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# DESIGN PRINCIPLES OF PRE-SERVICE TEACHER EDUCATION TO DEVELOP PROSPECTIVE TEACHERS' KNOWLEDGE OF MATHEMATICAL REASONING

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## Abstract

Mathematical reasoning (MR), highlighted in curricular orientations, is a necessary students' skill in mathematical learning. As the knowledge to promote this skill is limited and challenging for teachers practices, is required to use adequate design principles in pre-service teacher education to develop PTs mathematical and didactical knowledge to promote students' MR in teaching practices. This study aims to establish and understand positive contributes of adopted design principles and their operationalization in pre-service teacher education experiments, and the results revealed the potential of adopted design principles to promote the development of PTs' knowledge of MR and ability to promote students' MR in teaching practices.

**Keywords:** Design principles, Pre-service teacher education, Mathematical reasoning Prospective teachers´ knowledge

## Introduction

Mathematical reasoning (MR) is widely recognized, in current national and international curriculum guidelines for mathematics education, as an essential skill to be developed by all students, in order to successfully learn mathematics with understanding (Lannin, et al., 2011; ME, 2021; NCTM, 2000). The students' learning depends on the experiences that teachers provide them in lessons (Boston & Smith, 2011; Breen & O'Shea 2019; Stein et al., 1996). Particularly mathematical tasks involving reasoning are required in teaching as being one of the main sources necessary to give students opportunity to work on tasks that reason and that stimulate their learning (Watson & Ohtani, 2015) and engagement in this skill. However are not usually the mathematical reasoning processes to be explicitly addressed in the classroom, as teachers often face challenges in understanding what MR means and how to carry out teaching practices that provide opportunities to promote this students' skill, particularly the prospective teachers (Lannin et al., 2011). Considering these challenges and to follow the education recommendations it is required to highlight MR as a specific goal for an adequate pre-service teacher education experiment, providing opportunities for prospective teachers (PTs) develop their mathematical and didactical knowledge required to create learning contexts to promote the students' MR in their future teaching practices (Buchbinder & McCrone, 2020; Ponte & Chapman, 2015).

In order to study the understanding whether and how a training experience can support PTs' MR learning, it is necessary to organize the design of this process for this purpose (Fuentes & Ma, 2018). While the literature has reported on domains of MR and PTs development knowledge about those processes (Davidson et al., 2019; Oliveira & Henriques, 2021), more recent research is required as there are scarce studies specifically focused on pre-service teacher education design principles to prepare middle and secondary school prospective teachers' knowledge to promote students' MR. To fill this gap, it was pertinent conduct this study that aims to establish and understand the positive contributes of the adopted design principles and their operationalization in na innovative pre-service teacher education experiment, for developing middle school and secondary PTs' understanding of MR and the necessary knowledge and ability to create learning contexts concerning the promotion of students MR. Any recognized limitation of the described proposals also may help to establish new future design principles regarding developing MR. This focus of the study can also be useful for the scientific and educational community.

## Theoretical framework

### *Mathematical reasoning*

The term mathematical reasoning is widely "used in mathematics education with distinct and non-consensual meanings" (Oliveira & Henriques, 2021, p. 1301). In this study, the MR was considered a process of making justified inferences from previously known information, as remarked by diverse authors (Jeannotte & Kieran, 2017; Lannin et al., 2011; Mata-Pereira & Ponte, 2017; Rivera & Becker, 2009). This description implies mathematical reasoning as encompassing inductive and abductive kinds of reasoning to get new information as also the use of deductive reasoning to validate inferences made from that information. According to this mathematical reasoning conceptualization, and besides the aspects related to the kind of reasoning, the engagement in this activity involves a variety of processes identified in the literature. A mathematical reasoning model for school mathematics is presented by Jeannote and Kieran (2017), who integrate those processes in two categories, one related to the search for similarities and differences, and the other related to validation. The first category includes the processes of identifying patterns, comparing, conjecturing, generalizing, and classifying. In the second category are considered the processes of validation, including justifying and proving. Still, exemplifying is proposed as another process that is conceived as a support for

the other ones. From a convergent perspective, this study focuses essentially on the conjecturing, generalizing and justifying processes, which are recognized as central for mathematical reasoning, and which may be achieved by inductive, abductive and deductive reasoning.

The conjecturing process is common to be based on patterns identification through the exploration of examples using inductive reasoning, leading to statements believed to be true, and can also be developed by testing examples necessary to the require them validation or proving and check the working of the conjecture for other kinds of objects (Lannin et al., 2011). The generalization is based on a conclusion specific or a conjecture focus on a property, or a procedure that assert its validity or commonality to a set of objects (Jeannote & Kieran, 2017). Justification, as a process involving creating arguments and explaining their veracity, provide convincing reasons to establish conjectures that allow the students to make a clear reasoning and increase their conceptual understanding (NCTM, 2000).

To stimulate students´ MR it is essential to consider that the characteristics of the used tasks make possible to use multiple representations and solving strategies (Stein et al., 1996), and also to contemplate specific MR processes such as: the formulation of conjectures and generalizations, identifying counterexamples; questions that request explanations or justifications of answers (Brodie, 2010; Thompson, 2012); and classify mathematical objects using definitions (Breen & O´Shea, 2019).

**Pre-service teacher education and learning scenarios to Promote MR**

Pre-service teacher education is a complex process as it involves many interacting dimensions (Ponte & Chapman, 2015). These authors refer to the knowledge that mathematical teachers need as involving: knowledge about the mathematics nature and content, curriculum objectives, how to present mathematical ideas to be learned by the students, how students learn, and the classroom management. In order to integrate MR into their teaching practices, PTs themselves must have a solid understanding of MR (Lannin et al., 2011), being able to identify and understand what reasoning means and how to integrate learning tasks and experiences that promote the development of students' reasoning in their own classrooms.

Although the research on mathematics teacher education has recognized that it involves these closely related domains of content and pedagogy, several researchers (e.g., Ponte & Chapman, 2015) report the challenges that PTs face in using the knowledge they develop to implement mathematics teaching according to innovative curriculum guidelines, namely the ones that

emphasize mathematics reasoning. Considering these challenges, these authors suggest that teacher education learning objectives should be focused on how to develop mathematical reasoning within specific mathematical topics. Moreover, teacher education must emphasize the articulation between theory and practice and focus on students' learning, an approach that may positively influence teaching practice.

Therefore, for PTs develop MR knowledge, is essential in a first stage of teacher education to provide and discuss theoretical frameworks to increase the understanding of MR (meaning, kinds and processes), the recognition of characteristics of mathematical tasks and teachers' actions. And also asked to solve mathematical learning tasks focused on teaching professional learning and aimed at school student's perspective (Dempsey & O'Shea, 2020; Jeannote & Kieran, 2017; Mata-Pereira & Ponte, 2017). As a complement, the PTs engagement in analysing mathematical tasks including classroom episodes illustrating examples of students work on tasks focus on distinct ways of reasoning, and teacher actions to promote those processes, requiring them to noticing, justify and sharing interpretations of evidenced MR processes, which may help to recognize and understand the MR processes and increase the required knowledge throughout the training course (Park & Magiera, 2020).

In training, with the aim of mobilizing in an articulated way the PT's professional knowledge, both mathematical and didactic, they must work on those resources from the perspective of the exploratory teaching approach (Ponte & Quaresma, 2016). And the professional learning opportunities are characterized as collective moments focused on problems and cases of professional practice, in which teachers work and discuss articulately mathematical and didactic situations (Ribeiro & Ponte, 2020). Thus, it is recognized the importance of training PTs being guided by a practice and organized from materials that portray an authentic pedagogical practice, which allow creating opportunities for reflection on teaching practices and to develop professional knowledge through the analysis of situations to mobilize knowledge related to the mathematics contents, its teaching, and the way students learn Mathematics (Smith, 2001).

For the needed PTs' ability to integrate in their classes learning experiences based on tasks recognized as especially suited to promote the students' MR, is highlighted to provide PTs opportunities to select or adapt and enact tasks considering the required learning objectives and characteristics to promote that skill. And also, to plan appropriate teaching strategies, framing their goals in the students of the class in which they are proposed, to monitoring students' autonomous work and conducting whole class discussions, recognizing the role of

different teacher's actions to address the variety of students' conceptions and abilities (Hill, Ball & Schilling, 2008; Lannin et al., 2011; Mata-Pereira & Ponte, 2017; Stein et al., 2008). In addition to the writing of lesson plans, allowing PTs to analyse and discuss the students' mathematical activity and knowledge evidenced in the taught lessons based on the proposed tasks, may provide a progressive PTs ability to create or modify appropriated tasks and to involve students in the exploration and questioning to solve them to encourage their learning and MR knowledge and skills (Leavy & Hourigan, 2020; Lee et al., 2019; Kilic et al., 2017; Zazkis et al., 2009).

## Methodology

### Context and participants

This study, realized under the REASON project, is based on the adopted design principles and their operationalization in pre-service teacher education experiment carried out to develop middle school and secondary PTs' mathematical and didactical knowledge of MR to promote this essential skill for students in various mathematical themes as Algebra and Geometry. And follows a Design-Based Research contemplating two design cycles (Ponte et al., 2016) corresponding to two training experiences that took place in Mathematics Didactics courses of a master's degree in mathematics teaching for middle and secondary school levels at a university of Lisbon.

The first cycle was carried out in the 2nd year of the master's course, involving six PTs who were also already carrying out their supervised teaching practice in schools with students from the 3rd cycle and secondary education. In the second cycle were involved 13 PTs, but of the 1st year master's course that had not yet started the practice of supervised teaching. Those PTs (male and female) that participated in this study voluntarily (identified by pseudonyms) had already acquired some pedagogical knowledge in a previous course of pre-service teacher education, but without involving reasoning.

### Design principles of pre-service teacher education and their operationalization

The developed education experience for PTs learning, in both design cycles, involved a sequence of eight sessions with 2 hours each, taught and observed by two of this paper authors, mainly consisting of workshops that involve the PTs in solving instructional tasks whose aspects are ilustrated in Table 1, to point how to organize the design principles of the

training process.

**Table 1.** Developed learning trajectory: teaching sessions and their focus.

| **Teaching sessions work and focus** |
|---|
| Previous autonomous reading and subsequent collective discussion in the sessions of theoretical texts about mathematical reasoning (Ponte et al., 2020; REASON, 2022; Mata-Pereira & Ponte, 2016) focus on: Meaning of mathematical reasoning (concept, types and processes); Task design principles to promote MR; Teacher actions to promote students MR. |
| Autonomous resolution and discussion of instructional mathematical tasks focusing on MR, including examples of classroom practices ilustrating students work on MR processes and teachers actions to promote those processes. The resolutions are focused on: analysis of the tasks characteristics and potential as well technological resources to promote MR in algebra and geometry; the types and processes of MR revealed by students and their difficulties in solving tasks; teachers actions that contributed to the promotion of students MR. |
| Preparation in Groups of a lesson planning based on selected or adapted tasks focus on MR to lead in a teaching practice of mathematical reasoning to promote this students hability. And then, a final session is focus on oral presentation to provide a reflection and collective discussion about the proposed teaching practice. |

Those sessions were prepared incorporating a set of instructional design principles defined by research team members of the project, including the authors of this paper, within the three distinct categories that are informed by research literature and contributed to the design of the training process, described below:

(1) General framework. The pre-service teacher education was oriented by literature research about mathematical reasoning (meaning, types and processes), task design theoretical principles, and teachers' actions to promote students' MR in the classroom (Jeannote & Kieran, 2017; Mata-Pereira & Ponte, 2017). This literature was made available to the PTs, for a previous reading and discussion with them in the sessions, to later be used as analysis tool of the used materials and guide the selection or design of tasks.

(2) Instructional approach. The sessions follow an exploratory teaching approach of the pedagogical type (Menezes et al., 2012) supported on mathematical learning tasks to be explored autonomously by the PTs, working individually or in small groups, and discussed with whole class. The articulation between theory and practice was also valued in the teacher education (Ponte & Chapman, 2015). Some sessions are oriented to the practice, giving PTs opportunities to plan a lesson based on the selection or adaptation of a task aiming to promote

students' MR, to enact it in an actual classroom, and to reflect on and share their tasks and the taught experiences with their peers and the teacher educator, which support prospective teachers in analyzing the completeness and quality of the lesson plan that was applied in the classroom.

(3) Materials to support teacher education. The materials used in teacher education were framed on current curriculum documents and focus on mathematical and didactic knowledge about mathematical reasoning (Lee, Lee, & Park, 2019). So, the training tasks proposed in the sessions cover a diversity of middle and secondary school mathematical topics, such as Numbers, Algebra and Geometry. Some proposed training tasks included examples of school students' work on learning tasks and transcriptions of classroom episodes, to be analyzed by the PTs focus on the tasks' potential to promote MR, the types and processes of MR evidenced by students, and the teacher's actions that contribute to promote this students' skill. Complementarily, a typical training task is based on selecting and adapting a learning task from different sources (e.g., school textbook and professional or research journals), to be enacted with school students, and the PTs are asked to frame the task on the curriculum, solve it from a school student's perspective, and to justify how the task provide opportunities to promote MR attending to the adopted conceptual framework and the task design principles that are made available and discussed with the PTs in the sessions (Jeannote & Kieran, 2017; Lin et al., 2011). Finally, the pertinence of using technology for promoting students' MR was also be focused in the sessions (Ponte & Chapman, 2015).

## Method

Aiming to report the positive contributes of the design principles adopted in both cycles of the pre-service teacher education experience and their operationalization, in the development of the PTs´ MR understanding and knowledge to effectively integrate it in mathematics teaching and learning, data were collected during the training sessions, including: PTs' written solutions and discussion of the proposed tasks; PTs' produced document of lesson plan based on a selected task and reflection on the taught lesson; PTs' interviews at the end of the training sessions, conducted by an author trainer, focus on asking their developed understanding of MR meaning and its importance, potential or limitations of the task and teaching approach adopted into their practice to promote students' MR, and also conceptions about the work carried out as a contribution to their MR professional knowledge in future practice.

A qualitative and interpretative analysis (Erickson, 1986), of the collected PTs data excerpts, was used to present in the next section some results that evidence the identified PTs learning opportunities to acquire knowledge of MR (Ball et al., 2008), supported on effects of the established design principles.

Third Level Headings

## Results

The PTs' work (individually and in pairs) and group discussion in classes on proposed training mathematical tasks focused on MR meaning and processes involving Algebra and Geometry topics, that include their answers to questions involving analysis of reasoning processes ilustrated by students answers in solving the tasks showed in episodes, was recognized as essential to contribute to their clarification and better understand of the MR processes.

An analysis of two phases of that PTs work evidenced their growing knowledge regarding justification process. At the initial moment of the course most PTs were not yet familiar with MR knowledge, revealing in the first task lower levels (0 and 1) of knowledge about generalization and justification processes, since in their answers focused on students convincing justifications, they mainly confuse this process with the generalization, arguing: "The student used the generalization and verified its divisibility by 2" (CL); "They perceive the propriety and generalize" (MA); "He concluded…without exemplifying or generalizing" (CC). Only few PTs evidenced better knowledge levels (3 and 4), mentioning algebraic expression and mathematical properties that comply with the justification process definition: "Uses algebraic language and concept of parity" (CA); "Generalizes algebraically…and uses the property that says the sum of two even numbers is an even number" (JC).

The discussion between the PTs about the resolution of several tasks, which took place in the sessions, allowed a more detailed analysis and understanding of the processes and validity of the justifications presented by the students. At the final task the PTs' levels of justification knowledge have increased (2 to 5), recognizing that this process needs to be correct using mathematical logic and its relationship with generalization, responding: "he is not able to come to a logical" (MO); "part of a general case to justify this property" (CF); "justified through examples and search for patterns" (CL); "The justifications presented use inductions, for generic examples and formal demonstrations" (CB); "reveal understanding of the concepts

involved and necessary for the demonstration... general algebraic expressions are used" (CB); "He relies on geometric construction…, on knowledge that he already possesses… and gives a deductive generic example" (MA); "is an example of a justification through deductive reasoning characterizing justifications, justifies through examples… where its representation allowed reaching the justification" (IM); "ended up finding two regularities that justify his reasoning quite well…discovers the regularity, making a generalization" (B).The most of PTs elaboration of tasks and lesson plan to the intended teaching approach show that they were able to warrant the design of the task from a set of principles that entail coherence with the theoretical ones that are recognized as relevant to promote students' MR, addressed in the instructional unit. For example, they make explicit the intention to conduct the class using an exploratory teaching approach, allowing diverse solution strategies and the use of different representations, as well as specific MR principles to incentive the generalization based on the observation of similarities and differences between objects, and the justification, which is consistent with the task design principles: "The most important for students' learning success was an exploratory task with characteristics to promote mathematical reasoning: from knowledge acquired in initial questions they could generalize, reaching a general rule; other questions also encouraged justifications and to reflect on their colleagues' answers" (JOP1); "the task had different solving strategies; students, based on knowledge acquired in the initial questions, could generalize, reaching the general rule more easily; the questions always asked for justifications of the answers" (R); "in terms of privileged mathematical reasoning, the lesson was very rich, providing awareness of the importance of justification and explanation of strategies needed for the generalizing, to facilitate the understanding of what is observed. And we tried to construct questions that also covered these specific aspects of mathematical reasoning" (S).Besides the characteristics of the tasks, Pts also recognize the importance of teachers' actions (their role) in different moments of the lesson (task introduction, autonomous group work, and whole class discussion of the students' work), to create an environment that supports students' MR: "Helping students to unblock their MR through questioning, taking care not to do it themselves" (S); "The major factor that contributed to students' MR promotion was, during the collective discussion, gave them time to explain how they solved the question and justify it. I also gave space for colleagues to say whether they agreed or not with what was being presented and asked them to justify why" (JOP1); "the teacher should question the class, after each student, about opinion and favour the exchange of ideas and their justifications, playing the role of mediator of the discussion in which colleagues will be able to put their doubts and the student will be asked to explain" (LP).

In the reflection about teaching approach, the 2nd year PTs make a positive evaluation of the task's enactment to reinforce the learning in the topic of sequences, identifying that the students had a good involvement and performance, contrary to their usually difficulties in learning as: "they recognize the structure of the sequence and some groups were able to generalize algebraically the relationship" (Sandra); "most students, in their work, represent well the general rule algebraically, evidencing their adequate understood the relation right in the task's first question and the reasoning is all there" (JOP2).

After that, PTs claim that also recognized MR utility for students and reflected their positive assessment and confidence to continue working in the promotion of MR by implementing this kind of tasks developed under the project, in their future classrooms. showed to be aware of its meaning, regarding the involved kinds and processes of reasoning: "this work showed me the importance of MR in mathematics classes and that, in my future practices, it will be very enriching and indispensable for my students to acquire meaningful learning and develop these skills" (Sandra).

Please use 10-point font size. Please margin the text to the justified. Manuscripts should be 1.5 times spaced. A paragraph should have at least 3 sentences. Footnotes and endnotes are not accepted. All relevant information should be included in main text. Do not indent paragraphs; leave a 1.5 times space of one line between consecutive paragraphs. Do not underline words for emphasis. Use italics instead. Both numbered lists and bulleted lists can be used if necessary. Before submitting your manuscript, please ensure that every in-text citation has a corresponding reference in the reference list. Conversely, ensure that every entry in the reference list has a corresponding in-text citation.

## Discussion

Currently, as the mathematical reason is assumed as a required important student´s ability for mathematical learning, the teachers also need to develop this MR understanding and appropriate knowledge to promote a necessary intentional teaching for students. This study extends the scarce research about ways to develop the PTs mathematical reason knowledge, through two contributions: reported the set of established instructional design principles to enable the preparation of preservive teacher education experience, to develop mathematical reasoning knowledge of middle and secondary school PTs to promote students' MR, which is innovative in Portugal; and a specification of the PTs knowledge of MR content and the

potential of practices to promote students MR.

The enactment of the design principles adopted, in this prospective teacher education experience, provide new elements to guide the scientific and educational community to identify and understand the adequacy and potentialities of these principles for professional learning of future teachers involved in training.

The obtained results of the study evidenced that several of the instructional design principles, used in preservice teacher education sessions based on MR, in both design cycles, contributed positively for help PTs to build a deeper understanding of MR notions and developing significant aspects of their mathematical and didactical knowledge, which confirm the importance of providing opportunity to involve them in a pre-service teacher education experience. It was particularly identified the relevance of:

• The discussion that took place in the class, based on mathematical tasks illustrating students' MR, who favored a more detailed analysis and understanding of students' solution approaches, particularly the generalization and validation of justifications by the arguments presented by them. This work provided Pts growing knowledge of the MR nature and processes, particularly the justification, as it is common their difficulties in the interpretation of MR processes before working on this theme in a teacher education, as pointed out in other study (Buchbinder & McCrone, 2020).

• The desirable PTs training activity of selecting or adaptation of a mathematical task and create a learning scenario based on it, with potential to promote students' MR, and the essential teaching approach opportunity they have to implement this knowledge built in a school practice, as well a final discussion and reflection on the taught lesson. This work brings them closer to the expected adequate reality of its future practice, as increased their recognition and knowledge of the potential and relevance of promoting students´ MR ability in mathematics learning, using an exploratory teaching based on students´ autonomous work and collective discussion of the proposed mathematical tasks, in what they consider the necessary relevance of MR nature and processes as well specific task design principles addressed in the instructional unit. And also understand the revealed difficulties of students based on the practice records.

• The analysis and discussion of theoretical frameworks during the experience offered an opportunity, to better promote the PTs recognition of their acquired MR knowledge, and the potential of the specific design characteristics and objectives for mathematical tasks and

teachers' actions to promote their ability in teaching and learning situations in mathematics to promote students MR.

## Conclusion

To conclude, based on evident results, we emphasize in this study the characteristics of the training experience that allowed the PTs to develop and consolidate MR knowledge articulating the mathematical and didactic dimensions (Ponte & Quaresma, 2016) to integrate in future practices, in particular: allowing PTs to explore, analysis and discussion of mathematical tasks for teaching and learning MR; and being more favorable to also offered opportunity to PTs plan, reflect and discuss teaching and learning situations in mathematics practices to promote students' MR.

In summary, we recognize that the results of this study highlight valuable design principles for pre-service teacher education, particularly the type of work carried out in the instructional unit through the articulation of theory with teaching practice in school, which are confirmed to promote the MR knowledge of future teachers necessary to effectively integrate MR in the teaching and learning of mathematics, to using them on the expected reality of their future practice.

In view of the above, we can infer that our research contributes to consolidating the importance of the training process, to enable professional learning opportunities in the mathematical reasoning area.

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