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Based on International Understanding Education: An Analysis of the Integration of Logic and Mathematics Education in Junior and Senior High Schools

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Abstract

Logic, as a branch of the science of thinking, uncovers the essence and laws of thought. Mathematics demands a high level of logical thinking; thus, incorporating elements of logic into junior and senior high school mathematics curricula is crucial for fostering students' logical reasoning abilities. The teaching objectives and directions of junior and senior high school mathematics differ under the contexts of China's college entrance examination (Gaokao) and international understanding education. Through examples, this analysis explores the connection between logic and mathematics and the feasibility of their integration, emphasizing the importance of linking the two subjects to nurture logical thinking skills. Given this, it is essential to integrate logic with mathematics education within the framework of international understanding education.

Keywords: International Understanding Education; Logic; Junior and Senior High School Mathematics

Introduction

Professor Shing-Tung Yau of Harvard University once stated that plane geometry not only provides beautiful and significant theorems but also offers the sole logical training during secondary education, which is indispensable knowledge for every young person[1]. Logic stands as a fundamental discipline across humanities, social sciences, and natural sciences, as recognized by UNESCO in 1974.[2] With mathematics requiring substantial logical reasoning, embedding logic in math education in junior and senior high schools bolsters students' logical thinking capabilities.

Section I: Objectives and Directions of Junior and Senior High School Mathematics in the Context of International Understanding Education

In the context of the highly competitive college and high school entrance examinations (commonly known as the 'Zhongkao' and 'GaoKao'), junior and senior high school mathematics poses significant challenges in terms of depth and difficulty for students. These exams, serving as rigorous selection processes, inevitably lead to the elimination of some students, thereby creating intense competition. Both teaching pressures in mathematics and students' learning loads are substantial. To further facilitate student selection, the assessment of mathematical knowledge is stringent, requiring extensive exploration beyond textbook contents. As a result, the practice of solving a multitude of problems, or the so-called 'sea of questions' strategy, has become prevalent. Exams now not only demand comprehensive knowledge mastery but also emphasize computational abilities.

International Understanding Education, in simple terms, involves the localization of international education, the internationalization of domestic education, and bridging the gap between national and international educational systems. Regarding junior and senior high school mathematics, domestic education tends to focus more on problem-solving skills, whereas foreign education prioritizes the cultivation of logical thinking abilities. While multiple solutions to a single problem are seen as a means to train top students domestically, foreign mathematics education encourages this approach even for straightforward problems, valuing the thought process involved in finding alternative solutions. This disparity stems

[1] A speech delivered by Shing-Tung Yau during the celebration of the 110th anniversary of Beijing Normal University Affiliated High School, 2014.

[2] Zhou Chongli. The Combination Principle and Virtual Components in Natural Language [J]. Journal of Sichuan Normal University (Social Sciences Edition), 2017, Vol.44(No.1):5-9.

from differing educational philosophies and the varying intensity of competition in student selection, which allows students abroad more time to ponder mathematical concepts, contrasting with the emphasis in China on rapid problem-solving with limited time for reflection.

Consequently, the goals and orientations of teaching junior and senior high school mathematics within the framework of International Understanding Education should diverge from those driven solely by the demands of the Zhongkao and GaoKao, especially as International Understanding Education continues to evolve. Our aim is to cultivate individuals with a global perspective who are Chinese at heart. Therefore, in mathematics education, there is a need to adopt successful practices from abroad, shifting the focus from mere skill drills to fostering thinking abilities. Encouragingly, over the past three years, both the Zhongkao and GaoKao exams have shown a trend toward promoting students' cognitive skills. Such an approach ensures that students develop not only a solid foundation in mathematics but also possess open and logical thinking capabilities, better equipping them for the demands of their future lives.

Section II: The Connection and Feasibility of Integrating Logic and Mathematics

Logic is a branch of the science of thinking, which uncovers the essence and laws of thought. This scientific domain constitutes a group of disciplines, including psychology, neurophysiology, philosophical epistemology, artificial intelligence, among others, each examining the nature and principles of thought from distinct perspectives.[1]Logic specifically focuses on the study of thought forms as its unique subject matter. Given the high demand for logical thinking in mathematics, it is crucial for students in junior and senior high schools to learn elements of logic in their mathematics curriculum, which significantly contributes to enhancing their logical thinking abilities. In current junior and senior high school mathematics, students encounter basic propositions in textbooks, such as original propositions, negations, contrapositives, and contraries, alongside rudimentary reasoning methods involving compound propositions, and they acquire the ability to discern the truth value of such

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1 Logic (5rd Edition) (Textbook Series for 21st Century Philosophy) [M]. Published by Renmin University of China Press, Compiled by the Logic Teaching and Research Office, School of Philosophy, Renmin University of China, 2022.

propositions, along with an elementary understanding of proof by contradiction. These foundational concepts are prerequisites for studying logic, forming the basis for a systematic approach to nurturing logical thinking abilities.

Taking the textbook "Logic," compiled by the Department of Philosophy's Logic Research Office at Renmin University of China and published by Renmin University Press, as an example, certain chapters are selected to foster students' logical thinking skills. These include: 1) Relationships among concepts: compatible relationships (identity, genus-species, and intersection), incompatible relationships (contradiction and opposition), and corresponding Euler diagrams; 2) Division and decomposition, generalization, and restriction of concepts; 3) Compound propositions: conjunctive propositions, disjunctive propositions (both compatible and incompatible), conditional propositions (sufficient, necessary, and bi-conditional), and negated propositions, along with their corresponding truth tables; 4) Reasoning types: conjunctive, disjunctive, conditional, and dilemma reasoning, and methods for determining the validity of such reasoning; 5) Categorical propositions: their types (universal affirmative, universal negative, particular affirmative, and particular negative), extensionality of terms, and truth relationships among four categorical propositions with identical subjects and predicates; 6) Categorical syllogism: definition, structure, rules, figures and moods, and abbreviated forms; 7) Fundamental laws of logic: the law of identity, law of contradiction (paradox), law of excluded middle, and principle of sufficient reason; 8) Inductive logic: similarities and differences between inductive and deductive reasoning, forms of inductive reasoning (complete, incomplete, probabilistic, statistical, analogical, abductive, and hypothetical-deductive inference), and the five methods for establishing causality (method of agreement, method of difference, joint method of agreement and difference, method of concomitant variation, and residual method); 9) Proof and refutation: types of proof, rules of proof, refutation, informal arguments, and formalized methods; 10) Fallacies: psychological relevance fallacies, linguistic ambiguity fallacies, and inadequacy of evidence fallacies.

Using the section on compound propositions as an illustration, after studying Unit 2-1 in high school, students typically gain a basic understanding of several fundamental compound propositions.

(1)Conjunction proposition, for instance, affirms several states of affairs simultaneously. An example is: "The future is bright, yet the path is tortuous." This is a conjunction proposition, asserting that "the future is bright" and "the path is tortuous" coexist. The general form of a conjunction proposition is "p and q," symbolically represented as p^q, where "^" denotes "conjunction," abstracting the concept of "and." In "p^q," p and q are referred to as conjuncts. In everyday language, conjunction propositions can be expressed as "not only p but also q," "both p and q," "though p, yet q," or "not merely p but indeed q," among other formulations.

p	q	p^q
Truth(1)	Truth(1)	Truth(1)
Truth(1)	False(0)	False(0)
False(0)	Truth(1)	False(0)
False(0)	False(0)	False(0)

(2)Disjunctive Propositions: These are compound propositions that assert that at least one of several possible states of affairs exists. For example: "Tomorrow, I will either climb the Great Wall or visit Fragrant Hill." Disjunctive propositions are divided into two categories: compatible disjunctive propositions (e.g., "Mr. Zhang is either a poet or a painter. [Both can be true simultaneously]") and incompatible disjunctive propositions (e.g., "Mr. Zhang is either from Sichuan or from Hunan. [Both cannot be true at the same time]").

Compatible Disjunctive Proposition: p or q. Symbolic form: p ∨ q, where "∨" is read as "disjunction," an abstraction of the term "or." In "p ∨ q," p and q are also referred to as the disjuncts.

p	q	pVq
Truth(1)	Truth(1)	Truth(1)
Truth(1)	False(0)	Truth(1)
False(0)	Truth(1)	Truth(1)
False(0)	False(0)	False(0)

Incompatible Disjunctive Proposition: General form: Either p or q. (An incompatible disjunctive proposition is a disjunctive proposition that affirms that exactly one of the disjuncts is true.) Symbolic form: p $\dot{\vee}$ q, where "$\dot{\vee}$" is read as "exclusive disjunction," an abstraction representing the concept of "either...or..." but implying mutual exclusivity.

p	q	p $\dot{\vee}$ q
Truth(1)	Truth(1)	False(0)
Truth(1)	False(0)	Truth(1)
False(0)	Truth(1)	Truth(1)

False(0)	False(0)	False(0)

(3) Conditional Propositions: These are compound propositions that determine the conditional relationship between states of affairs.

① The general form of a sufficient conditional proposition is: If p, then q. The symbolic representation is: p → q. The symbol "→" is read as "implies," which is an abstraction of the phrase "if...then...". This signifies that whenever p holds true, q necessarily follows; p being true guarantees q's truth.

p	q	p→q
Truth(1)	Truth(1)	Truth(1)
Truth(1)	False(0)	False(0)
False(0)	Truth(1)	Truth(1)
False(0)	False(0)	Truth(1)

②Necessary Conditional Proposition (e.g., "Only those who are 18 years old or above have the right to vote.") The general form is: Only if p, then q. The symbolic representation is: p ← q. The symbol "←" is read as "inverse implication," abstracting the concept of "only...then...". This implies that q is true only when p is true; without p, q cannot be true.

p	q	p←q
Truth(1)	Truth(1)	Truth(1)
Truth(1)	False(0)	Truth(1)
False(0)	Truth(1)	False(0)
False(0)	False(0)	Truth(1)

(4) Negative Proposition: This is a proposition derived by negating another proposition (for example: "Not all people are selfish."). The general form is: Not p. The symbolic representation is: ¬p. The symbol "¬" is read as "not," expressing the negation or denial of the proposition. In other words, it signifies that p is false or does not hold true.

p	¬ p
Truth(1)	False(0)
False(0)	Truth(1)

Negative Compound Proposition's Equivalent Propositions：

¬（p^q）↔（¬p v ¬q）

¬（p v q）↔（¬p^¬q）

¬¬p↔p

Once we have mastered these propositions and their truth tables, we can delve into the content of logic, grasp the inherent logical relationships of the problem, and analyze whether specific inferences are valid.

For instance, to determine the validity of the following reasoning using methods such as truth tables: "(Xiao Zhang) Only with excellent exam results can one be awarded the title of 'Three Good Student.' In reality, Xiao Zhang was not awarded the title of 'Three Good Student'; therefore, her exam results did not meet excellence."

Let p represent "Xiao Zhang's exam results are excellent," and q denote "Xiao Zhang is awarded the title of 'Three Good Student'." The logical form of this inference is then: $((p \rightarrow q) \wedge \neg q) \rightarrow \neg p$. Below is the constructed truth table for this logical expression:

p	q	((p ← q)^¬ p)→¬ q
Truth(1)	Truth(1)	Truth(1)
Truth(1)	False(0)	False(0)
False(0)	Truth(1)	Truth(1)
0	0	1

The above truth table illustrates that this truth-functional form is not a tautology; hence, the reasoning is invalid.

From the above two examples, it becomes evident that within the framework of logic, students' logical reasoning shifts from addressing numerous specific problems to adopting a method that examines patterns of thought and their underlying principles. By substituting concrete phrases with letters and transforming textual explanations into alphabetic segments, we generalize particular issues, thereby grasping their logical relationships in a formal logical manner. This approach of perceiving the essence through phenomena—focusing on cultivating students' logical thinking abilities—carries broad applicability.

Section III: The Importance of Linking Logic and Mathematics Education in the Context of International Understanding Education

The renowned physicist Max von Laue once stated, "What matters is not acquiring knowledge but developing the ability to think. Education is essentially what remains after one has forgotten everything one has learned."[1] This is particularly pertinent in mathematics education, where teaching students how to fish (i.e., fostering logical thinking) is far more valuable than merely giving them fish (memorizing problem-solving templates).

Against the backdrop of International Understanding Education, the importance of integrating logic with junior and senior high school mathematics education rests on two main points:

1.Fundamental Position of Logic: Logic occupies a foundational role, serving as the base for all fields of knowledge. All human cognitive activities and domains of knowledge necessitate the application of logic. Every scientific discipline employs logic, as they all involve the use of concepts, judgments, and reasoning. Logic, as the science that investigates these forms, rules, and methods of thought, is of immense benefit to future international students, laying a strong mental foundation for their undergraduate and potentially graduate studies.

2.Universality of Logic: Logic transcends ethnic and regional boundaries. It is a universally applicable discipline, not confined to any specific ethnicity or region. Despite linguistic diversity across the globe, all rely on the same logical principles. Logic is a shared intellectual wealth of humanity, independent of any specific language, thought, or cultural idiosyncrasies. Thus, proficiency in logic is a locally adaptable skill for international students, requiring no adjustment or adaptation, and can be utilized in any country they choose to study or research in, highlighting its broad applicability—a critical asset for any global learner.

In summary, within the context of International Understanding Education, connecting logic with junior and senior high school mathematics is imperative.

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