depends on three parameters: the firm's economic rate of return, the effectiveness of the investments to be implemented in terms of prevention, and finally the minimum acceptable security investment rate for the residents. The authors show that an increase in the economic rate of return leads to an increase in the safety investment implemented, as does an increase in the minimum acceptable investment rate for the resident. The increase in the rate of return allows the firm to increase internal financing for these investments (see evidence in Appendix 1). Increased cash flow therefore allows the firm to invest more in prevention.

A downward revision of the efficiency of the investments to be implemented (a decrease in the "productivity" of the safety investment) increases the level of safety to compensate for the loss of utility to local residents, but more importantly to compensate for the increased risk of accidents (Piluso and Rau, 2016).

It should also be recalled that this probabilistic approach is also adopted in the basic model of civil liability. The latter puts forward an economic agent whose activity may cause an accident and damage to another agent. However, he can reduce the probability of the accident occurring by his level of effort. Moreover, the agent cannot escape responsibility for two reasons. First, he has the financial capacity to compensate the victims in case of an accident; second, the information about the level of prevention of the agent and the legal standard that is determined *ex post* by the court is perfect. If we adopt all of these hypotheses, the fault-based liability rule and the no-fault liability rule are compared in terms of their ability to

[1] This consultation was institutionalized by the Bachelot law of July 31, 2003 on major technological and natural risks. The framework for this consultation is the "site monitoring commissions", formerly called "local information and consultation committees"

generate sufficient incentives for prevention on the part of the potentially responsible economic agent. Since the social objective is the minimization of the social cost of the agent's activity, the socially optimal effort is the one that minimizes the sum of the cost of prevention and the expectation of damages (Calabresi, 1970; Deffains, 2000; Kambia-Chopin, 2007).

Like any analysis, however, this method has a weakness related to the difficulty economists have in estimating the probabilities of occurrence of future events. It is particularly difficult to specify an accident probability function. The latter is most often the result of empirical studies on frequencies of occurrence, which are exogenous (Levêque, 2013, Gollier, 2004). Moreover, it can be modified according to the experts who are commissioned (ICSI, 2009). In other words, there is by definition no microeconomic basis for the establishment of such a function, which is likely to call into question the confidence that people (industrialists, residents) can place in probabilistic calculations.

Thus, the social science literature shows that local residents perceive a *danger*, while firms and shareholders perceive a *risk* (Luhmann, 1993). According to sociologists, there is an asymmetry of perception between industrialists and local residents, in other words, between those who produce the risk and those who are involuntarily exposed to it: the former carry out a cost-benefit calculation of the installations on the basis of probabilities of occurrence, whereas local residents only perceive the danger represented by the existence of these facilities independently of any probability of occurrence (Chaskiel, 2008, Piluso, 2013). To clarify this analysis, it should be remembered that a firm frequently has the possibility of pooling the risks of several establishments (which makes it possible, in certain cases, to reduce the overall risk). On the other hand, local residents often cannot use this type of pooling and are critical of the threat (or potential loss) associated with the facility. They express, in particular with regard to the "long term" threat, a fear of its diffuse nature, of its capacity to have consequences on future generations. In the shorter term, from a material point of view, the local residents fear the threat to their homes, most often acquired over the long term.

This is why, in addition to this approach, we will try to put aside the probabilistic dimension of the calculations and ask ourselves whether, independently of all the benefits linked more or less directly to security, the costs generated by the security investments really harm the financial health of the company.

The approach proposed here is to focus only on the costs associated with the safety investment, independently of the reduction in the probability of an accident and other benefits resulting from the safety measures. In the same way that local residents only perceive the danger of the installations, independently of the probability of an accident occurring, it is possible to make the hypothesis that the industrialist perceives above all else the immediate cost of a safety investment (Piluso, 2020)[2] . The question that then arises is whether the investment in safety is likely to jeopardize the profit dynamic, and therefore the very existence of the company? Indeed, if we are faced with costs with no expectation of profit, the corollary is a drop in profits, a negative impact on the profit dynamic and a drop in productivity. The *objective of the article is to know if industrialists have an interest in investing in safety, independently of the benefits derived from a greater protection against accidents.*

To answer this question, we have developed a dynamic model tracing the interactions between capital accumulation (i.e. the dynamics of investments) and the distribution of dividends to shareholders (which depend directly on the firm's profits). It will nevertheless be necessary to identify what distinguishes security investment from traditional investment. We shall see that, in certain circumstances, it may be in a firm's interest to invest in security insofar as the dynamics of its profits are not durably affected by the financial shock that this generates.

The interest of the article is precisely to show that even if there is no immediate expectation of profit, the negative shock that constitutes this type of investment does not jeopardize the survival of the firm, nor the distribution of dividends or profits, under certain conditions that we identify. This allows us to reinforce the argument that risky industries can and should invest in safety for the good of all, if certain economic conditions are met.

In the first part of the paper, we set out the main lines of reasoning. We then introduce, without developing it, the economic model put forward to link security investments and company profits. Using different case studies, we show how, according to this model, it is in a company's interest to invest in security in the sense that its profits will not be durably affected. At the end of the article, the interested reader will find a precise presentation of the mathematical model on which the described approach is based.

Does the security investment jeopardize the company's profits? A first approach

[2] It should be remembered that frequently an industrialist can mutualize risks with the ownership of several facilities (Piluso, 2020).

1- Investments in security have a negative impact on profitability

No one will ever dispute that security investments represent a cost that the company must bear. There is thus an obvious tension between security and profitability.

"As soon as safety has a productively inefficient dimension, since it represents everything that could be removed without lowering the performance of a workshop, it has a negative influence on financial results, causing specific tensions within company management, between industrialists and financiers, or between industrialists themselves depending on whether a commercial or technical approach is favored" (Chaskiel, 2008).

Security investments can penalize the company doubly because of:

- a weakening of productivity: the firm increases its capital but this is not intended to produce more but to secure it. As a consequence, more capital does not produce more, and the ratio between the quantity produced and the quantity of capital decreases.
- an increase in the rate of capital depreciation: every year, some of the company's equipment depreciates, either because of wear and tear or because of obsolescence caused by technological progress. Investing in new, more secure equipment can lead to a sudden depreciation of the equipment already installed.

Security investment is thus perceived here as an accumulation of capital leading to a negative shock on the rate of depreciation of capital and/or productivity. However, it seems that an accelerated depreciation of existing capital and/or a fall in productivity has a largely negative effect on dividends, which would radically dissuade managers from undertaking such a security investment.

2- But the negative financial shock can be cushioned over time

In this paper, we attempt to shed new light on the consequences of an increase in the rate of capital depreciation or a decline in productivity on dividend dynamics. The elements that we will introduce in the course of this paper are likely to relativize the point of view presented above.

In fact, security investments have many similarities with the company's advertising expenses. They are not intended to increase the company's production capacity or its productivity, i.e. its productive efficiency. Although they incur costs, they increase potential sales because they help attract new customers to the goods or services sold by the company. In the same way, security investment is not a productive investment, it represents a cost for the company, but it allows to guarantee the continuity of the company's activity. In this way, it secures the shareholder's income: an activity that is safer for the population and the environment is also the assurance of more secure profits.

The reason is that the economic activity of a company is not reduced to a static creation of wealth, over a given and finite period. It creates a flow of wealth in continuous time. In other words, if a firm experiences an increase in its production costs at time t, and thus a decline in profits, a new flow of wealth will be created at time t+1, which will at least partially restore profits. The process continues at time t+2, so that the increase in costs is eventually amortized. In the end, it can be said that the short-term loss of profit generated by the security investment is offset by the potential gain realized over the duration of its activity.

Continuing this line of reasoning, we see that the firm is able to generate a cost-reducing flow of wealth if the productivity of its production apparatus (how much wealth is generated on average by each unit of capital input) is greater than the cost of that apparatus, i.e. the interest rate at which the firm pays its loan (which is used to acquire the capital) plus the rate of depreciation. The safety investment decreases productivity and/or increases the cost of the depreciation rate, but as long as the spread is positive, the firm can continue to make money. The ongoing process of wealth creation eventually amortizes the costs incurred. A company that invests in safety will prefer less profit in each production period (i.e. in the short term), but these profits are more secure. Therefore, it will earn more profits over the long term.

We propose here both a problematic and an approach different from the microeconomic approach. It is no longer a question of knowing whether a decision is efficient or not, i.e. whether it leads to a waste of resources or not. It is more simply a question of knowing whether the safety investment, which is indispensable from the moment there is a danger, calls into question the existence of the industry and its jobs. We will now develop our model without going into the details of the mathematical formalization.

Does security investment put company profits at risk? A presentation of the main results of the model

As indicated at the beginning of the document, it is possible to affirm that the adoption of new technologies in the field of industrial security accelerates the depreciation of the capital and/or decreases its productivity within the company. Indeed, in terms of security investments, two possibilities are possible:

- or securing involves the acquisition of new equipment that does not call into question the production technology and the existence of the equipment already installed (*type 1 investments*); in this case, such an acquisition constitutes an accumulation of capital that does not allow any increase in the volume of production; as a result, **the productivity of the capital as a whole** (the ratio between production and the quantity of capital that was necessary to obtain it) **decreases**;
- or securitization calls into question the production technology and equipment in place (*type 2 investments*); it therefore implies a premature obsolescence of the installed capital. The rotation of capital[3] accelerates while productivity also decreases. The **result is both a fall in productivity and an increase in the rate of capital depreciation**, i.e. the speed at which capital wears out and/or becomes obsolete.

In the face of this tension between profitability and security, the **question that arises is whether the shock generated by security investment is detrimental to the company's continued activity.** To answer this question, it seems important to analyze the effect of a safety investment on the dynamics of profits or, more precisely, on the dividends distributed to shareholders.

The following development therefore introduces a dynamic model of the "prey-predator" type[4] **that makes it possible to quantify the effects of a depreciation and/or a decline in capital productivity**. This type of model makes it possible to formalize the interaction between firms and shareholders: the latter take a share of the wealth created by the firm, while also contributing to its financing. The present model does not pretend to model risk and security investment as such; it describes a continuous process of capital accumulation. If during accumulation a safety investment is made, a shock to productivity and/or the rate of depreciation occurs. Security investment is thus understood here through its effects.

Two configurations will be analyzed. First, the analysis will focus on a representative firm with a capital input whose returns are decreasing (i.e. the increases in production resulting from increases in capital are increasingly small). In a second step, we will represent a firm whose production function is similar to that of the "AK" model (Rebelo, 1991: returns are constant, in the sense that production and capital increase in the same proportions). In the first

3 See definition in the following box.
4 Lotka-Volterra type model (Bacaer, 2008)

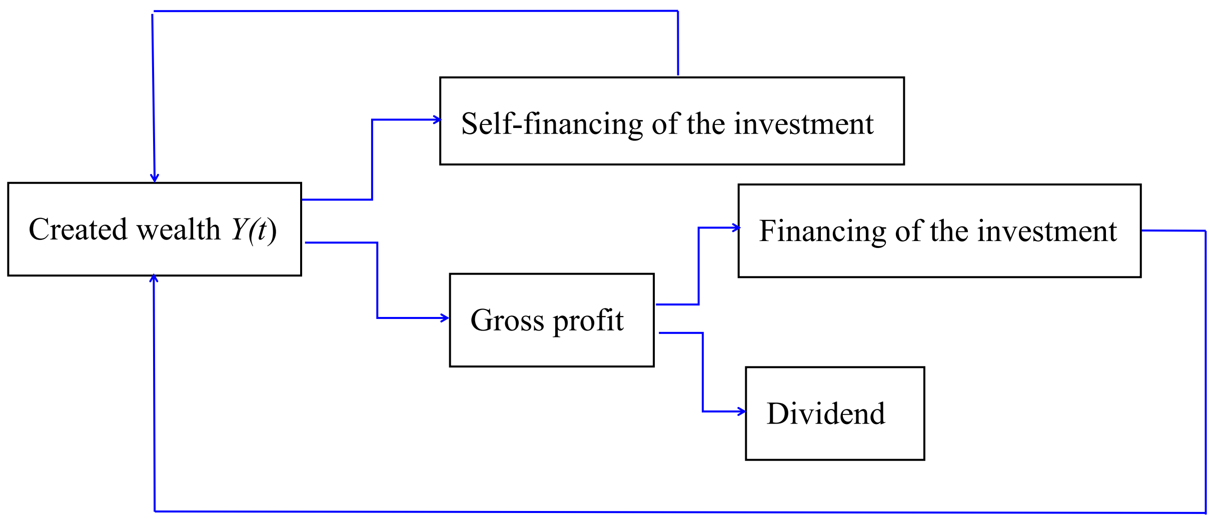
case, it is not possible to obtain an analytical solution and only numerical simulations are possible. In the second case, it is perfectly possible to solve the model.

1 Case of a firm with decreasing returns

Consider an economy composed of a representative firm and a financial market that allows its financing. An assembly of shareholders owns the firm. The latter are remunerated in proportion to the firm's results. Nevertheless, they must also participate in the financing of productive investment. Thus, the final dividend to the shareholder is equal to the difference between the gross profit and the contribution to the financing of the capital[5] .

The model describes an economy in which a representative firm produces wealth continuously by acquiring capital. This wealth is either distributed to shareholders (or to employees, in the form of profit-sharing), or capitalized in order to acquire new equipment (self-financing of the investment). The shareholders can either participate in the financing of the investment or consume their income.

We are thus in the presence of the following circuit:



The higher the productivity of capital, the higher the level of production, and therefore of wealth created.

The continuous production of wealth depends on the rate of accumulation of the production factor "capital". The more resources the firm devotes to self-financing and the more shareholders participate in financing the investment, the greater the accumulation. It also

[5] If the legal form of the firm does not imply the existence of shareholders, one can assume that profit remains split between profit reinvested in the productive activity and profit distributed (to employees or partners).

depends on a third factor: the rate of depreciation of capital. This rate, as mentioned earlier, indicates the rate at which existing capital must be renewed in order to keep production capacity constant.

It is immediately clear that in the model, an increase in productivity and a decrease in the rate of depreciation lead to an increase in the equilibrium value of accumulated capital. In other words, the equilibrium value of capital (i.e. that for which the model reproduces itself identically over time) depends positively on its productivity and negatively on its rate of depreciation.

On the other hand, savings in circulation on the financial market (i.e., the net profit from financing investment) depend, in equilibrium, on the rate of depreciation of capital but not on productivity. Indeed, it appears that an increase in the rate of depreciation leads to a greater fall in gross dividends than the fall in investment to be financed. On the other hand, in the case of a fall in productivity, the distribution of dividends and the amount of investment to be financed fall in the same proportions.

The consequences of such an outcome are significant. They mean that:

- when a safety investment implies a decrease in productivity without an increase in the rate of depreciation (*type 1 investment which does not call into question the existing installations*), the net value of the profit at equilibrium is not affected, although the stationary value of the accumulated capital decreases; the firm therefore has every interest in undertaking such a safety investment to reduce the probability of an industrial accident
- when, on the contrary, the security investment implies an increase in capital turnover (*type 2 investment that calls into question existing facilities*), the dividends paid out decrease on a permanent basis, so that the company may fear that its stock price will decrease[6] .

According to our model, when the returns to production are decreasing, it is always possible to make type 1 investments without jeopardizing the ability of firms to generate profits over time.

2 Case of a firm with constant (or increasing) returns

After solving the constant efficiency model (not shown here), it is appropriate to identify three

[6] In the model, we do not take into account the fact that the shareholder receives income when he obtains a capital gain when he resells his stock, for the sake of simplification.

possible configurations:

- situation A where the productivity of capital is higher than its rate of depreciation;
- situation B where the two quantities are equal;
- situation C where the productivity of capital is lower than its depreciation rate.

In situation C, by definition, capital depreciates faster than its productivity increases. No project to secure the production site can be implemented as soon as the depreciation of capital is too high. In such a configuration, capital accumulation is bound to decrease, bringing with it a fall in the overall product. The loss of capital due to depreciation cannot be compensated for by additional product, given the low productivity.

In configuration B, the firm can no longer accumulate capital to the extent that it depreciates at the same rate as productivity. It is therefore inconceivable that the firm would make a securing investment such that the rate of depreciation and the productivity of capital are equalized.

Situation A is the most interesting. It shows that in the long run, capital accumulation and dividends grow indefinitely. Nevertheless, it reveals a phase of decreasing capital accumulation and dividend in the short run, the longer the differential [productivity/depreciation rate] is reduced. In other words, when the rate of capital depreciation increases for a given productivity, the firm faces a phase of reduced capital accumulation and reduced dividend payouts in the short run, which is longer the higher the rate of depreciation increases:

- when the differential between productivity and the rate of depreciation is large and positive, accumulation follows a constantly increasing trajectory;
- a reduction in the productivity/depreciation rate differential inflects the growth curve.

This result is easy to understand: when a firm suffers a depreciation shock, the volume of production and dividends distributed decreases; the firm must accumulate capital to compensate for the depreciation, which takes time. The accumulation and thus the compensation of the "depreciation effect" will be all the faster the higher the productivity of capital.

To the extent that returns are constant, a securitization investment that creates a shock to productivity and/or the rate of depreciation similarly decreases the accumulated capital and the net dividend distributed. Nevertheless, it is important to stress that this decrease is only temporary. Indeed, we observe that accumulation and dividend distribution eventually increase once the effects of the shock have been absorbed.

It is therefore perfectly possible for a company to decide to undertake a type 1 or type 2 security investment without permanently jeopardizing its profitability prospects.

However, shareholders are often particularly sensitive to short-lived developments (Aglietta and Rébérioux, 2014); these have a significant impact on the dynamics of stock prices. When competition pushes firms to innovate and introduce new equipment, they reduce the effective life of capital, which leads to an increase in the depreciation rate. Nevertheless, this depreciation shock is cushioned by an increase in the productivity of the capital that precisely justified such innovation. In the case of high-risk industries, the actual implementation of innovative security technologies is not aimed at increasing productivity. These security investments have two opposing effects on profit prospects:

- they reduce the probability of an industrial disaster, and therefore of a sudden drop in profit and production levels;
- in the short term, they penalize the possibilities of accumulation and growth of the dividend.

Nevertheless, security investments do not jeopardize the existence of the company and the possibilities of increasing dividends in the medium/long term as long as the differential between marginal productivity of capital and the rate of depreciation remains positive after investment.

Conclusion

Industries at risk must constantly balance the need to secure their facilities with the need to maintain or increase profitability through investment.

It is possible to examine the problem in two distinct ways: either from a microeconomic point of view, by examining the way in which decisions are made (Guesnerie, 2004), or from a macroeconomic point of view, by analyzing the possible structural problem posed to companies by security investments.

The two approaches may be complementary, but they certainly lead to completely different diagnoses. From a microeconomic point of view, it may turn out that the decision-making method used by industry is not satisfactory, or, more precisely, insufficiently efficient. This is suggested by technology risk economists (Gollier, 2011, Treich, 2006, Rushton, 2006) who promote cost-benefit analysis. Our macroeconomic analysis suggests a more structural problem: any calculation method can potentially serve as a justification for not investing in security. These only mask short-term profitability issues. We have seen that the particularity of security investments is that they accelerate the depreciation of capital and existing technologies and/or put downward pressure on productivity. The effects of such a shock are such that the decision not to invest in security is not inevitable. In a situation of diminishing returns, safety investments that reduce productivity alone do not lead to a lasting reduction in net dividends. In a situation of constant returns, these same investments only lead to a temporary decrease in distributed profits, so that in the long term, the latter continue to grow.

Thus, it is possible to say that in certain configurations, companies have all the latitude to implement safety investments since they do not have to fear a lasting decline in their stock price. The determining factor in the decision is then the manager's degree of preference for the present: how much importance does he attach to the limited and temporary decline in dividend distribution that the investment induces?

Appendix 1: The Mathematical Model

1) Introduction

Let the following matrix system X be :

$$\begin{pmatrix} \dot{s} \\ \dot{K} \end{pmatrix} = A \begin{pmatrix} s \\ K \end{pmatrix} \text{ with } A = \begin{pmatrix} -\gamma & \rho\beta \\ \gamma & \rho(1-\beta)-\delta \end{pmatrix}$$

To find the eigenvectors of the matrix, it is necessary to calculate the characteristic polynomial P of A :

$$P(\lambda) = \begin{vmatrix} -\gamma-\lambda & \rho\beta \\ \gamma & \rho(1-\beta)-\delta-\lambda \end{vmatrix} = \lambda^2 - (\rho(1-\beta)-\delta-\gamma)\lambda - \gamma(\rho-\delta).$$

The determinant of the polynomial P is given by :

$$\Delta = (\rho(1-\beta)-\delta-\gamma)^2 + 4\gamma(\rho-\delta) \text{ or :}$$

$$\Delta = (\gamma+\rho(1-\beta)-\delta)^2 + 4\gamma\rho\beta$$

is not negative since $\gamma>0$ and $\rho\beta>0$.

Case where Δ *=0*

$\frac{\rho(1-\beta)-\delta-\gamma}{2}$ is the only solution in this configuration. The matrix *A* is not diagonalizable but triangularizable. It is possible to solve the system explicitly in this basis. Two solutions are possible:

-if γ=0 and ρ(1-β)=δ which corresponds to a situation where the financial market does not finance the real sphere (see Figure A). The system becomes:

$$\begin{cases} \dot{s}(t) = \rho\beta K(t) \\ \dot{K}(t) = 0 \end{cases}$$

and the solution is given by :

$$\begin{cases} s(t) = s_0 + \rho\beta K_0 \\ K(t) = K_0 \end{cases}$$

-if β=0 and γ+ρ=δ, all the aggregate proceeds are captured by the financial market (Figure B). The system becomes:

$$\begin{cases} s(t) = -\gamma s(t) \\ K(t) = -K(t) + \gamma s(t) \end{cases}$$

and the solution is given by

$$\begin{cases} s(t) = s_0 e^{-\gamma t} \\ K(t) = (K_0 + \gamma s_0) e^{-\gamma t} \end{cases}$$

Case where Δ *>0*

The characteristic polynomial has two distinct roots:

$$r_+ = \frac{\rho(1-\beta)-\delta-\gamma+\sqrt{\Delta}}{2} \; , \; r_+ = \frac{\rho(1-\beta)-\delta-\gamma-\sqrt{\Delta}}{2}$$

To find a system of eigenvectors, we must write that the coordinates (x, y) of an eigenvector associated with r+ satisfies : $(-\gamma - r_+)x + \rho\beta y = 0$, which leads for example to the vector $v_+(\rho\beta, \gamma + r_+)^T$. In the same way, an eigenvector associated to r- can be chosen : $v_- = (b, \gamma + r_-)^T$. It is thus possible to write that (x(t), y(t)) is the solution of the differential system *X* in the basis (v+,v). This satisfies :

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} r_+ & 0 \\ 0 & r_- \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

The solution is given by :

$$\begin{cases} x(t) = x_0 e^{r_+ t} \\ y(t) = y_0 e^{r_- t} \end{cases}$$

Where x_0 and y_0 are respectively the values of x and y at time t0. To give information about the solutions of the form (s(t), K(t)), we need to examine the solutions of the form (x(t), y(t)). For this, it is possible to write y(t) as a function of x(t) :

$$y(t) = \frac{y_0}{x_0^{r_- / r_+}} x(t)^{r_- / r_+}$$

The solutions depend on the value of r_- / r_+ .

Cases where $\rho(1-\beta)-\delta \geq \gamma$

In this configuration, it is easy to see that r+ is positive and that r-<r+. We can then distinguish the following situations:

(a) $r_- < 0$: the solutions (x(t), y(t)) have a hyperbolic form

(b) $r_- > 0$: the solutions have a parabolic shape

(c) $r_- = 0$: the solutions are given by:

$$\begin{cases} x(t) = x_0 e^{r_+ t} \\ y(t) = y_0 \end{cases}$$

Case where $\rho(1-\beta)-\delta \leq \gamma$. In this situation, $r_- < 0$ and the sign of r_+ must be studied. Three possibilities can be identified:

(d) $r_+ > 0$ .The solution has a hyperbolic form as in the previous case (a).

(e) $r_+ < 0$ The solution has a parabolic form as in the previous case (b).

(f) $r_+ = 0$ The solution is given by: $\begin{cases} x(t) = x_0 \\ y(t) = y_0 e^{r_- t} \end{cases}$

Now it is possible to observe the solutions in the original basis (s(t), K(t)) since we have found a system of eigenvectors $v_+(\rho\beta, \gamma + r_+)^T$ and $v_+(\rho\beta, \gamma + r_+)^T$.

2) Decreasing productivity model

Consider the following differential system:

$$\begin{cases} Y(t) = f(K(t)) \\ \dot{s}(t) = \beta Y(t) - \gamma s(t) \\ \dot{K}(t) = (1-\beta)Y(t) + \gamma s(t) - \delta K(t) \end{cases}$$

where *Y(t)* denotes the level of output per capita in volume at time *t*, *s(t)* the amount of the dividend at time *t*, *K(t)* the physical capital per capita used in *t*. The function *f* is increasing and has the usual Cobb Douglas properties.

The coefficients are as follows: γ is the share of the gross dividend that provides financing for productive investment. The rest of the dividend (1-γ) goes to the shareholder. β is the share of the created wealth used to finance the shareholder's gross dividend. The rest of the wealth created 1- β is devoted to self-financing investment. δ is the rate of capital depreciation.

It is impossible to give this system an explicit solution. On the other hand, it is possible to carry out numerical simulations by giving a value for each parameter (figure 1). Moreover, the value of the stationary equilibrium points is given by :

$$\begin{cases} K(t) = \left(\dfrac{\rho}{\delta}\right)^{\frac{1}{1-\alpha}} \\ s(t) = \left(\dfrac{\gamma}{\beta\delta}\right)^{\frac{\alpha}{1-\alpha}} \end{cases}$$

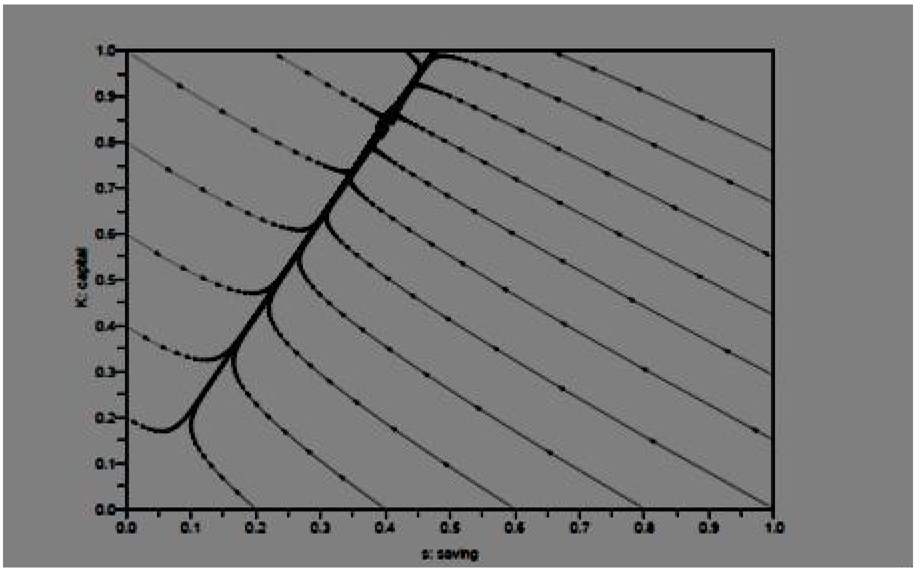


Figure 1

Solution (s, K) of system (1) for α=0.1, γ=0.2, ρ=0.5, β=0.7 and δ=0.4

Figure 1 shows that the amount of capital decreases and then increases to reach a stationary equilibrium value. The dividend paid out grows all along the path to its stationary value. The lower the equilibrium point, the higher the rate of capital depreciation (decrease in equilibrium accumulated capital). On the other hand, the value of the rate of depreciation does not affect the equilibrium position of the net dividend distributed. A safety investment that only affects productivity therefore has no lasting impact on the firm's profits.

2)-Constant productivity model

As in the "AK" model of Rebelo (1991), we assume here that the factor return is constant. The production function becomes: Y(t)=ρK(t) , giving the following differential system:

$$\begin{cases} \dot{s}(t) = \rho\beta K(t) - \gamma s(t) \\ \dot{K}(t) = (\rho(1-\beta) - \delta)K(t) + \gamma s(t) \end{cases}$$

After solving the system in the eigenvector basis (see mathematical appendix), it is possible to identify the solutions of the system (2) in the basis (s(t), K(t)). Three configurations emerge:

-Situation A where ρ>δ (marginal productivity of capital is greater than the rate of depreciation);

-the situation B where ρ=δ ;

-the situation C where ρ<δ.

As these configurations have already been explained, we will limit ourselves to presenting the numerical simulations that we have carried out:

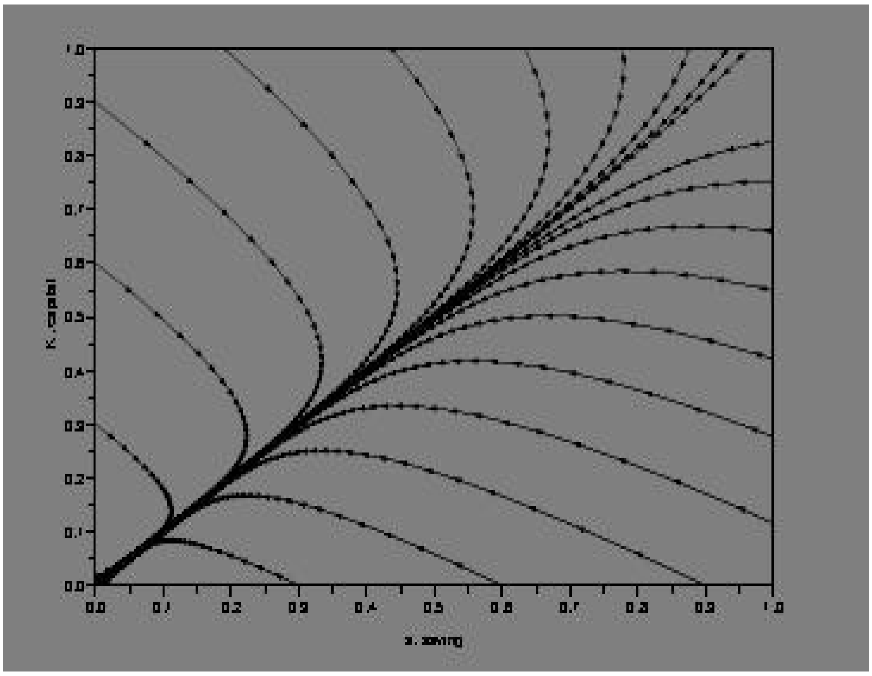


Figure 2: the situation C

Solution (s, K) of system (2) for γ=0.2, ρ=0.5, β=0.7 and δ=0.7

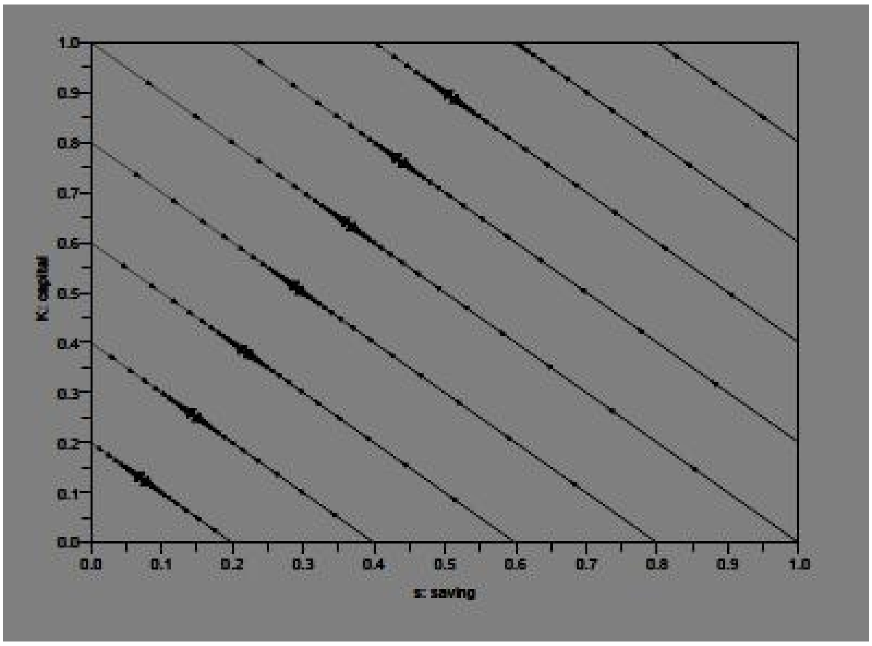


Figure 3: Situation B

Solution (s, K) of system (2) for γ=0.2, ρ=0.5, β=0.7 and δ=0.5

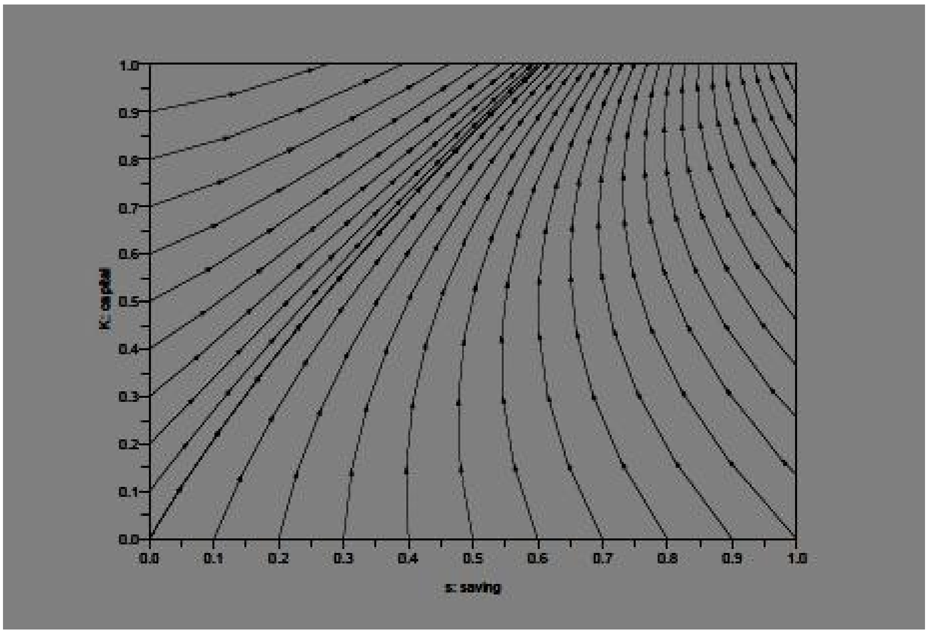


Figure 4: Situation A

Solution (s, K) of system (2) for γ=0.2, ρ=0.5, β=0.7 and δ=0.1

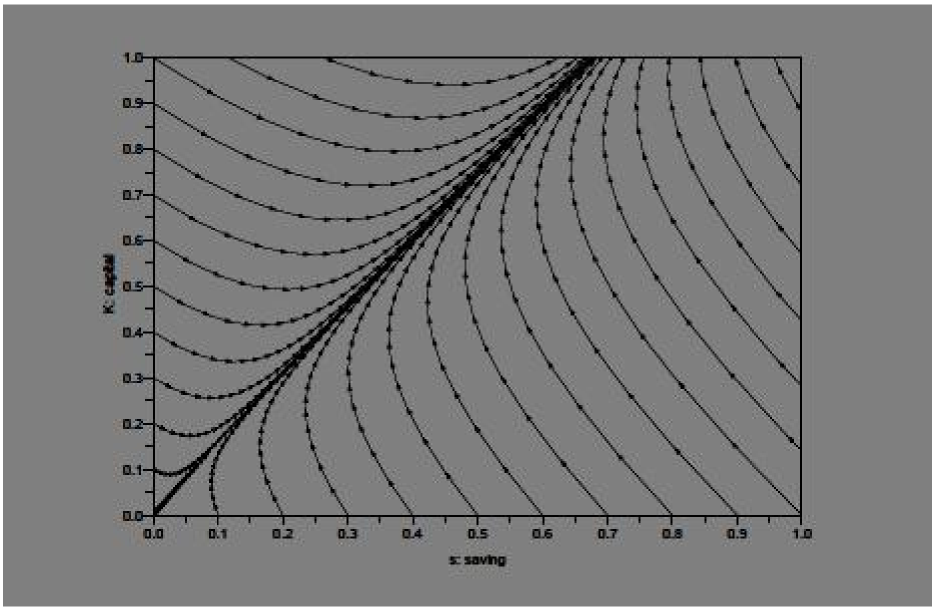


Figure 5: Situation A'

Solution (s, K) of system (2) for γ=0.2, ρ=0.5, β=0.7 and δ=0.3

Bibliography

[1] Aglietta M., Rébérioux A., 2014, Les dérives du capitalisme financier, Albin Michel, Paris.

[2] Bacaer N., 2008, Histoires de mathématiques et de populations, Éditions Cassini, coll. " Le sel et le fer ", Paris.

[3] Barro R.J, 2006, "Rare disasters and asset markets in the twentieth century", The Quarterly Journal of Economics, August, pp.823-866.

[4] Calabresi G (1970) *The Cost of Accident*, New Haven, Yale University Press.

[5] Chaskiel P., 2008, " Luhmann et le mystère du risque technologique, Un retour de l'opinion publique ? ", *Réseaux*, mai, n°151.

[6] Deffains B., (2000) L'évaluation des règles de droit : un bilan de l'analyse économique de la responsabilité, *Revue d'Économie Politique*, 110(6), pp. 751-785.

[7] Ehrlich, Isaac & Becker, Gary S, 1972, "Market Insurance, Self-Insurance, and Self-Protection," *Journal of Political Economy*, University of Chicago Press, vol. 80(4), pp 623-48.

[8] Etnet J., 2006, "Les risques associés à l'énergie nucléaire et leurs perceptions", Economie et Société, Série Economie de l'énergie, pp. 10-11.

[9] Farmer, F.R, 1977, "Today's risks: thinking the unthinkable," Nature, 267, pp. 92-93.

[10] ICSI, 2009, " Etudes de danger et ouvertures au public ", Cahiers de la sécurité industrielle, February.

[11] Fiore K, 2006, " Irréversibilité Mixte, valeur d'option et modularité : une application à l'investissement électronucléaire ", Economy and Society, Energy Economics Series, pp. 10-11.

[12] Gollier C, 2011, "Le calcul du risque dans les investissements publics", Rapport du Conseil d'analyse stratégique, June.

[13] Gollier C., 2007, " Comment intégrer le risque dans le calcul économique ", Revue d'économie politique, vol 117, n°2, pP.209-223.

[14] Guesnerie R. (2004), " Calcul économique et développement durable ", Revue économique, vol. 55, n° 3, May, p. 363-382.

[15] Kambia-chopin D (2007) Civil liability rules and environmental risk prevention. *Revue économique* 2(117), pp. 285-308.

[16] Levêque F (2013) Le risque d'accident nucléaire majeur : calcul et perception des probabités. *working paper 13-ME-02*, CERNA, Mines, Paris.

[17] Luhmann, N., 1993. *Risk. A Sociological Theory,* New-York, Walter de Gruyter.

[18] Ministère de l'Écologie, du Développement et de l'Aménagement durables, 2007, *Le risque industriel*, Dossier informatif.

[19] Piluso N., Rau C., 2020, "The effects of the consultation with residents on the prevention of industrial risks", *American Journal Of Management*, forthcoming

[20] Piluso N., Rau C. , 2016, " Les investissements de sécurisation des sites industriels à risque et la concertation entre firmes et riverains: une approche théorique ", *Région et Développement*, n°64.

[21] Piluso N., 2013, "Technological choices of firms and contested consumption equilibria", *Revue d'économie industrielle, n°143, 3rd quarter.*

[22] Rebelo S, 1991, "Long-Run Policy Analysis and Long-Run Growth," Journal of Political Economy, 99(3), pp. 500-521.

[23] Rushton A, 2006, "CBA, ALARP and industrial safety in the United Kingdom," *Cost Benefit Risk Analysis,* April. http://www.icsi-eu.org/francais/news/2006/04/journee-ACB/ACB-10-questions.pdf.

[24] Solow R, 1956, "A contribution to the theory of economic growth", *Quarterly Journal of Economics*, vol 70, pp 65-94.

[25] Treich N, 2008, " L'analyse coût-bénéfice de la prévention des risques ", Les Cahiers de la Sécurité Industrielle, ICSI, March.