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A cross-national comparison of curricula in China and the US in terms of cognitive complexity: The case of one dimensional equation

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Abstract

The holistic level of conceptual understanding and teaching have been an issue of national concern in China. To describe the results of years of improvement in Chinese textbooks, this research examined the California textbook named Algebra1(A1) and Chinese textbooks of people's edition(C1) and Zhejiang edition(C2). This study refers to Webb's depth of knowledge framework and Son and Senk's cognitive expectation feature, which is used to examine the level and kind of problems presented in the three textbooks. Referring to the breadth of cognitive complexity, C1 and C2 focus more on the cultivation of students' way of thinking and logic, and A1 prefers letting students to perform an algorithm. Besides, referring to the depth of cognitive complexity, A1 is more challenging than C2, but less challenging than C1. Finally, we discuss the inspiration of these results for teachers, students, related researchers and material developers.

Keywords: Textbook analysis; One dimensional equation; Comparative study; Cognitive

demands; Problems.

1 Introduction

Since entering the 21st century, more and more countries attach importance to the role of education, mathematics education is the top priority. In the International math tests PISA and TIMSS, East Asia scores higher than the

West. We hope to explore some of these reasons, so we choose to find the cause of the problem through the comparison of textbooks.

We chose the representative of the East China and the representative of the developed west the United States. Moreover, mathematics textbooks for junior high schools with high usage rate in the two countries are selected for comparative study. They are California textbook named Algebra1(A1) and Chinese textbooks of people's edition (C1) and Zhejiang edition (C2). Through the comparison of textbooks, we can understand the influence of different textbooks on students, and also bring reference value to the comparative study of textbooks between the two countries. The focus of our study is on the linear equation in one unknown in numbers and algebra. And carrying on cognitive research to the exercises in the three textbooks, mainly from the depth and breadth of cognitive complexity. Through the search of the CNKI, we find that there are few cognitive studies on exercises in Chinese and foreign textbooks. In addition, the linear equation in one unknown is a very important course in junior high school mathematics. It is the transformation of students from concrete image thinking to abstract logical thinking. It has great influence on the following equation and function learning and has strong comparability. Therefore, we choose to conduct research of cognitive on the exercise part of metafile degree equation in the middle school mathematics textbooks in China and the United States. We hope to discover the advantages and disadvantages of the three textbooks through cognitive research. This study investigated the breadth and depth of the cognitive complexity of the mathematical problems presented in mathematics textbooks of the three curricula. Cognitive complexity here refers to the kind and level of thinking required from students in order to successfully engage with and solve mathematical problems, what Stein and her colleagues (2000) called the "cognitive demands" of the mathematics problem (p.11). Drawing on Webb's (1999) depth of knowledge framework and Son and Senk's (2010) cognitive expectation, this study set out to address the similarities and differences of students' opportunities to learn the linear equation in one unknown among the three sets of learning materials. Cognitive expectation here means the mathematical knowledge and skills which students should acquire while doing mathematics (Son and Senk 2010). The research questions which guided this study were:

(1) What expectations are evident with respect to the breadth and depth of cognitive complexity in the lessons of the Chinese textbooks of people’s edition and Zhejiang edition in comparison to Algebra 1?

(2) What are the advantages and disadvantages of Zhejiang province uses Zhejiang education edition textbooks compared with most other regions which use people’s edition textbooks?

2 Theoretical framework

2.1 Related research on textbook analysis

Knowledge points and topics are the most core components of mathematics teaching materials. In this study, we focus on problem types which have two aspects, cognitive expectation and cognitive demand. A mathematical problem is a mathematical object that requires logical thinking to solve or answer. Mathematical problems presented in textbooks form the basis of students’ opportunities to learn mathematics. Thus, study and analyze the different characteristics of mathematics problems in Chinese and American textbooks in order to understand their impact on students' mathematics achievement which can help the compilation of mathematics teaching materials in China. Therefore, we will carry out analysis and research according to the following three aspects of teaching materials:

(1) Teaching emphasis (the key and difficult points of teaching content)

(2) Cognitive expectation (kinds of know ledge and skills required in solving problems)

(3) Cognitive demand (levels of cognition required in solving problems)

Among these factors, cognitive expectation and cognitive demand are more concerned by textbook researchers and they are more valuable for research. Thus, the two factors will be emphatically analyzed. And through the comparative analysis of these aspects, to see the similarities and differences between Chinese and American teaching materials, and analyze the reasons for their impact on students' learning effect that will clearly show the advantages and disadvantages of Chinese and American textbooks.

Webb developed the depth of knowledge framework which will be used for statistical analysis of cognitive demand, the depth of knowledge level. And the framework, with three cognitive complexity levels (low, moderate, and high) and four levels of knowledge depth (see Table 1) has been widely applied in math and science in the USA. Thus, we classified the exercises according to Webb classification standard. As for cognitive expectation, we use the classification of Son and Senk. In this category, problems had the potential of falling into one of five different areas, conceptual knowledge, procedural knowledge, mathematical reasoning, representation and problem solving. Divide the problem sets into five categories：

(1) Conceptual knowledge is the most basic level, requiring the student to do nothing more than answer a question about the concept itself.

(2) Procedural knowledge requires the student to perform an algorithm, but does not require any kind of reasoning or justification.

(3) Mathematical reasoning requires the student to do more than simply solve a problem: the student must reason through the solution process and justify the answer.

(4) Representation requires the student to represent the problem or answer using a model.

(5) Problem solving requires the student to come up with and apply a strategy to solve a word problem, typically in context.

These two models are important ways to study problem types, and we can reach scientific conclusions through model statistics.

This study mainly starts from the depth and breadth of mathematics cognition in Chinese and American textbooks, analyzes the proportion of different problem types, and obtains the relationship between problem distribution and learning effect. Through such

comparative research, promote the reform and development of junior middle school mathematics in China.

2.2 Research on linear equation in one variable

As students make the meaning of a work based on the problem’s background (Carpenter et al.1999;Reel and Smith)，the CGI (cognitively guided instruction) framework classifies

word problems of addition and subtraction lied on the semantic formation of the problems, was applied.

The Principles and Standards for School Mathematics (NCTM, 2000) declares that: Representation is important to mathematics learning. Students can open up and intensify their apprehension of mathematical notions and correlations when they make, match, and use multifarious representations. Representations contains of physical substances, pictures, diagram, graphs, and indications can assist students convey their idea. Although we have maintained the manipulative methods, study has presented the use of physical substances may become an obstruction to mathematical advancement under some circumstances. (Uttal, Scudder and DeLoache 1997) told “part of the difficulty that children encounter when using manipulatives stems from the need to interpret the manipulative as a representation of something else” (p. 38). Carpenter and Hiebert (1992) debated that the closer the match between prominent characteristics of the mediums and the mathematical correlations, the more contextual support students have for constructing the expected connections.[4]

Table 1 Webb’s (1999,2002) depth of knowledge levels

Complexity	Depth of knowledge levels

Low	Level 1: Recall/reproduce Focus on knowledge recall and reproduction, requiring students to be able to apply facts, definitions, expressions, or simple processes as well as simple steps and procedures. It usually only takes one mental step. For example: State the events, characters, setting and so on in the story Make a list of key words about a concept Find and identify information on maps, charts, graphics, catalogues Solving a one-step problem
Moderate	Level 2: Basic application of skill/concept Emphasizes skills and concepts that go beyond recall and reproduction of knowledge to think, observe, and derive inferences and explanations. For example: Use timelines, cartoons, summaries, or flowcharts to order a series of events with specific details Explain concepts in various forms such as words, objects and pictures Construct or deduce a model of how a mathematical problem works Develop appropriate learning strategies for conducting research
High	Level 3: Strategic thinking The use of strategic thinking and reasoning, including complex and abstract, even logical reasoning cognitive requirements, often requires a multistep thought process. For example: Use evidence to form opinions or ideas Explain complex concepts and relate them to the real world Use concepts to solve unconventional problems Use indirect translation between a problem situation and symbolic notation
	Level 4: Extended thinking Developing extended thinking requires higher order cognitive behavior and the use of higher order thinking patterns, such as analysis, synthesis and reflection; Involves complex concepts and cross disciplinary areas of thinking and practice. Design mathematical models to solve real or abstract problems Combine reality, solve the problem in a new way Use the best of many methods to solve the problem

3 method

3.1 Material analyzed

The data used for the study come from the lessons on linear equations of one variable in algebra in the China and US mathematics textbooks. We choose the people's education edition mathematics textbooks(C1) and Zhejiang education edition mathematics (C2), and Algebra1(A1). The central work on the linear equation of one variable occurs in the seven grades of 3th and 5th unit in the C1 and C2, also occurs on the 2 unit in the A1. Although there has some knowledge about linear equation in Algebra2, but we considering which does not take linear equation as the important content, so we don't take it into account.

According to Google search we obtain that the market occupancy of C1、C2 and A1 .For C1, it has about 65% market occupancy. Besides, C2 only used in Zhejiang province and A1 has a relatively large market occupancy. Because the study aims to research the advantages and disadvantages of Zhejiang education edition textbook compared with other editions, so we choose C2. Furthermore, C1 is the most widely used textbook in China and it has good learning system and knowledge structure compact, so we choose it to compare with C2. One of the reasons for choosing a dimensional equation is a quite important module in middle school mathematics. Besides, the problem design of one equation is representative, which can fully reflect the teaching purpose of the textbook editor. In conclusion, the study attaches importance to the learning task, or the exercise

Table 2 one dimensional equation in C1 and C2

C1	C2
Unit 3. Linear equation in one unknown	Unit 5. Linear equation in one unknown
3.1.1 Linear equation in one unknown	5.1 Linear equation in one unknown
3.1.2 Properties of equality	5.2 Basic properties of equations
3.2 Solve an equation of one dimension merge congeners and transference	5.3 Solution of one-dimensional equation
3.3 Solve an equation of one dimension remove the parentheses and the denominator	5.4 Application of equations of one-dimensional equation
3.4 Practical problems with one dimensional equation	

Table 3 One dimensional equation in Algebra

Algebra1	
Unit1 Foundations for Functions	Chapter2 Solving Linear Equation

21 Writing Equations
22 Solving Addition and Subtraction Equations
23Solving Equations by Using Multiplication and Division
24 Solving MultiStep Equations
25Solving Equations with the Variable on Each Side
26 Ratios and Proportions
27 Percent of Change
28 Solving for a Specific Variable
29 Weighted Averages
Extend29 Finding a Weighted Averages

3.2 Analytical coding scheme and analysis

The study analyzed the mathematical problems presented in the lessons. In this study, mathematical problems are mathematical tasks and exercises problems or mathematical activities where students are asked to apply the linear equation of one variable. Although different textbooks provide different content to meet the needs of every student, the nature of the offered is awfully different. Hence, only content designed was analyzed, specially focusing on the mathematical problems presented in the core of each lesson. Thua we can see what the essential difference that different textbooks have. Label 4 shows the analytical framework that used along with the research question that used.

In the research, problems are coded into two categories. Firstly, we take the cognitive expectation, which was come up with Son and Senk. In this category, problems are taken part into five parts. *Conceptual knowledge* is the basic level, only requiring students know the concept itself. Therefore, students just required to call upon the meaning of linear equation of one variable or any underlying concept. *Procedural knowledge* linear need students to perform an algorithm, but without any types of reasoning or justification. *Mathematical reasoning* requires student reason through the process and justify the answer. *Representation* needs students to represent the problem or answer using the model that they have learned. *Problem solving* acquire students to consider and apply a strategy to solve a word problem. Cognitive expectation requires students determine the breadth of cognitive complexity. These problems were coded relying on the depth of knowledge framework (see Table 1) as the second category for exploring the range of difficulty in problems in the first category. Table 5 and 6 show some typical example problems including the categories and their codes

Table 4 Framework used for analysis of mathematical problems in textbooks

Focus questions	Aspects investigated
1. What is expected in terms of the breadth of cognitive complexity?	Cognitive expectation Conceptual knowledge (C) Procedural knowledge (P) Mathematical reasoning (MR) Representation (R) Problem solving (PS)
2. What is expected in terms of the depth of cognitive complexity?	Depth of knowledge (DOK) Level 1: recall/reproduce Level 2: skill/concept Level 3: strategic thinking Level 4: extended thinking

Table 5 (A1) Sample textbook problem coding with respect to the DOK framework

Examples	Coding	
	Depth of knowledge	Cognitive expectation
1、 Five times the number a squared is three times the sum of b and c.	Level 1	C
2、 Translate each equation into a verbal sentence. 2 = x-9	Level 1	C
3、 Solve each equation. Check your solution. 3p = 18	Level 2	P
4、 Determine whether each pair of ratios forms a proportion. Write yes or no. $\frac{4}{11}, \frac{12}{33}$	Level 3	P/MR
5、 Lucy drove 248 miles in 4 hours. At that rate, how long will it take her to drive an additional 93 miles?	Level 2	PS
6、 Consider the proportion a:b:c = 3:1:5. What is the value of $\frac{2a+3b}{4b+3c}$? (Hint: Choose different values of a, b, and c for which the proportion is true and evaluate the expression.)	Level 4	P

In addition, we check the consistency of the code. Firstly, we randomly selected one third of the exercises in C2 as the text value. Then we asked a classmate outside the group to classify the problems after reading the classification criteria for the exercises. Finally, we conducted one-way anova and correlation analysis for the two groups of data

Table 6 (C1andC2) Sample textbook problem coding with respect to the DOK framework

Examples	Coding	
	Depth of knowledge	Cognitive expectation
1.Which of the following types is an equation? What are the equations of one dimension? (1) $5x=0$ (2) $1+3x$ (3) $y^2=4+y$ (4) $3m+2=1-m$	Level 1	C
2.Use the properties of the equation to solve the following equation and write out the test. (1) $x-5=6$ (2) $8-2x=10$	Level 2	P
3.Give an example of how to "transpose" when solving an equation. Do you know the basis for doing so?	Level 2	MR
4.Can we change from this equation $\frac{a}{3}-\frac{b}{4}=0$ to $4a=3b$? If yes, please state the deformation process and the basis for each step.	Level 3	R
5.Please make up a practical word problem that requires the equation to be $15x+45x=180$	Level 4	PS\R

4 Results

4.1 Consistency text

The results of one-way anova for the two groups of data are shown in Table7.

According to the Label7, the p-value equals 0.00, which less than 0.05, so there's no significant difference. Also, the correlation coefficient of the two sets of data is 0.679, it shows there is a greater correlation. Above all, the coding has a high accuracy.

Label7 One-way anova table

Source of Variation	SS	Df	MS	F	P-value
Between groups	43.35	3	14.45	25.22	0.00
Within groups	30.37	53	0.57		
Total	73.72	56			

4.2 Breadth of cognitive complexity

Figure 1 shows the distribution of the five domains of cognitive expectation in the types of mathematical problems presented in the three textbooks.

The proportion of mathematical reasoning and representation in the three textbooks is low, basically less than 15%. However, procedural knowledge and problem solving have the largest total proportion, accounting for more than half of the problem sets. This shows that the types of questions that require students to judge and explain reasons and that require the students to represent the problem or answer using a model are not the focus of exercises in Chinese and American textbooks. But procedural knowledge and problem solving as the main points of knowledge practice. In other words, both Chinese and American textbooks are more inclined to cultivate students' computing ability and the application of knowledge points, combining mathematics with practical problems and laying a good foundation for computing. Conceptual knowledge is at an intermediate position, and the setting of basic questions is more aimed at enabling students to understand knowledge. In order to master the knowledge, other types of questions are needed. Therefore, the number of such questions needs to be set at a medium position.

In addition to the comparison between the three textbooks, we can observe that there are great differences in the proportion of each field in the textbooks themselves. Mathematical reasoning and representation account for comparison as consistent in Chinese textbooks, while conceptual knowledge and problem solving differ greatly, accounting for 17.22% and 19.66% respectively. It is observed that C1 tends to be applied, while C2 tends to be conceptual, indicating that C1 emphasizes the application of students' knowledge and reality, while C2 pays more attention to the exercise of basic questions. When we look at A1, procedural knowledge and problems solving account for most of them, and we can clearly see what American textbooks training purpose. They have a great exercise for students' logic, starting from practical problems to apply mathematical knowledge to solve the problem.

In general, Chinese textbooks focus on the cultivation of students' way of thinking and logic, while American textbooks tend to make students to perform an algorithm.

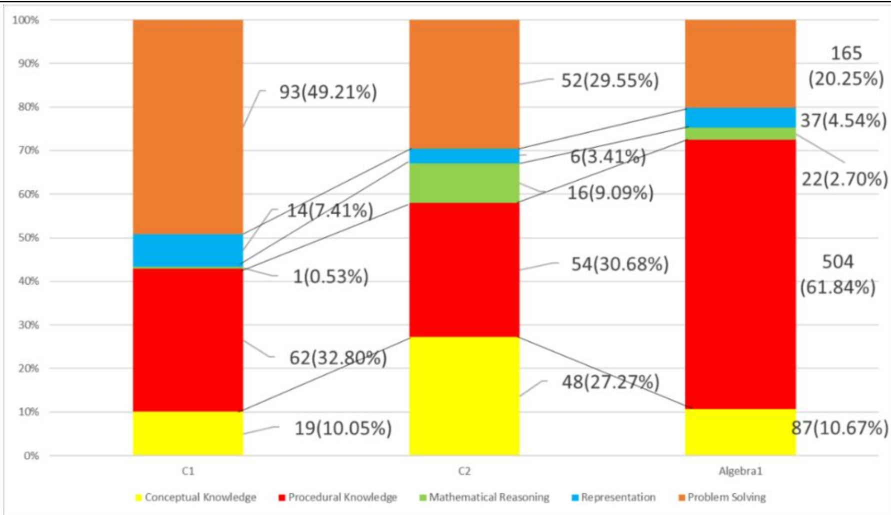

Fig. 1 One dimensional equation problems according to cognitive expectation

4.3 Depth of cognitive complexity

When classifying these problems, breadth is not enough, we need depth to indicate the level of problems. For example, among items categorized as word problems, some problems require only one step, others require multiple steps. Therefore, mathematical problems in the three sets of materials were further analyzed based on the depth of knowledge levels required. Figure 2 presents the results for the linear equation in one unknown.

Overall, all three sets of materials pay attention to Level 2 (basic application of concepts and skills). C2 pay more attention to Level 1 (recall/reproduce) than C1, but a slightly higher percentage of mathematical problems were categorized Level 2 in the C1. Moreover, for the mathematical problems at Level 3 and Level 4, the proportion of C1 is much higher than C2. Furthermore, the findings show that A1 provided a more balanced level of depth of knowledge than the C1. In A1, the percentage of Level 1 more than C1 and C2, but the percentage of Level 2 less than C1 and C2. The percentage of Level 3 and Level 4 are more than C2, but it is less than C1. In addition, in the mathematical problems at Level 3 and Level 4, the proportion of Level 3 in the A1 are the most of these materials, and Level 4 are the least. This distribution indicates that A1 not only does it focus on basic problems, but also provides more opportunities for students to improve their thinking level. All in all, the findings from the analysis of the depth of knowledge levels show that the A1 is more challenging than C2, but less challenging than C1.

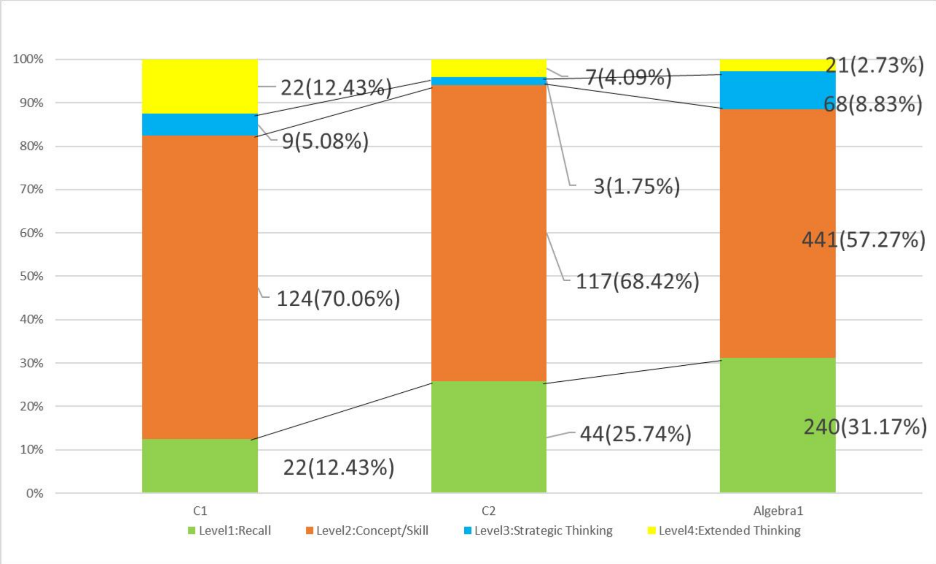

Fig.2 One dimensional equation problems per depth of knowledge level

Table 8 (A1) Sample textbook problem coding with respect to the DOK framework

Examples	Coding	
	Depth of knowledge	Cognitive expectation
7、 Five times the number a squared is three times the sum of b and c.	Level 1	C
8、 Translate each equation into a verbal sentence. 2 = x – 9	Level 1	C
9、 Solve each equation. Check your solution. 3p = 18	Level 2	P
10、Determine whether each pair of ratios forms a proportion. Write yes or no. $\frac{4}{11}, \frac{12}{33}$	Level 3	P/MR
11、Lanette drove 248 miles in 4 hours. At that rate, how long will it take her to drive an additional 93 miles?	Level 2	PS
12、Consider the proportion a:b:c = 3:1:5. What is the value of $\frac{2a+3b}{4b+3c}$? (Hint: Choose different values of a, b, and c for which the proportion is true and evaluate the expression.)	Level 4	P

Table 9 (C1andC2) Sample textbook problem coding with respect to the DOK framework

Examples	Coding	
	Depth of knowledge	Cognitive expectation
1.Which of the following types is an equation? What are the equations of one dimension? (1) $5x=0$ (2) $1+3x$ (3) $y^2=4+y$ (4) $3m+2=1-m$	Level 1	C
2.Use the properties of the equation to solve the following equation and write out the test. (1) $x-5=6$ (2) $8-2x=10$	Level 2	P
3.Give an example of how to "transpose" when solving an equation. Do you know the basis for doing so?	Level 2	MR
4.Can we change from this equation $\frac{a}{3}-\frac{b}{4}=0$ to $4a=3b$? If yes, please state the deformation process and the basis for each step.	Level 3	R
5.Please make up a practical word problem that requires the equation to be $15x+45x=180$	Level 4	PS\R

Label 10 Number of problems and lessons about linear equation with one unknown

	No. of problems	No. of lessons	Ave. no. of problems/lessons
C1	171	19	9
C2	177	10	18
A1	1547	30	52

5 Summary and discussion

5.1 Summary

Our research compared and analyzed the cognitive complexity of mathematical problems' depth and breadth which presented in three different versions of mathematical textbooks: educational edition of seventh grade mathematical textbook(C1), Zhejiang edition of seventh grade mathematical textbook(C2), and America's mathematical textbook: Algebra 1(A1).

Initially, the discovers of the research suggest that the these three textbooks are all pay high attention on procedural knowledge and problems solving according to their proportions are more than half of the problem sets in textbook C1,C2 as well as in A1.However, these three textbooks all put insufficient emphasis on the problems about mathematical reasoning and representation, which requires students to explore or use a model to represent and settle the problem more than merely solve a simple question. Such similar problem sets choreography indicates that both China and America mathematical education are believed that it is monumentally essential to cultivate student's ability to solve linear equations in one variable and solve problems by linear equations in one unknown.

Although there are plenty of similarities in these three textbooks' sets on linear equation in one variable, it still has disparities. In terms of knowledge breadth, in textbooks C1 and C2, we found that the problems' proportions on conceptual knowledge and problem solving differ tremendously whereas proportions on mathematical reasoning and representation are comparison consistent, which directly reveals that C1 trends to application while C2 lays more emphasis on theoretical concept. Moreover, Algebra 1's procedural knowledge and problem solving occupy a huge part comparing to C1 and C2, which manifests A1 trends to provide better opportunities for students to perform an algorithm and Chinese textbooks better focus on the cultivation of students' manner to thinking and logic. Another important finding of this research is that A1 is more challenging than C2 whereas easier than C1 in terms of comparison on knowledge depth of three sets of materials.

5.2 Discussion

With respect to the depth of cognitive complexity of textbooks, this study finds that all three sets of materials give very large portion on Level 1 and Level 2, more than 80% of problems are still at Level 1 and Level 2, which is similar to results reported by JiWon Son (2012) in her research about the comparison of reform curricula in Korea and US[10].

On top of that, the syllabus referring to "gradually from a sense of innovation", the innovative consciousness to be cultivated in junior school mathematics mainly refers to: the strange heart to nature and social phenomenon, the constant pursuit of new knowledge, independent thinking, will discover and put forward problems from the perspective of mathematics, and use mathematical methods to explore, research and solve. (Nine-year compulsory education full-time junior middle school mathematics teaching syllabus, 2000.) It's worth noting that C2 reach up to nearly 95%, which indicates that Zhejiang edition of seventh grade mathematical textbook emphasis tremendous strength on students' cognition and application ability of basic knowledge about linear equation in one variable. It may help most students grasp the basic application of skills and concept better, but the lack of strategic and extending thinking cultivation is not conductive to the improvement of student's mathematical thinking ability, which is not conform to the requirements of the syllabus.

Moreover, only about 18,6,12% of questions are at Levels 3 and 4 respectively in the C1，C2,A1.The aim should be more than twice. According to Kadijevic’s(2002,p.98) statement in the TIMSS video study， 8th grade’ s cognitive areas of the chosen objective percentage: comprehending facts and process (15%), using conceptions (20%), settling regular question (40%) and reasoning (25%)—are quite advantageous and balanceable.

This research has some implication for textbook and material developers, teachers, researchers and professional developers. First of all, developers of C1 and C2 should attach importance to problems’ logical thinking. Moreover, C2 may should increase the number of creative questions. (These enlightenments are also same with A1 developers.)

As is well known, textbooks are tremendously essential for teacher’s teaching and students’ learning. As a matter of fact, textbooks serve as the intermediary nor only for teacher's teaching but also teachers’ changes. (Ball&Cohen,1996; Cai & Howson, 2013). In other words, teachers may can make up part of the defects in teaching material arrangement. In this way, teacher trainer and professional developers ought to offer extra instructions for teachers to raise teachers’ awareness of the cognitive need of activities, exercises and problems presented in curriculum materials, which can assist them to inspect and reshape textbook advice to meet classroom requirements. And teachers can consciously ask student to do some training with strategic and extended thinking class [11].

Last but not least, as our research just computatively analyzed one fraction linear equation in one variable, researchers might compare other fractions in different version mathematical textbooks with analytical framework and DOK framework. Such an analysis is supposed to conduct by comparative analysis to possibly contemporaneously offer alternate educational designs or syllabus advancement for particular mathematical conception and skills (Silver 2009). More study on textbook analysis can help teachers and students learning mathematics in a more efficient way.

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