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Knowledge Representation Based on the Aristotelian Modal Syllogism □AE◇E-2

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Abstract:

This paper firstly presents knowledge representations of Aristotelian modal syllogisms based on the structure of modal categorical propositions, and proves the validity of the syllogism □AE◇E-2 by using set theory, modal logic, Aristotelian quantifiers theory and so on, and then illustrates that the other 30 valid syllogisms are derived from □AE◇E-2. In other words, it shows that there are reducible relationships between/among them. Owing to Aristotelian quantifiers (that is, *all*, *some*, *no*, *not all*) can be mutually defined, as well as so can the possible modality (◇) and necessary modality (□), there are reducible relationships between/among valid Aristotelian modal syllogisms. This formal study not only provides new insights for knowledge mining in artificial intelligence, but also provides ideas for studying modern logic.

Keywords: Aristotelian modal syllogisms, Aristotelian quantifiers, reducible relationship, possible worlds

1.Introduction

Syllogistic reasoning, as a form of reasoning, plays an important role in human society and human thinking. In natural language, there are many types of syllogisms, such as Aristotelian syllogisms [1], Aristotelian modal syllogisms [2], generalized syllogisms [3], rational syllogisms [4], and so on. This paper mainly focuses on Aristotelian modal syllogisms and discusses the reducible relationships between the Aristotelian modal syllogism □AE◇E-2 and other Aristotelian modal ones. Unless otherwise specified, the following syllogisms refer to Aristotelian modal syllogisms.

Since Aristotle, scholars such as McCall [5], Thomas [6], Johnson [7], Malink [8], Xiaojun [9] have mainly studied the validity of Aristotelian modal syllogisms, but there are inconsistency in their results. This paper aims to study the reducible relationships between/among the syllogism □AE◇E-2 and other valid syllogisms. Specifically, it utilizes relevant knowledge to infer the validity of other syllogisms from that of □AE◇E-2.

2.Knowledge Representation about Aristotelian modal syllogisms

In the following, let *r*, *v* and *z* be lexical variables, *D* be the domain of lexical variables, *Q* be any of the four Aristotelian quantifiers (namely, *all*, *some*, *no*, and *not all*). Let π, λ, θ and β be propositions. '⊢π' shows that π is provable. '$\lambda =_{def} \theta$'stands for that λ is defined by θ. Others are similar.

Aristotelian syllogisms contain the following four types: 'all *r*s are *z*s', 'no *r*s are *z*s', 'some *r*s are *z*s', and 'not all *r*s are *z*s', which can be respectively expressed as follows: *all*(*r*, *z*), *no*(*r*, *z*), *some*(*r*, *z*), and *not all*(*r*, *z*). They are respectively noted as Proposition *A*, *E*, *I* and *O.*

Aristotelian modal syllogisms are obtained by adding modalities to Aristotelian syllogisms, and they have at least one possible modality (◇) or necessary one (□) and at most three ones. For instance, the Aristotelian syllogism *AEE-2* adds one □ and one ◇ to obtain the modal syllogism □AE◇E-2. The modal syllogism □AE◇E-2 denotes 'all *z*s are necessarily *v*s, and no *r*s are *v*s, so no *z*s are possibly *r*s', which can be formalized as □*all*(*z, v*)∧*no*(*r, v*)→◇*no*(*r, z*).

An instance of the Aristotelian modal syllogism □AE◇E-2 is as follows:

Major premise: All fishes are necessarily aquatic animals.

Minor premise: No birds are aquatic animals.

Conclusion: No birds are possibly fishes.

Let r be the lexical variable for a fish in the domain, v be the lexical variable for an aquatic animal in the domain, and z be the lexical variable for a bird in the domain. Then, this example of syllogism can be formalized as $\Box all(z, v) \wedge no(r, v) \rightarrow \Diamond no(r, z)$, which can be abbreviated as □AE◇E-2. The others are similar.

2.1 Relevant Definitions

Definition 1 (inner negation): $Q\neg(r, z) =_{def} Q(r, D-z)$.

Definition 2 (outer negation): $(\neg Q)(r, z) =_{def}$ It is not that $Q(r, z)$.

Definition 3 (truth value definition):

(3.1) $all(r, z)$ is true just in case $R \subseteq Z$ is true in any real world.

(3.2) $some(r, z)$ is true just in case $R \cap Z \neq \varnothing$ is true in any real world.

(3.3) $no(r, z)$ is true just in case $R \cap Z = \varnothing$ is true in any real world.

(3.4) $not\ all(r, z)$ is true just in case $R \not\subseteq Z$ is true in any real world.

(3.5) $\Box all(r, z)$ is true just in case $R \subseteq Z$ is true in any possible world.

(3.6) $\Diamond all(r, z)$ is true just in case $R \subseteq Z$ is true in at least one possible world.

(3.7) $\Box some(r, z)$ is true just in case $R \cap Z \neq \varnothing$ is true in any possible world.

(3.8) $\Diamond some(r, z)$ is true just in case $R \cap Z \neq \varnothing$ is true in at least one possible world.

(3.9) $\Box no(r, z)$ is true just in case $R \cap Z = \varnothing$ is true in any possible world.

(3.10) $\Diamond no(r, z)$ is true just in case $R \cap Z = \varnothing$ is true in at least one possible world.

(3.11) $\Box not\ all(r, z)$ is true just in case $R \not\subseteq Z$ is true in any possible world.

(3.12) $\Diamond not\ all(r, z)$ is true just in case $R \not\subseteq Z$ is true in at least one possible world.

2.2 Relevant Facts

F1 (inner negation):

(1.1) $\vdash all(r, z) \leftrightarrow no\neg(r, z)$;　　(1.2) $\vdash no(r, z) \leftrightarrow all\neg(r, z)$;

(1.3) $\vdash some(r, z) \leftrightarrow not\ all\neg(r, z)$; (1.4) $\vdash not\ all(r, z) \leftrightarrow some\neg(r, z)$.

F2 (outer negation):

(2.1) $\vdash \neg not\ all(r, z) \leftrightarrow all(r, z)$; (2.2) $\vdash \neg all(r, z) \leftrightarrow not\ all(r, z)$;

(2.3) $\vdash \neg no(r, z) \leftrightarrow some(r, z)$; (2.4) $\vdash \neg some(r, z) \leftrightarrow no(r, z)$.

F3 (dual): (3.1) $\vdash \neg \Box Q(r, z) \leftrightarrow \Diamond \neg Q(z, r)$; (3.2) $\vdash \neg \Diamond Q(r, z) \leftrightarrow \Box \neg Q(z, r)$.

F4 (symmetry): (4.1) $\vdash some(r, z) \leftrightarrow some(z, r)$; (4.2) $\vdash no(r, z) \leftrightarrow no(z, r)$.

F5 (subordination):

(5.1) $\vdash \Box Q(r, z) \rightarrow Q(r, z)$; (5.2) $\vdash \Box Q(r, z) \rightarrow \Diamond Q(r, z)$;

(5.3) $\vdash Q(r, z) \rightarrow \Diamond Q(r, z)$; (5.4) $\vdash all(r, z) \rightarrow some(r, z)$;

(5.5) $\vdash no(r, z) \rightarrow not\ all(r, z)$.

According to modal logic [10], generalized quantifier theory [11], the above facts can be proved.

2.3 Relevant Inference Rules

R1 (subsequent weakening): From$\vdash(\pi \wedge \beta \rightarrow \lambda)$ and $\vdash(\lambda \rightarrow \theta)$ infer $\vdash(\pi \wedge \beta \rightarrow \theta)$.

R2 (anti-syllogism): From $\vdash(\pi \wedge \beta \rightarrow \lambda)$ infer $\vdash(\neg \lambda \wedge \pi \rightarrow \neg \beta)$.

R3 (anti-syllogism): From $\vdash(\pi \wedge \beta \rightarrow \lambda)$ infer $\vdash(\neg \lambda \wedge \beta \rightarrow \neg \pi)$.

3. Reducible Relationships between the Other 30 Valid Syllogisms and □*AE*◇*E-2*:

The following Theorem 1 is a proof for the validity of the syllogism □*AE*◇*E-2*. '□*AE*◇*E-2* →□*AE*◇*E-4*' in Theorem 2 indicates that the validity of syllogism □*AE*◇*E-4* can be inferred from that of the syllogism □*AE*◇*E-2*. In other words, there is a reducible relationship between them. The same goes for others.

Theorem 1 (□*AE*◇*E-2*): $\Box all(z, v) \wedge no(r, v) \rightarrow \Diamond no(r, z)$ is valid.

Proof: According to the above, □*AE*◇*E-2* is a short form of the second figure syllogism

□*all*(*z, v*)∧*no*(*r, v*)→◇*no*(*r, z*). Suppose that □*all*(*z, v*) and *no*(*r, v*) are true, then $Z\subseteq V$ is true at any possible world and $R\cap V=\varnothing$ is true at any real according to Definition (3.5) and (3.3) respectively. Because all real worlds are possible worlds. It can be seen that $R\cap Z=\varnothing$ is true in at least one possible world. Therefore, ◇*no*(*r, z*) is true in line with Definition (3.10). It follows that ⊢□*all*(*z, v*)∧*no*(*r, v*)→◇*no*(*r, z*) is valid, as required.

Theorem 2: The following 30 valid Aristotelian syllogisms can be deduced just from the syllogism □AE◇E-2:

(2.1) □AE◇E-2→□AE◇E-4
(2.2) □AE◇E-2→□AE◇E-4→E□A◇E-1
(2.3) □AE◇E-2→E□A◇E-2
(2.4) □AE◇E-2→□A□II-1
(2.5) □AE◇E-2→□A□II-1→□A□II-3
(2.6) □AE◇E-2→□A□II-1→□A□II-3→□I□AI-3
(2.7) □AE◇E-2→□A□II-1→□I□AI-4
(2.8) □AE◇E-2→□AE◇O-2
(2.9) □AE◇E-2→□AE◇O-2□→AE◇O-4
(2.10) □AE◇E-2→□AE◇E-4→E□A◇E-1→E□A◇O-1
(2.11) □AE◇E-2→E□A◇E-2→E□A◇O-2
(2.12) □AE◇E-2→E□A◇E-2→E□A◇O-2→□A□AI-3
(2.13) □AE◇E-2→□AE◇O-2→□A□AI-1
(2.14) □AE◇E-2→□AE◇O-2→□A□AI-1→□A□AI-4
(2.15) □AE◇E-2→E□I◇O-3
(2.16) □AE◇E-2→E□I◇O-3→E□I◇O-1
(2.17) □AE◇E-2→E□I◇O-3→E□I◇O-1→E□I◇O-2
(2.18) □AE◇E-2→E□I◇O-3→E□I◇O-4
(2.19) □AE◇E-2→□EA◇E-2
(2.20) □AE◇E-2→□EA◇E-2→□EA◇E-1
(2.21)□AE◇E-2→□EA◇E-2→□EA◇E-1→A□E◇E-4
(2.22) □AE◇E-2→□EA◇E-2→A□E◇E-2
(2.23) □AE◇E-2→□EA◇E-2→□EA◇O-2
(2.24) □AE◇E-2→□EA◇E-2→□EA◇O-2→□EA◇O-1
(2.25) □AE◇E-2→□EA◇E-2→□EA◇E-1→A□E◇E-4→A□E◇O-4
(2.26) □AE◇E-2→□EA◇E-2→A□E◇E-2→A□E◇O-2
(2.27) □AE◇E-2→□A□II-1→□E□IO-1

(2.28) □AE◇E-2→□A□II-1→□E□IO-1→□E□IO-3

(2.29) □AE◇E-2→□A□II-1→□E□IO-1→□E□IO-3→□E□IO-4

(2.30) □AE◇E-2→□A□II-1→□E□IO-1→□E□IO-2

Proof:

[1] ⊢□*all*(*z, v*)∧*no*(*r, v*)→◇*no*(*r, z*) (i.e.□AE◇E-2, basic axiom)

[2] ⊢□*all*(*z, v*)∧*no*(*v, r*)→◇*no*(*r, z*) (i.e.□AE◇E-4, by [1] and F(4.2))

[3] ⊢□*all*(*z, v*)∧*no*(*v, r*)→◇*no*(*z, r*) (i.e.E□A◇E-1, by [2] and F(4.2))

[4] ⊢□*all*(*z, v*)∧*no*(*r, v*)→◇*no*(*z, r*) (i.e.E□A◇E-2, by [1] and F(4.2))

[5] ⊢¬◇*no*(*r, z*) ∧□*all*(*z, v*)→¬*no*(*r, v*) (by [1] and R2)

[6] ⊢□¬*no*(*r, z*) ∧□*all*(*z, v*)→*some*(*r, v*) (by [5], F(3.2) and F(2.3))

[7] ⊢□*some*(*r, z*) ∧□*all*(*z, v*)→*some*(*r, v*) (i.e.□A□II-1, by [6] and F(2.3))

[8] ⊢□*some*(*z, r*) ∧□*all*(*z, v*)→*some*(*r, v*) (i.e.□A□II-3, by [7] and F(4.1))

[9] ⊢□*some*(*z, r*) ∧□*all*(*z, v*)→*some*(*v, r*) (i.e.□I□AI-3, by [8] and F(4.1))

[10] ⊢□*some*(*r, z*) ∧□*all*(*z, v*)→*some*(*v, r*) (i.e.□I□AI-4, by [7] and F(4.1))

[11] ⊢◇*no*(*r, z*)→◇*not all*(*r, z*) (by F(5.5))

[12] ⊢□*all*(*z, v*)∧*no*(*r, v*)→◇*not all*(*r, z*) (i.e.□AE◇O-2, by [1], [11] and R1)

[13] ⊢□*all*(*z, v*)∧*no*(*v, r*)→◇*not all*(*r, z*) (i.e.□AE◇O-4, by [12] and F(4.2))

[14] ⊢□*all*(*z, v*)∧*no*(*v, r*)→◇*not all*(*z, r*) (i.e.E□A◇O-1, by [3], [11] and R1)

[15] ⊢□*all*(*z, v*)∧*no*(*r, v*)→◇*not all*(*z, r*) (i.e.E□A◇O-2, by [4], [11] and R1)

[16] ⊢¬◇*not all*(*z, r*)∧□*all*(*z, v*)→¬*no*(*r, v*) (by [15] and R2)

[17] ⊢□¬*not all*(*z, r*)∧□*all*(*z, v*)→*some*(*r, v*) (by [16], Fact (3.2) and F(2.3))

[18] ⊢□*all*(*z, r*)∧□*all*(*z, v*)→*some*(*r, v*) (i.e.□A□AI-3, by [17] and F(2.1))

[19] ⊢¬◇*not all*(*r, z*)∧□*all*(*z, v*)→¬*no*(*r, v*) (by [12] and R2)

[20] ⊢□¬*not all*(*r, z*)∧□*all*(*z, v*)→*some*(*r, v*) (by [19], F(3.2) and F(2.3))

[21] ⊢□*all*(*r, z*)∧□*all*(*z, v*)→*some*(*r, v*) (i.e.□A□AI-1, by [20] and F(2.1))

[22] ⊢□*all*(*r, z*)∧□*all*(*z, v*)→*some*(*v, r*) (i.e.□A□AI-4, by [21] and F(4.1))

[23] ⊢¬◇*no*(*r, z*) ∧*no*(*r, v*)→¬□*all*(*z, v*) (by [1] and R3)

[24] ⊢□¬*no*(*r, z*) ∧*no*(*r, v*)→◇¬*all*(*z, v*) (by [23], F(3.2) and F(3.1))

[25] ⊢□*some*(*r, z*) ∧*no*(*r, v*)→◇*not all*(*z, v*) (i.e.E□I◇O-3, by [24], F(2.3) and F(2.2))

[26] ⊢□*some*(*z, r*) ∧*no*(*r, v*)→◇*not all*(*z, v*) (i.e.E□I◇O-1, by [25] and F(4.1))

[27] $\vdash\Box some(z, r) \wedge no(v, r)\rightarrow\Diamond not\ all(z, v)$ (i.e.E□I◇O-2, by [26] and F(4.2))

[28] $\vdash\Box some(r, z) \wedge no(v, r)\rightarrow\Diamond not\ all(z, v)$ (i.e.E□I◇O-4, by [25] and F(4.2))

[29] $\vdash\Box no\neg(z, v)\wedge all\neg(r, v)\rightarrow\Diamond no(r, z)$ (by [1], F(1.1) and F(1.2))

[30] $\vdash\Box no(z, D-v)\wedge all(r, D-v)\rightarrow\Diamond no(r, z)$ (i.e.□EA◇E-2, by [29] and Definition 1)

[31] $\vdash\Box no(D-v, z)\wedge all(r, D-v)\rightarrow\Diamond no(r, z)$ (i.e.□EA◇E-1, by [30] and F(4.2))

[32] $\vdash\Box no(D-v, z)\wedge all(r, D-v)\rightarrow\Diamond no(z, r)$ (i.e.A□E◇E-4, by [31] and F(4.2))

[33] $\vdash\Box no(z, D-v)\wedge all(r, D-v)\rightarrow\Diamond no(z, r)$ (i.e.A□E◇E-2, by [30] and F(4.2))

[34] $\vdash\Box no(z, D-v)\wedge all(r, D-v)\rightarrow\Diamond not\ all(r, z)$ (i.e.□EA◇O-2, by [30], [11] and R1)

[35] $\vdash\Box no(D-v, z)\wedge all(r, D-v)\rightarrow\Diamond not\ all(r, z)$ (i.e.□EA◇O-1, by [34] and F(4.2))

[36] $\vdash\Box no(D-v, z)\wedge all(r, D-v)\rightarrow\Diamond not\ all(z, r)$ (i.e.A□E◇O-4, by [32], [11] and R1)

[37] $\vdash\Box no(z, D-v)\wedge all(r, D-v)\rightarrow\Diamond not\ all(z, r)$ (i.e.A□E◇O-2, by [33], [11] and R1)

[38] $\vdash\Box some(r, z) \wedge\Box no\neg(z, v)\rightarrow not\ all\neg(r, v)$ (by [7], F(1.1) and F(1.3))

[39] $\vdash\Box some(r, z) \wedge\Box no(z, D-v)\rightarrow not\ all(r, D-v)$ (i.e.□E□IO-1, by [38] and Definition 1)

[40] $\vdash\Box some(z, r) \wedge\Box no(z, D-v)\rightarrow not\ all(r, D-v)$ (i.e.□E□IO-3, by [39] and F(4.1))

[41] $\vdash\Box some(z, r) \wedge\Box no(D-v, z)\rightarrow not\ all(r, D-v)$ (i.e.□E□IO-4, by [40] and F(4.2))

[42] $\vdash\Box some(r, z) \wedge\Box no(D-v, z)\rightarrow not\ all(r, D-v)$ (i.e.□E□IO-2, by [39] and F(4.2))

The above 30 valid syllogisms have been derived from the valid syllogism □AE◇E-2 by utilizing relevant definitions, facts, and rules.

4.Conclusion

This paper firstly presents knowledge representations of Aristotelian modal syllogisms based on the structure of modal categorical propositions, and proves the validity of the syllogism □AE◇E-2 in Theorem 1 by using set theory, modal logic, Aristotelian quantifiers theory and so on, and then illustrates that the other 30 valid syllogisms are derived from □AE◇E-2 in Theorem 2. In other words, it proves that there are reducible relationships between/among the syllogism and the other 30 valid syllogisms.

This formal study not only provides new insights for knowledge mining in artificial intelligence, but also provides ideas for studying other kinds of syllogisms, such as rational syllogisms and generalized modal syllogisms.

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